Capacity Scaling of Wireless Networks

Alla Merzakreeva

LTHI, I&C, EPFL

***Abstract*—We have n randomy located nodes forming a wireless network. Nodes want to communicate with each other by transmitting signals which are subject to a power loss with a distance. We analyze the problem of capacity scaling of such networks. We overview two papers establishing upper bounds for network throughput under different assumptions, and consider their close-to-optimal schemes. Using this background information we propose our future research directions which focus on constant throughput scaling and on capacity scaling under various regimes.**

We also consider a problem of information security in which data is to be reliably transmitted over a discrete memoryless channel that is subjected to a wire-tap at the receiver. We overview the paper which establishes the upper bound for the transmission rate in perfect secrecy. We indicate how this problem can be extended and how it is related to our previous work.

***Index Terms*—wireless networks, capacity scaling, transport capacity, throughput, multiple-input multiple-output (MIMO), discrete memoryless channel, wire-tapper, perfect secrecy.**

I. Introduction

WIRELESS networks consist of a number of nodes which communicate with each other over a wireless channel. Since wireless networks play an important role in future communication networks, it is important to understand what such networks are capable of doing, and how to operate them to maximize their capabilities. In our work we will analyze the problem of capacity scaling of wireless networks, i. e. scaling of the aggregate throughput when the number of users gets large.

Further we will focus on a network model called *dense* networks, where the total area of a network is fixed and the density of nodes increases. The other model is *extended* networks, which scale to cover an increasing area with the density of nodes fixed. This model is beyond the scope of the current work but will play an important role in the future research of capacity scaling of wireless networks.

For the dense case, one way to approach the problem is due to Gupta and Kumar [1], who proposed to study the simpler case in which all the nodes in the network are required to transmit at the same bit-rate, and to look at the scaling limit of the achievable rate, as the number of nodes in the network grows. The model of Gupta and Kumar considers n nodes randomly located in the unit disk, where each node wants to communicate to a random destination node at a rate $R(n)$ bits per second. Signals transmitted from one node to another at a distance r apart are subject to a power loss of $r^{-\alpha}$. In their paper it was shown that multihop architectures cannot achieve a scaling better than $O(\sqrt{n})$, and that a scheme that uses only nearest neighbor communication can achieve a throughput that scales as $\Theta(\sqrt{n/\log n})$. The scheme will be described in section II.

However, the model of Gupta and Kumar makes certain assumptions on the physical-layer communication technology: it assumes that the signals received from nodes other than one particular transmitter are interference to be regarded as noise degrading the communication link. The arising question is whether the Gupta-Kumar scaling law is a consequence of the physical-layer technology or one can do better using more sophisticated physical-layer processing.

The paper of Ozgur, Leveque and Tse [2] shows that indeed, for dense networks one can do better using more sophisticated physical-layer processing. The main result is that for any $\alpha \geq 2$ one can achieve arbitrary close to *linear* scaling: for any $\epsilon > 0$ there exists a scheme that achieves a rate of $\Theta(n^{1-\epsilon})$. As one cannot get a better capacity scaling than $O(n \log n)$, their scheme is close to optimal. To achieve linear scaling, one must be able to perform many simultaneous long-range communications using multiple transmit and receive antennas (MIMO). We will describe the scheme in section III.

The hierarchical scheme proposed by Ozgur, Leveque and Tse [2], used both in dense networks and

Proposal submitted to committee: September 3rd, 2010; Candidacy exam date: September 10th, 2010; Candidacy exam committee: Suhas Diggavi, Emre Telatar, Olivier Leveque, Bixio Rimoldi.

This research plan has been approved:

Date: September 10th, 2010

Doctoral candidate: _____
(A. Merzakreeva) (signature)

Thesis director: _____
(E. Telatar) (signature)

Thesis co-director: _____
(O. Leveque) (signature)

Doct. prog. director: _____
(R. Urbanke) (signature)

in extended networks, gives an inspiration for future research directions. These research directions will be described in section V. However, the multihop scheme proposed by Gupta and Kumar [1] should not be underestimated. In fact, in the case of extended networks when path loss exponent $\alpha \geq 3$, multihop scheme performs better than hierarchical scheme, i. e. nothing more than $\Theta(\sqrt{n})$ can be done.

Wyner's paper "The Wire-Tap Channel" [6] deals with information security. It considers the situation when data is reliably transmitted over a discrete memoryless channel which is subjected to a wire-tap at the receiver. The purpose is to build the encoder-decoder such that the transmission rate R and the equivocation d of the data seen by wire-tapper are maximized. The paper establishes the trade-off curve between R and d. In case when perfect secrecy is desired, the maximum transmission rate that can be achieved is

$$C_s = \max_{p_X}[I(X;Y) - I(X;Z)].$$

Wyner's problem can be extended to the case of more than one wire-tapper, such that it is desired to build an encoder-decoder in a way as to maximize the transmission rate R while keeping all wire-tappers as ignorant of the private message as possible. We worked on this extended problem during the last half-year, and some results were obtained in this direction. However, our future research directions will concentrate on wireless networks and their capacity scaling.

II. Gupta and Kumar Model

The model supposes that n nodes are located in a region of a unit area. Each node can transmit at W bits per second over a common wireless channel. Packets are sent from node to node in a multihop fashion until they reach their final destination. Due to spatial separation, several nodes can make wireless transmission simultaneously, provided there is no destructive interference of a transmission by others.

Gupta and Kumar consider two types of networks, *Arbitrary Networks*, where the node locations, destinations of sources, and traffic demands, are all arbitrary, and *Random Networks*, where the nodes and their destinations are randomly chosen. There are also two possible models for successful reception of a transmission over one hop, called the *Protocol Model* and the *Physical Model*.

Let us denote X_i the location of a node, and in some cases X_i can also be used to refer to the node itself.

The Protocol Model. Suppose node X_i transmits over the mth subchannel to a node X_j. Then this transmission is successfully received by node X_j if

$$|X_k - X_j| \geq (1+\Delta)|X_i - X_j| \tag{1}$$

for every other node X_k simultaneously transmitting over the same subchannel.

The Physical Model. Let $\{X_k\ ;\ k \in \mathcal{T}\}$ be the subset of nodes simultaneously transmitting at some time instant over a certain subchannel. Let P_k be the power level chosen by node X_k for $k \in \mathcal{T}$. Then the transmission from a node X_i, $i \in \mathcal{T}$ is successfully received by a node X_j if

$$\frac{\frac{P_i}{|X_i - X_j|^\alpha}}{N + \sum\limits_{\substack{k\in\mathcal{T}\\ k\neq i}} \frac{P_k}{|X_k - X_j|^\alpha}} \geq \beta \tag{2}$$

This models a situation where a minimum signal-to-interference ratio (SIR) of β is necessary for successful receptions, the ambient noise power level is N, and signal power decays with distance r as $\frac{1}{r^\alpha}$.

A. Arbitrary Networks

In the arbitrary settings n nodes are arbitrarily located in a disk of unit area in the plane, each node has an arbitrarily chosen destination to which it wishes to send traffic at an arbitrary rate. Each node can chose an arbitrary power level for each transmission.

It is assumed that the network transports λnT bits over T seconds and the average distance between the source and destination of a bit is $\bar{L}$.

The Transport Capacity is the sum of products of bits and the distances over which they are carried. The above assumption implies that a transport capacity of $\lambda n\bar{L}$ bit-meters per second is achieved.

The following theorem shows the upper bound for the transport capacity of Arbitrary Networks.

Theorem 2.1: 1. In the Protocol Model, the transport capacity $\lambda n\bar{L}$ is bounded as follows:

$$\lambda n\bar{L} \leq \frac{\sqrt{8}}{\pi}\frac{1}{\Delta}W\sqrt{n} \text{ bit-meters per second.} \tag{3}$$

2. In the Physical Model

$$\lambda n\bar{L} \leq \left(\frac{2\beta+2}{\beta}\right)^{1/\alpha}\frac{1}{\sqrt{\pi}}Wn^{(\alpha-1)/\alpha} \text{ bit-meters per second.} \tag{4}$$

3. If the ratio $\frac{P_{\max}}{P_{\min}}$ between the maximum and minimum powers that transmitters can employ is strictly bounded above by β, then

$$\lambda n\bar{L} \leq \sqrt{\frac{8}{\pi}}\frac{1}{\left(\frac{\beta P_{\min}}{P_{\max}}\right)^{1/\alpha} - 1}W\sqrt{n} \text{ bit-meters per second.} \tag{5}$$

4. When the domain is of A square meters rather than 1 m^2, then all the upper bounds above are scaled by $\sqrt{A}$.

In fact, the order of the upper bound is sharp for Protocol Model, which shows the following theorem.

Theorem 2.2: There is a placement of nodes and an assignment of traffic patterns such that the network can achieve

$$\frac{W}{1+2\Delta}\frac{n}{\sqrt{n}+\sqrt{8\pi}} \tag{6}$$

bit-meters per second under the Protocol Model, and

$$\frac{1}{\left(16\beta\left(2^{\frac{\alpha}{2}} + \frac{6^{\alpha-2}}{\alpha-2}\right)\right)^{\frac{1}{\alpha}}}\frac{Wn}{\sqrt{n}+\sqrt{8\pi}} \tag{7}$$

bit-meters per second under the Physical Model, both whenever n is a multiple of 4.

B. Random Networks

In random settings n nodes are randomly located either on a surface of a three-dimentional sphere of unit area or in a disk of unit area in the plane. Each node has an arbitrary chosen destination to which it wishes to send $\lambda(n)$ bits per second. $\lambda(n)$ is called throughput capacity and is defined as the time average of the number of bits per second that can be transmitted by every node to its destination. The destination for each node is independently chosen as the node nearest to a randomly located point. All transmissions employ the same nominal range of power.

Theorem 2.3: 1. For Random Networks in the Protocol Model, there is a deterministic constant $c > 0$ not depending on n, Δ, or W, such that

$$\lambda(n) = \frac{cW}{(1+\Delta)^2\sqrt{n \log n}} \tag{8}$$

bits per second is feasible with high probability.
2. For Random Networks in the Physical Model, there are deterministic constants c' and c'' not depending on n, N, α, β, or W, such that

$$\lambda(n) = \frac{c'}{(2(c''\beta(3+\frac{1}{\alpha-1}+\frac{2}{\alpha-2}))^{\frac{1}{\alpha}}-1)^2}\frac{W}{\sqrt{n \log n}} \tag{9}$$

bits per second is feasible with high probability.

In their paper Gupta and Kumar constructed a routing scheme which achieves the above-mentioned throughput.

C. Routing Scheme for Random Networks

1. The routing scheme utilizes a Voronoi tesselation of a sphere with n nodes such that every Voronoi cell contains a disk of area $100 \log n/n$ - here let us denote the radius of a disk of area $\frac{100 \log n}{n}$ on the sphere as $\rho(n)$ - and is contained in a disk of radius $2\rho(n)$. It is shown in the paper that each cell contains at least one node.

The transmission range $r(n)$ is chosen so that

$$r(n) = 8\rho(n). \tag{10}$$

This range allows direct communication within a cell and between adjacent cells.

2. Let us say that two cells are *interfering neighbors* if there is a point in one cell which is within a distance $(2+\Delta)r(n)$ of some point in the other cell. This implies that if two cells are not interfering neighbors, then in the Protocol Model a transmission from one cell cannot collide with a transmission from the other cell.

In the constructed Voronoi tesselation the number of interfering neighbors of a cell is uniformly bounded, which allows the construction of a spatial transmission schedule of bounded length with one opportunity for each cell in the tesselation to transmit.

3. Each node X_i wishes to communicate with the node nearest to a randomly chosen location Y_i. Let us denote by L_i the straight-line segment connecting X_i and Y_i. The routes of packets are chosen to approximate these straight-line segments. Every straight-line segment intersects many cells in the tesselation. Packets originating at X_i will be relayed from the cell where X_i lies to the destination cell in a sequence of hops. In the each hop, the packet is transferred from one cell to another in the order in which they intersect the line. Here let us note that the number of routes passing through each cell is bounded. Finally, after reaching the cell containing Y_i, the packets are sent on to their final destination, which is no more than one hop away with hight probability.

The traffic is routed efficiently over nearly straight-line path, and no node is overloaded. This routhing scheme is the constructed proof of the lower bound on the throughput capacity for Random Networks.

D. An Upper Bound on Throughput Capacity for Random Networks

The capacity for Random Networks is upper-bounded. The key to the upper bound, as in the case of Arbitrary Networks, is the fact that each transmission consumes valuable area.

Theorem 2.4: 1. For Random Networks in the Protocol Model, there is a deterministic constant $c' < +\infty$ not depending on n, Δ, or W, such that

$$\lim_{n\to\infty} \text{Prob}\left(\lambda(n) = \frac{c'W}{\Delta^2\sqrt{n \log n}} \text{ is feasible}\right) = 0. \tag{11}$$

2. For Random Networks under the Physical Model, there is a deterministic sequence $\epsilon(n) \to 0$, not depending on N, α, β, or W, such that

$$\lim_{n\to\infty} \text{Prob}\left(\lambda(n) = \sqrt{\frac{8}{\pi}}\frac{W}{\bar{L}(\beta^{1/\alpha}-1)}\frac{1+\epsilon(n)}{\sqrt{n}} \text{ is feasible}\right) = 0. \tag{12}$$

E. Main Results

Under the Protocol Model the transport capacity is $\Theta(W\sqrt{n})$ bit-meters per second if the nodes are optimally placed, the traffic pattern is optimally chosen, and if the range of each transmission is chosen optimally. For the Physical Model and Arbitrary Networks, $cW\sqrt{n}$ bit-meters per second is feasible, while $c'Wn^{(\alpha-1)/\alpha}$ is not, for appropriate c, c'.

For Random Networks under the Protocol Model the order of the throughput capacity is $\lambda(n) = \Theta\left(\frac{W}{\sqrt{n \log n}}\right)$ bits per second.

The main result in the case of Random Networks allows for a perfect scheduling algorithm which knows the locations of all nodes and all traffic demands, and which coordinates wireless transmissions temporally and spatially to avoid collisions which would otherwise result in loss packets.

For Arbitrary Networks, if each source has its destination about the same distance of 1 m away, then throughput per node is $\Theta(\frac{W}{\sqrt{n}})$ and decreases with n. This may be regarded as unacceptable. However, when nodes need to communicate only with nearby nodes, the scaled distance between sources and destinations is only $O(\frac{1}{\sqrt{n}})$ meters. In this case all nodes can transmit data to nearby neighbors at a bit rate that does not decrease with n.

III. Hierarchical Cooperation Model

The dense scaling model in the paper of Ozgur, Leveque and Tse [2] assumes that there are n nodes uniformly and independently distributed in a square area. Every node is both a source and a destination, and each source has the same traffic rate $R(n)$ to send to its destination node with a common average transmit power budget of P Joules per symbol. In this case $T(n) = nR(n)$ is the total throughput of the system.

Communication takes place over a flat channel of bandwith W hertz and the complex baseband-equivalent channel gain between node i and k at time m is given by

$$H_{ik}[m] = \sqrt{G} r_{ik}^{-\alpha/2} \exp(j\theta_{ik}[m]). \tag{13}$$

The signal received by node i at time m is given by

$$Y_i[m] = \sum_{k=1}^{n} H_{ik} X_k[m] + Z_i[m] \tag{14}$$

where $X_k[m]$ is the signal sent by node k at time m and $Z_i[m]$ is white circularly symmetric Gaussian noise of variance N_0 per symbol.

There is an information-theoretic upper bound on the achievable scaling law for the aggregate throughput in the network, which is shown in the following theorem.

Theorem 3.1: The aggregate throughput in the network with n nodes is bounded above by

$$T(n) \leq Kn \log n \tag{15}$$

with high probability.

The paper [2] states that in fact, arbitrary close to linear scaling can be achieved, which is shown in the following theorem.

Theorem 3.2: Let $\alpha \geq 2$. For any $\epsilon > 0$, there exists a constant $K_\epsilon > 0$ independent of n such that with high probability, an aggregate throughput

$$T(n) \geq K_\epsilon n^{1-\epsilon} \tag{16}$$

is achievable in the network for all possible pairings between sources and destinations.

The proof of this theorem relies on the construction of an explicit hierarchical scheme that realizes the promised scaling law. The idea is to use MIMO concept in such a way that both source nodes and destination nodes cooperate in clusters and form distributed transmit and receive antenna arrays, respectively. In this way, mutually interfering signals can be turned into useful ones that can be jointly decoded at the receive cluster, and spatial multiplexing gain can be realized. Also, the construction is based on recursively using the following lemma.

Lemma 3.3: Consider $\alpha > 2$ and a network with n nodes subject to interference from external sources. The signal received by node i is given by

$$Y_i = \sum_{k=1}^{n} H_{ik} X_k + Z_i + I_i \tag{17}$$

where I_i is the external interference signal received by node i. Assume that $\{I_i,\ 1 \leq i \leq n\}$ is a collection of uncorrelated zero-mean stationery and ergodic random processes with power P_{I_i} upper-bounded by

$$P_{I_i} \leq K_I, \qquad 1 \leq i \leq n \tag{18}$$

for a constant $K_I > 0$ independed of n. Let us assume that there exists a scheme such that for each n, with probability at least $1 - e^{-n^{c_1}}$ achieves an aggregate throughput

$$T(n) \geq K_1 n^b \tag{19}$$

for every possible source-destination pairing in this network of n nodes. K_1 and c_1 are positive constants independent of n and the source-destination pairing, and $0 \leq b < 1$. Let us also assume that the per-node average power budget required to realize this scheme is upper-bounded by P/n as opposed to P.

Then one can construct another scheme for this network that achieves a *higher* aggregate throughput

$$T(n) \geq K_2 n^{\frac{1}{2-b}} \tag{20}$$

for every source-destination pairing in the network, where $K_2 > 0$ is another constant independent of n and the pairing. Moreover, the failure rate for the new scheme is upper-bounded by $e^{-n^{c_2}}$ for another positive constant c_2 while the per-node average power needed to realize the scheme is also upper-bounded by P/n.

For the case when $\alpha = 2$, the aggregate interference scales as $\log n$, and there is an equivalent lemma given in [2].

A. Hierarchical Scheme

The network is divided into clusters of M nodes. Consider a particular source node s and its destination node d. s sends LM bits to d in three steps.

1. Node s distributes its LM bits among the M nodes in its cluster, L bits for each node.

2. These nodes together form a distributed transmit antenna array, sending the LM bits simultaneously to the destination cluster where d lies.

3. Each node in the destination cluster obtained one observation from the MIMO transmission, and it quantizes and ships the observation back to d, which can then do joint MIMO processing of all the observations and decode the LM transmitted bits.

The first and the last steps involve local communication and can be parallelized across source-destination pairs. The middle step is long-range communication and only one source-destination pair can operate at a time.

As all source-destination pairs have to accomplish these three steps, this leads to three phases in the operation of the network.

Phase 1. Setting Up Transmit Cooperation. In this phase clusters work in parallel, and within a cluster each source node distributes LM bits to the other nodes, L bits for each node. At the end of the phase each node has L bits from each of the source nodes in the same cluster. It gives a traffic demand of exchanging LM^2 bits. This problem of exchanging LM^2 bits

is similar to the original problem of communicating between n source and destination pairs, but on a smaller network of size M. Suppose that the aggregate rate of exchanging these bits is M^b. It is shown in the paper that this phase can be completed in less than $(18L/K_1)M^{2-b}$ channel uses all over the network with high probability.

Note that during this phase, the interference power received by a node from the simultaneously operating clusters is upper-bounded by a constant K_{I_1} for $\alpha > 2$ and by $K_{I_2} \log n$ for $\alpha = 2$.

Phase 2. MIMO Transmissions. There are long-distance MIMO transmissions between source-destination pairs, one at a time. In each of the MIMO transmissions, for example, between s and d, the M nodes in the cluster of s independently encode L bits-long subblocks into C symbols by using a randomly generated Gaussian code that respects an average transmit power constant $\frac{P(r_{\mathrm{SD}})^\alpha}{M}$, and simultaneously transmit them to the M nodes in the cluster of d. This phase is completed in n time slots. It is important to note that the power received by each node in the destination cluster is bounded below and above by constants that are independent of M. In the next phase this will allow to encode the observations at a fixed rate Q bits per observation.

Phase 3. Cooperate To Decode. In this phase again clusters work in parallel. Each cluster has CM received observations from MIMO transmissions in Phase 2. Nodes quantize each observation into fixed CQ bits, so there are now a total of at most CQM^2 bits to exchange inside each cluster. Similar to Phase 1, Phase 3 can be concluded in $(18CQ/K_1)M^{2-b}$ time slots.

The aggregate throughput of the scheme is:

$$\begin{aligned} T(n) &= \frac{nML}{(18L/K_1)M^{2-b} + 2Cn + (18CQ/K_1)M^{2-b}} \\ &\geq K_2 n^{\frac{1}{2-b}}. \end{aligned} \tag{21}$$

bits per time slot. Maximizing this throughput by choosing $M = n^{\frac{1}{2-b}}$ yields $T(n) = K_2 n^{\frac{1}{2-b}}$ which is the result in the *Lemma 3.3*.

It is shown in the paper [2] that if per-node average power used by old scheme was bounded above by P/n, then per-node average power used by new scheme is also bounded above by P/n. This concludes the proof of *Lemma 3.3*.

B. Conclusion

The paper shows that by achieving near global MIMO cooperation between nodes without introducing significant co-operation overhead, interference can be successfully removed as a limitation, when scaling laws are concerned. This result highlights the fact that interference limitation in the Gupta and Kumar [1] scaling is not a fundamental one and can be alleviated by more complicated physical-layer processing.

IV. The Wire-Tap Channel

In Wyner's [6] classical setup of communication with secrecy constraints we are given two discrete memoryless channels. It is desired to transmit a private message to receiver 1 at rate R and probably a common message to both receivers while keeping receiver 2 (wire-tapper) as ignorant of the private message as possible. The attempt is to encode the data in such a way that the wire-tapper's level of confusion will be as high as possible.

A. Wyner Scheme

Before turning to the main result, let us outline the scheme.

The original source is discrete and memoryless. Let us denote $W = S^K$ as a message to be encoded, and H_W is the entropy of the message. The main channel and the wire-tap channel are discrete memoryless channels with transition probabilities $p_M(\cdot|\cdot)$ and $p_W(\cdot|\cdot)$ respectively. The encoder is a channel with input W and output X^N, X^N is in turn the input to the main channel. The main channel output and the wire-tap channel input is Y^N , the wire-tap channel output is Z^N. The decoder input is Y^N and output is $\hat{W}$.

The error probability P_e is defined as

$$P_e = \Pr\{W \neq \hat{W}\}, \tag{22}$$

the equivocation is defined as

$$\Delta = \frac{1}{K} H(S^K|Z^N), \tag{23}$$

and the transmission rate is KH_S/N source bits per channel input symbol.

A pair (R, d) is called to be achievable if it is possible to build an encoder-decoder with an arbitrarily small error probability P_e such that it is possible to reliably transmit information at a rate R with equivocation d. The problem is to characterize the set $\mathscr{R}$ of achievable (R, d) pairs.

Let us denote the capacity of the main channel as C_M, the capacity of the cascade of the main channel and the wire-tap channel as C_{MW}, and p_X is a probability mass function, where $\mathscr{X}$ is a finite input alphabet. Let $\mathscr{P}(R)$ be the set of p_X such that $I(X;Y) \geq R$.

B. Main Result

The main Wyner's result is the following theorem.

Theorem 4.1: The set $\mathscr{R}$ is equal to $\bar{\mathscr{R}}$, where

$$\bar{\mathscr{R}} = \{(R, d) :\ 0 \leq R \leq C_M,\ 0 \leq d \leq H_S,\ Rd \leq H_S \Gamma(R)\}. \tag{24}$$

and $\Gamma(R)$ is defined as

$$\Gamma(R) = \sup_{p_X \in \mathscr{P}(R)} I(X;Y|Z). \tag{25}$$

Let us remark that, for any distribution p_X, the corresponding X, Y, Z forms a Markov chain, so that

$$I(X;Y|Z) = I(X;Y) - I(X;Z) \tag{26}$$

and $\Gamma(R)$ can be rewritten as

$$\Gamma(R) = \sup_{p_X \in \mathscr{P}(R)} [I(X;Y) - I(X;Z)]. \tag{27}$$

An important question is whether or not it is possible to transmit at a rate bounded away from zero, and yet achieve

approximately perfect secrecy. By perfect secrecy we mean that knowledge of Z^N does not add any additional information to the wire-tapper about the source, i. e. $H(W|Z^N) = H(W) = H_W$, or $I(W;Z^N) = 0$. In case of perfect secrecy a transmission rate

$$C_s = \max_{(R,H_S)\in\bar{\mathscr{R}}} R \tag{28}$$

is therefore achievable. C_s is called the "secrecy capacity".

The following theorem answers the question.

Theorem 4.2: If $C_M > C_{MW}$, there exists a unique solution C_s of

$$C_s = \Gamma(C_s). \tag{29}$$

Further, C_s satisfies

$$0 < C_M - C_{MW} \le \Gamma(C_M) \le C_s \le C_M, \tag{30}$$

and C_s is the maximum R such that $(R, H_S) \in \mathscr{R}$.

In other words, when perfect secrecy is desired, the maximum transmission rate that can be achieved is

$$C_s = \max_{p_X}[I(X;Y) - I(X;Z)]. \tag{31}$$

C. Conclusion

The original problem of information transmission solved by Wyner is the special case when the second channel is a degraded version of the main channel. The problem was extended by Csiszar and Korner [7], where both two discrete memoryless channels have a common input but they are not degraded versions of each other.

V. Research Proposal

The papers [1] and [2] discussed above form the basis of our future research. Further we will work with both dense and extended networks.

A. Constant Throughput Scaling for Dense Networks

In the case of dense networks we plan to investigate how close to linear throughput one can achieve. By other words, if $T(n) = K_\epsilon n^{1-\epsilon}$, how small $\epsilon > 0$ can actually be.

Recall that if one starts with $T(n) = C_0 n^0$ and applies *Lemma 3.3* recursively h times, one gets a scheme achieving

$$T(n) = C_h n^{\frac{h}{h+1}} = C_h n^{1-\frac{1}{h+1}}$$

aggregate throughput. Note that $h \ge \sqrt{\log n}$ is in fact useless and therefore

$$T(n) \le n^{1-\frac{1}{\sqrt{\log n}}}.$$

In this case the throughput per node is

$$\frac{T(n)}{n} = e^{-\sqrt{\log n}} \to 0 \ (n \to \infty) \ ,$$

i. e. decreases when the number of nodes gets large.

The question arising is whether it is possible to achieve the aggregate throughput

$$T(n) = n^{1-\frac{1}{\log n}} \ ?$$

In this case the throughput per node is

$$\frac{T(n)}{n} = e^{-\frac{1}{\log n}\cdot \log n} = \text{const},$$

which might be desirable.

B. Capacity Scaling under Various Operating Regimes

The other potential research direction is based on modification of the original problem discussed in [2].

Let us remind that MIMO transmissions in the hierarchical scheme in [2] were based on the assumption that the phase shifts θ_{ik} in (13) are independent and uniformly distributed on $[0, 2\pi]$. However, in the reality the phase shifts are derived from Maxwell's equations and actually given by

$$\theta_{ik} = 2\pi r_{ik}/\lambda \ ,$$

where λ is the carrier wavelength. In this new model the phase shifts are not independent and not uniformly distributed on $[0, 2\pi]$. Therefore, it is not clear that MIMO transmissions can achieve the full spatial degrees of freedom, and as consequence, the aggregate throughput scaling of wireless networks might be different from that in [2].

It appears that in some operating regimes one can still achieve the same performance as under previous model. By operating regimes we mean power budget and network area. Indeed, when network area and power budget are large enough, one can still optimally use hiererchical cooperation scheme described above.

In case of small power budget the question of capasity scaling is still open. Although there are some schemes to operate in this regime, it is not known whether they are optimal or one can do better.

In the regime of small power budget we plan to establish an upper bound for network throughput by considering a cut-set bound on how much power can flow across a network. The idea is to divide a network area into two equal halves and to look at the number of communication requests with sources on the left half network and destinations on the right half network. One can bound above the sum of the rates of communications passing through the cut from left to right, and therefore establish the upper bound for the aggregate throughput.

Although in our research we plan to investigate the problem of capacity scaling of wireless networks when the number of users gets large, one might also want to consider the case of a real network with a given number of users. Performance analysis for such network would involve investigation many additional parameters and may require modification of the scheme to optimize the pre-constant in the aggregate throughput.

References

[1] P. Gupta, P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Trans. Inf. Theory*, vol. 42, no. 2, pp. 2313-2328, June 2006.

[2] A. Ozgur, O. Leveque, D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in Ad-Hoc Networks", *IEEE Trans. Inf. Theory*, vol 53, no. 10, pp. 3549-3572, Oct 2007.

[3] E. Telatar, Capacity of multi-antenna Gaussian channels, *Europ.Trans. Telecommun.*, vol. 10, no. 6, pp. 585596, Nov. 1999.

[4] L. L. Xie, P. R. Kumar, "A Network Information Theory for Wireless Communications: Scaling Laws and Optimal Operation", *IEEE Trans. Inf. Theory*, vol. 50, no. 5, pp. 748-767, May 2004.

[5] M. Franceschetti, M.D. Migliore, P. Minero, "The Capacity of Wireless Networks: Information-Theoretic and Physical Limits", *IEEE Trans. Inf. Theory*, vol. 55, no. 8, pp. 3413-3424, Aug 2009.

[6] A. D. Wyner, "The Wire-Tap Channel", *Bell Syst. Tech. J*, vol. 54, no. 8, pp. 1355-1387, Oct 1975.

[7] I. Csiszar, J. Korner, "Broadcast Channels with Confidential Messages", *IEEE Trans. Inf. Theory*, vol. IT-24, no. 3, May 1978.