W/ STOKES-EINSTEIN, WE RELATED VELOCITY OF A PARTICLE TO TEMPERATURE:

$$kT \frac{\partial C}{\partial x} = -m\beta C \cdot v$$

FROM FICK'S LAW:

$$\frac{kT}{m\beta} = D,$$ β IS A DRAG COEFFICIENT.

STOKES CALCULATION GAVE EXPRESSION FOR β.
- PARTICLE SIZE & VISCOSITY.

HOW DO WE GET D FROM PARTICLE TRAJECTORIES?

TO MAKE THE CONNECTION B/T TRAJECTORIES & D, WE HAVE TO ANALYZE A RANDOM WALK.

A 1-D RANDOM WALK:

SUPPOSE WE HAVE A COLLECTION OF PARTICLES, MOVING ALONG THE x-AXIS. ALL PARTICLES START AT $x=0$, $t=0$.

EACH WALKER OBEYS SOME SIMPLE RULES:

1. ALL WALKERS TAKE A STEP, EITHER LEFT OR RIGHT ALONG x AT EACH TIME INTERVAL τ. THE STEP SIZE IS FIXED BY PARTICLE VELOCITY:

$$\delta = v_x \tau.$$

2. THE PROBABILITY OF A LEFT / RIGHT STEP IS 1/2.

3. THE PARTICLES DO NOT INTERACT.

WE'LL ANALYZE THE BEHAVIOR OF OUR COLLECTION OF PARTICLES, ACCORDING TO THESE RULES.

DEFINE $x_i(n)$ AS THE i^{th} PARTICLE'S POSITION AFTER n STEPS.

ACCORDING TO OUR RULES, WE EXPRESS i's POSITION AS A FUNCTION OF IT'S PRIOR POSITION:

$$x_i(n) = x_i(n-1) \pm \delta$$

NOW WE CALCULATE THE MEAN POSITION OVER ALL N PARTICLES AT TIME STEP n:

$$\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n)$$

IN TERMS OF PRIOR POSITIONS:

$$\langle x(n) \rangle = \frac{1}{N} \sum_{i=1}^{N} [x_i(n-1) \pm \delta]$$

$$= \frac{1}{N} \sum_{i=1}^{N} x_i(n-1) = \langle x(n-1) \rangle$$

THUS, THE MEAN POSITION DOESN'T CHANGE.

TO MEASURE THE "SPREAD" OR SMEARING OF PARTICLES, WE'LL CALCULATE THE ROOT MEAN SQUARED DISPLACEMENT

$$\langle x^2(n) \rangle^{1/2}$$

THE SQUARE DISPLACEMENT OF PARTICLE i:

$$x_i(n)^2 = x_i(n-1)^2 \pm 2\delta x_i(n-1) + \delta^2$$

$$\Rightarrow \langle x_i(n)^2 \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i(n)^2 = \frac{1}{N} \sum_{i=1}^{N} [x_i(n-1)^2 \pm \underbrace{2\delta x_i(n-1)}_{0} + \delta^2]$$

$$= \langle x(n-1)^2 \rangle + \delta^2.$$

TO DETERMINE $\langle x(n)^2 \rangle$, CONSIDER DISCRETE TIME STEPS:

$$\langle x(0)^2 \rangle = 0 \,; \quad \langle x(1)^2 \rangle = \delta^2, \quad \langle x(2)^2 \rangle = 2\delta^2, \dots$$

$$\langle x(n)^2 \rangle = n\delta^2$$

FROM OUR RULES, $t = n\tau \Rightarrow n = \frac{t}{\tau}$

$$\Rightarrow \langle x(n)^2 \rangle = \frac{t}{\tau}\,\delta^2 = \frac{\delta^2}{\tau}\cdot t$$

DIMENSIONS OF $\frac{L^2}{T}$

EQUIVALENT TO DIMENSIONS OF D.

IN FACT, $\boxed{D = \frac{\delta^2}{2\tau}}$ <u>EXACTLY</u>.

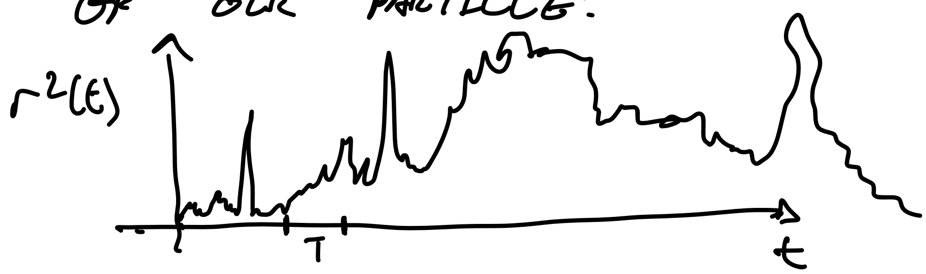
$$\Rightarrow \boxed{\langle x^2(t) \rangle = 2Dt}$$

APPLYING THE SAME RULES TO x- & y-AXES:

$$\langle x^2 \rangle = 2Dt, \quad \langle y^2 \rangle = 2Dt$$

$$r^2 = x^2 + y^2 \rightsquigarrow \langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = \underline{4Dt}.$$

OUR IMAGING DATA WILL GENERATE A TRAJECTORY OF OUR PARTICLE:



$\rightsquigarrow \langle r^2 \rangle$