

# **GANT CONCLUDING WORKSHOP**

**6 TO 10 JUNE 2011**



## Monday 6 June

10h30- 11h30

### **Cycle integrals of modular functions**

**Ozlem Imamoglu**

In this talk I will report on some results on cycle integrals of modular functions. These are based on two joint works, one with W. Duke and A. Toth, the other is with J. Bruinier and J. Funke.

13h30 – 14h30

### **Computing values of modular forms on Shimura curves**

**Paul Nelson**

We shall describe a practical method for computing (numerically) the values of a modular form on a compact Shimura curve in terms of its Hecke eigenvalues. The primary motivation here is to compute Heegner points arising from modular parametrizations via complex-analytic means in the absence of a  $q$ -expansion and in contexts where known methods of  $p$ -adic uniformization do not apply. We apply one of the ideas involved to the restriction problem of showing that as  $f$  traverses a sequence of weight  $(k, -k)$  base change forms on a Hilbert modular surface with  $k$  tending to infinity, the twisted periods of  $f$  along the diagonally embedded modular curve decay.

14h45 – 15h45

### **Applications of the Kuznetsov formula on $GL(3)$**

**Valentin Blomer**

We develop a fairly explicit Kuznetsov formula on  $GL(3)$  and discuss the analytic behaviour of the test functions on both sides. Applications to Weyl's law, exceptional eigenvalues, a large sieve and  $L$ -functions are given.

16h15 – 17h15

### **Uniform spectral gap bounds and applications**

**Alexander Gamburd**

TBA

## Tuesday 7 June

09h00 – 10h00

### **Entropy and quantum unique ergodicity**

**Elon Lindenstrauss**

The quantum unique ergodicity conjecture posed by Rudnick and Sarnak states loosely that eigenfunctions of the Laplacian on negatively curved manifolds should become equidistributed as the eigenvalue goes to infinity. Much of the progress towards his conjecture involves the basic ergodic theoretic notion of the entropy. I will survey some of this progress, and in particular will discuss recent joint work with S. Brooks which establishes this conjecture in compact arithmetic surfaces provided the Laplacian eigenfunctions are also eigenfunctions of at least one Hecke operator.

**10h30 – 11h30**      **Square-free points on quadratic surfaces**  
**Alexander Gorodnik**

We establish an asymptotic formula for the number square free-points on quadratic surfaces. Our method is based on the uniform spectral gap property of automorphic representations of the orthogonal group. This is joint work with T. Browning and A. Nevo.

**14h00 – 15h00**      **On the continued fraction expansion of quadratic irrationals (effective results)**

**Uri Shapira**

I will discuss a recent result regarding the evolution of the period of the continued fraction expansion of  $2^n \sqrt{3}$  (for example) as  $n$  goes to infinity. We prove that the statistics of the period approaches the one given by the Gauss-Kuzmin measure in an effective manner. The actual argument goes through an equidistribution theorem of certain sequences of (single) geodesic loops on the modular surface. This is joint work with Menny Aka.

**15h30 – 16h30**      **Counting integral binary quadratic forms in an  $SL_2(\mathbb{Z})$ -orbit and limits of translates of divergent geodesics**

**Nimish Shah**

For any nonuniform lattice  $\Gamma$  in  $SL(2, \mathbb{R})$  we describe the limit distribution of orthogonal translates of a divergent geodesic in  $\Gamma \backslash SL(2, \mathbb{R})$ . It turns out that the problem is very closely related to equidistribution of long closed horocycles. Using the results of Sarnak and also Strombergsson, we obtain precise estimate on smoothed count for the number of binary quadratic forms of discriminant  $D$  in a single  $SL(2, \mathbb{Z})$ -orbit and of norm bounded by  $T$ .

**Wednesday 8 June**

**09h00 – 10h00**      **Special cycles and cohomology of certain arithmetic manifolds**  
**Nicolas Bergeron**

In a joint work with John Millson and Colette Moeglin we show that special cycles generate a large part of the cohomology of locally symmetric spaces associated to orthogonal groups. We prove in particular that classes of totally geodesic submanifolds generate the cohomology groups of degree  $n$  of compact congruence  $p$ -dimensional hyperbolic manifolds "of simple type" as long as  $n$  is strictly smaller than  $(p-2)/4$ . We also prove that for connected Shimura varieties associated to  $O(p, 2)$  the Hodge conjecture is true for classes of degree  $< (p-1)/4$ . I will explain the strategy of the proof.

**10h30 – 11h30**

**The local and global meaning of "unfolding"**  
**Yiannis Sakellaridis**

The "unfolding" technique, i.e. a sequence of inverse Fourier transforms, has been used to study a wide range of integrals of automorphic forms (for example: Hecke integrals or Rankin-Selberg integrals of cusp forms on  $GL_2$ ) and to show that they represent L-functions. I will explain how the same method can be used to solve a problem of local harmonic analysis; and how this leads to a uniform way of understanding the local Euler factors of those global integrals. This is part of joint work with Akshay Venkatesh.

**Thursday 9 June**

**09h00 – 10h00**

**Random walks on linear groups**  
**Peter Varju**

I will talk about a joint work with Jean Bourgain that establishes spectral gaps for random walks on  $SL_n(\mathbb{Z}/q\mathbb{Z})$ . Let  $S$  be a fixed finite and symmetric subset of  $SL_n(\mathbb{Z})$  which generates a Zariski dense subgroup. We show that words of length  $C \log(q)$  are almost uniformly distributed among congruence classes modulo  $q$ . Unlike in previous results,  $q$  is arbitrary and not restricted to any special subset of the integers.

**10h30 – 11h30**

**Growth in linear groups in positive characteristic**  
**Laszlo Pyber**

By the Product Theorem, proved independently by Breuillard-Green-Tao and Pyber-Szabo', generating sets of finite simple groups of Lie type of bounded rank grow quickly. We consider extensions of this result by investigating subsets  $A$  of linear groups that satisfy  $|A^3| < K|A|$ .

**13h30 – 14h30**

**Generic properties of groups**  
**Elena Fuchs**

In recent years, there have been great advances in understanding families of graphs that are expanders. For example, given a subgroup  $G$  of  $GL_n(\mathbb{Z})$  with finite generating set  $S$ , Salehi-Golsefidy-Varju give necessary and sufficient conditions for the family of graphs  $\text{Cay}(G=d;S)$  for squarefree  $d$  to be an expander family. These results have found unexpected applications in number theory. However, their most important applications are to groups  $G$  which are thin, or of infinite index in their Zariski closure. In this talk we will discuss how generic such a group  $G$  is.

14h45 – 15h45

**Functional Transcendence Results and the André-Oort Conjecture**  
**Jacob Tsimerman**

The recent approach to the André-Oort Conjecture by J.Pila requires as a key ingredient a functional transcendence result about the uniformization map for Shimura varieties. This is a generalization of the Lindemann-Weirstrass part of the Ax-Schanuel theorem for the exponential function, and was thus named the "Ax-Lindemann-Weirstrass" conjecture by Pila.

We will explain why this conjecture is relevant to Andre-Oort and also sketch the methods (such as o-minimality) that come up in its proof for products of modular curves and more recently for the moduli space  $A_{2,1}$  of principally polarized abelian surfaces.

16h15 – 17h15

**Partial measure classification for torus actions on homogeneous spaces**  
**Manfred Einsiedler**

We will discuss the ergodic theoretic theorems that partially classify measures that are invariant under higher rank torus subgroups. Even though there are similarities in the (desired) conclusions there are also important differences between the theorems concerning unipotent flows resp. torus actions. Most notable is the "entropy assumption", which we will discuss, and the lack of a "linearization technique", where the latter is partially compensated by the topological argument known as isolation theorems. We will also discuss the case of joinings of higher rank torus actions where a complete classification is known. The discussed theorems are the work of Katok, Lindenstrauss, Barak Weiss, and myself in various combinations.

Friday 10 June

09h00 – 10h00

**Rank in families of elliptic surfaces**  
**Christopher Hall**

Let  $K$  be the rational function field  $\mathbb{Q}(t)$  and let  $E/K$  be an elliptic curve. By varying an auxiliary parameter  $c$  we may form a one-parameter family of twists by the quadratic extensions  $K(\sqrt{t-c})$ . The central theme of this talk is to vary  $c$  over  $\overline{\mathbb{Q}}$  and to consider the rank of the respective Mordell-Weil group over the finite extension  $K(c)$ . More precisely, we will show that if  $E$  has non-constant  $j$ -invariant and if the divisor  $D \subset \mathbb{P}_{\mathbb{Q}}^1$  of bad reduction for  $E/K$  has sufficiently large degree, then there are only finitely many  $c$  of bounded degree such that over  $K(c)$  the Mordell-Weil group of the quadratic twist has positive rank.

10h30 – 11h30

**On the size of the fundamental solution of pell equation**  
**Etienne Fouvry**

We present several results describing the cardinality of the set of  $D \leq X$ , such that the fundamental solution  $x + y\sqrt{D}$  of Pell equation  $x^2 - Dy^2 = 1$  is less than  $D^\gamma$ , where  $\gamma$  is a fixed, but not large, constant.

This is a joint work with F. Jouve.