Particle Image Velocimetry – PIV

Quantitative Flow Visualisation
Origins of Particle Image Velocimetry

'Poohsticks'

1928
Overview of Particle Image Velocimetry

Typical experimental arrangement for PIV

Principle of operation for PIV

\[
\text{Velocity vector at } \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)_{\Delta t} = \left( \frac{x_2 - x_1}{\Delta t}, \frac{y_2 - y_1}{\Delta t} \right)
\]

- A non-intrusive velocity measurement technique
- An accurate whole field technique – Provides the velocity field over a large flow area as a function of time to an accuracy of above 95%
- Indirect measurement of flow velocity – Velocity is determined from the motion of suspended tracer particles and not the true flow velocity
- Well suited to a fully digital implementation – DPIV
- Performance governed by a number of parameters – Need to optimize a PIV system to its application
Figure 2.4. Streamlines of steady flow (from left to right) past a sphere at various Reynolds numbers (from Taneda 1956, reproduced by permission of the author).
**momentum response time, relaxation time**

**Stokes number**

\[ S_t = \frac{\text{particle response time}}{\text{time characteristic of flow}} = \frac{\tau_v}{\tau_F} \]

- \( S_t << 1 \): The particles and fluid will be in near equilibrium.
- \( S_t >> 1 \): Particles will be unaffected by the fluid.

**Tracking particle trajectory**

Large scale vortex (free shear layer)  
Crowe and co-workers (1989)

\( S_t \approx 1 \)
Lift generation from a rotating sphere

\[ p + \frac{1}{2} \rho v^2 = p_\infty + \frac{1}{2} \rho V^2 = \text{const.} \quad \text{Bernoulli's equation} \]

Velocity gradients in the fluid cause the tracer particles to rotate and thus generate lift. The particles then migrate across the streamlines.

Therefore, the motion of the particles does not represent the velocity of the surrounding fluid.
Basset-Boussinesq-Oseen equation (BBO equation): low Re

Equation of motion of a single particle (flow curvature effects are neglected)

\[ m \frac{d\vec{v}}{dt} = 3\pi\mu D (\vec{u} - \vec{v}) \quad \text{steady state drag} \]

\[ + V_d ( -\nabla p + \nabla \tau ) \quad \text{external forces} \]

\[ + \frac{\rho_c V_d}{2} (\vec{u} - \vec{v}) \quad \text{virtual mass} \]

\[ + \frac{3}{2} D^2 \sqrt{\pi \rho_c \mu} \left[ \int_0^t \frac{\vec{u} - \vec{v}}{\sqrt{t-t'}} dt' + \frac{(\vec{u} - \vec{v})_0}{\sqrt{t}} \right] \quad \text{Basset force} \]

\[ + m \vec{g} \quad \text{Body force} \]

\[ m = \frac{\pi}{6} D^3 \rho_d \quad V_d = \frac{\pi}{6} D^3 \]
Mie light scattering from spherical particles

Fig. 2.5. Light scattering by a 1 μm glass particle in water

Fig. 2.6. Light scattering by a 10 μm glass particle in water

Fig. 2.7. Light scattering by a 30 μm glass particle in water

Light sources: Lasers, collimated white light...

- Colour matches the spectral sensitivity of the imaging sensor
- Able to pulse at the required Δt
- Stable pulse intensities
Tracer Particles for PIV

![Image of glass particles: ×500 and ×5000](image)

Fig. 2.8. Glass particles: ×500 and ×5000

<table>
<thead>
<tr>
<th>Type</th>
<th>Material</th>
<th>Mean diameter in μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>Polystyrene</td>
<td>10 - 100</td>
</tr>
<tr>
<td></td>
<td>Aluminum</td>
<td>2 - 7</td>
</tr>
<tr>
<td></td>
<td>Glass spheres</td>
<td>10 - 100</td>
</tr>
<tr>
<td></td>
<td>Granules for synthetic coatings</td>
<td>10 - 500</td>
</tr>
<tr>
<td>Liquid</td>
<td>Different oils</td>
<td>50 - 500</td>
</tr>
<tr>
<td>Gaseous</td>
<td>Oxygen bubbles</td>
<td>50 - 1000</td>
</tr>
</tbody>
</table>

Table 2.1. Seeding materials for liquid flows

<table>
<thead>
<tr>
<th>Type</th>
<th>Material</th>
<th>Mean diameter in μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid</td>
<td>Polystyrene</td>
<td>0.5 - 10</td>
</tr>
<tr>
<td></td>
<td>Aluminum</td>
<td>2 - 7</td>
</tr>
<tr>
<td></td>
<td>Magnesium</td>
<td>2 - 5</td>
</tr>
<tr>
<td></td>
<td>Glass micro-balloons</td>
<td>30 - 100</td>
</tr>
<tr>
<td></td>
<td>Granules for synthetic coatings</td>
<td>10 - 50</td>
</tr>
<tr>
<td></td>
<td>Dioctyl phthalate</td>
<td>1 - 10</td>
</tr>
<tr>
<td>Smoke</td>
<td></td>
<td>&lt; 1</td>
</tr>
<tr>
<td>Liquid</td>
<td>Different oils</td>
<td>0.5 - 10</td>
</tr>
</tbody>
</table>

Table 2.2. Seeding materials for gas flows
Vélocimétrie par images de particules

Principe de fonctionnement

\[
V_i = \frac{\Delta x_i}{\Delta t}
\]

Plan d'observation illuminé par Laser

Faible densité de particules: Vélocimétrie par poursuite de particules (Particle Tracking Velocimetry - PTV)

Haute densité de particules: Vélocimétrie par images de particules (Particle Image Velocimetry - PIV)

• Pas d'identification de particules individuelles
Exemple d'image de particules mesurée

Volume d'interrogation typique
Numerical processing flowchart for DPIV

The digitised image sequence is spatially sub-sampled into smaller interrogation tiles of size \( W \times W \) pixels.

2 tiles, centered on the same flow location \((x_c, y_c)\) but separated in time by \(\Delta t\), are then cross-correlated to estimate the mean local velocity.

Tile size: \( W \times W \) pixels
Tile center: \((x_c, y_c)\)
Time: \( t \)

From Nyquist theorem: 
\[
|\Delta x|_{\text{max}} = M \Delta t |u|_{\text{max}} < \frac{W}{2}
\]

Determine location \((\Delta x^*, \Delta y^*)\) of the maximum peak of \( R(\Delta x, \Delta y) \) to sub-pixel accuracy

Velocity vector at:
\[
(x_c + \frac{\Delta x^*}{2}, y_c + \frac{\Delta y^*}{2})_{\text{sub-pixel}} = \left( \frac{\Delta x^*}{M \Delta t}, \frac{\Delta y^*}{M \Delta t} \right)
\]
Cross correlation of two samples

Example of the formation of the correlation plane
image particule < pixel

image particule couvre environ 4 pixels ou plus

Fig. 3a–c. Cross-correlation estimate e of image pair a, b which consist of randomly positioned particles represented as Dirac delta functions: the relative spatial shift is -4 pixels along the y-axis.

max de corrélation localisé à 1 pixel près

max de corrélation peut être trouvé à une fraction de pixel par interpolation (typ. fit Gaussien)

Fig. 4a–c. Same as Fig. 3, but for samples taken from recorded images (particle displacement ≈ 8 pixels along y-axis).
La corrélation croisée est définie par :

\[ R_{12}(\xi) = \int_{-\infty}^{\infty} f_1(x) f_2(x + \xi) \, dx \]
PIV Optimisation

In plane particle image pair loss $F_i$

- Choose sampling interval $\Delta t$ and optical magnification factor $M$ so that the maximum image displacement is less than a quarter of the interrogation tile size $W$.

  $$|\Delta x_i|_{\text{max}} = M \Delta t |u|_{\text{max}} < 0.25W \quad \therefore F_{i_{\text{min}}} = 0.75$$

- Shift the interrogation tile at $t+\Delta t$ by the estimated mean local image displacement and repeat the cross-correlation, thus $F_i \rightarrow 1$.

  Added advantage that measurement errors decrease significantly for particle shifts less than half a pixel.

Out of plane particle image pair loss $F_o$

- Choose a suitable observation plane in the flow such that $u_z \approx 0$, thus $F_o \rightarrow 1$.

- Choose a sampling interval $\Delta t$ such that the maximum particle shift normal to the observed plane is less than a quarter of the light sheet thickness $\Delta z_{\text{light}}$

  $$\Delta t < \frac{\Delta z_{\text{light}}}{4 |u_z_{\text{max}}|} \quad \therefore F_{o_{\text{min}}} = 0.75$$

![](image)

**Fig. 4.10.** Effect of reducing the time between two pulses as observed in the light sheet (figure is not to scale)

Particle image density $N_i$

- Since we require $N_i F_i F_o > 5$ for optimal detection, a conservative estimate of $F_i = F_o = 0.75$ implies that $N_i > 10$.

- Typically choose $N_i = 20$ particle images per tile to minimise measurement errors.
Optimal parameters for DPIV operation

\[
\Delta t_{\text{Detection}} < \min \left\{ \frac{W}{4M u_{\max}} \right. \\
\left. \frac{\Delta z_{\text{light}}}{4u_{\max}} \right. \\
\left. 0.03 \right. \\
\left. \tau_{xy} \right. \right\} 
\]

or

\[
\Delta t_{\text{Accuracy}} < \frac{1}{2M u_{\max}}
\]

\[\rho_p = \rho_{\text{Fluid}} \quad \text{Neutrally buoyant particles}\]

\[D_I = M D_p \approx 2.5 \text{ pixels}\]

\[N \approx 20 \text{ particle images per interrogation window}\]
Key parameters for DPIV operation

\[ W \] 
Interrogation window size [pixels]

\[ N F_i F_o \] 
‘Particle retention factor’
Indicates how many particles remain in an interrogation tile after a time step of \( \Delta t \).

Where:

\[ N \] 
Number of particle images per interrogation tile

\[ F_i \] 
In-plane retention factor

\[ F_o \] 
Out-plane retention factor

Eg: if the particles are uniformly displaced by \( W/4 \) pixels after a time step \( \Delta t \), then 75% of the original particles remain in the window, thus the in-plane retention factor \( F_i = 0.75 \)

\[ D_I = M D_P \] 
Particle image diameter [pixels]

\[ |\Delta x|_{max} = M \Delta t |u|_{max} \] 
Maximum in-plane particle image displacement in time \( \Delta t \) [pixels]

\[ |u_z|_{max} \] 
Maximum out-plane velocity
Effect of particle image diameter on measurement errors

![Graph showing RMS uncertainty vs. particle image diameter](image)

**Fig. 5.23.** Measurement uncertainty (RMS random error) in digital cross-correlation PIV evaluation with respect to varying particle image diameter.

Optimal particle image diameter: \( D_i \approx 2.5 \) pixels

- Using the optical magnification factor \( M \), determine the minimum tracer particle diameter \( D \) to achieve this, thus minimising the response time
- Determine \( D_i \) in situ from recorded particle image intensity profiles
Effect of particle image density on measurement errors

Interrogation validity

For a valid detection probability >95% :  \( N_f F_i F_o > 5 \)

Interrogation accuracy

Increasing \( N_f \) for a given tile size \( W \) decreases the measurement error.
The effect of velocity gradients on measurement accuracy

Strong local rotation

Strong local shear

Image displacement gradients lead to a widening of the correlation peak, thus reducing valid detection probability and accuracy.
Effect of displacement gradients on measurement errors

Interrogation validity

Displacement gradient: \( G = 100 \Delta t \frac{\partial u}{\partial y} \) [pixels/pixel]

For a valid detection probability >95% : \( G = \Delta t \frac{\partial u}{\partial y} < 0.03 \) pixels/pixel

Interrogation accuracy

Displacement gradient: \( G = \Delta t \frac{\partial u}{\partial y} \) [pixels/pixel]

For a given displacement gradient \( G \) and particle image density \( N_t \), the error decreases with decreasing interrogation tile size \( W \)

For optimal \( N_t = 20 \) use \( W = 32 \times 32 \) pixel tiles
Effect of image noise on measurement errors

Fig. 5.32. Measurement uncertainty as a function of displacement and various amounts of white background noise (simulation parameters: $d = 2.2$ pixel, $N_T = 10.2, 32 \times 32$ pixel, optimum exposure, top-hat light sheet profile)