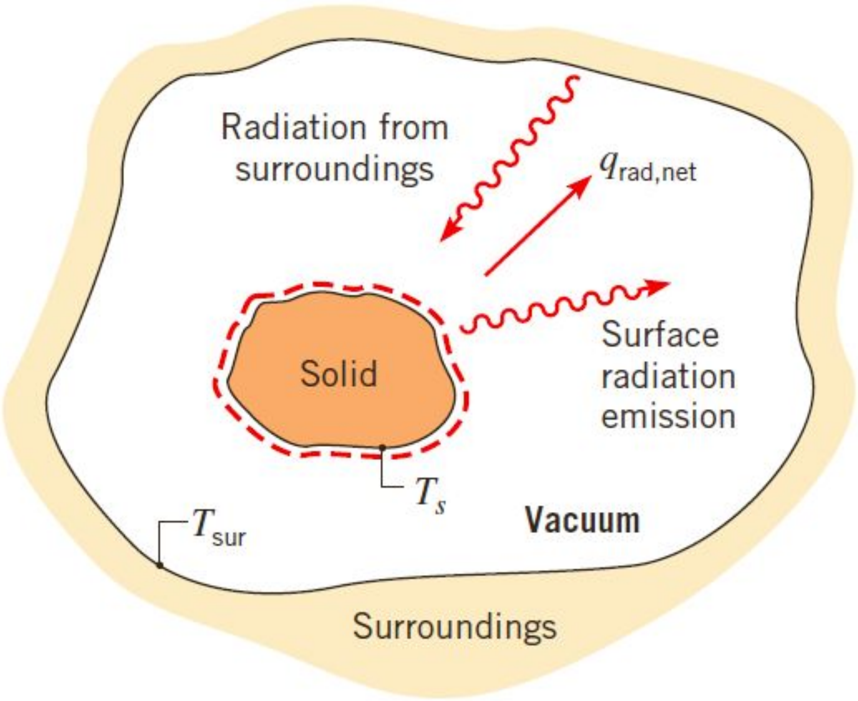


Consider a solid that is initially at a higher temperature T_s than that of its surroundings T_{sur}, but around which there exists a vacuum (Figure 12.1). The presence of the vacuum precludes energy loss from the surface of the solid by conduction or convection. However, our intuition tells us that the solid will cool and eventually achieve thermal equilibrium with its surroundings. This cooling is associated with a reduction in the internal energy stored by the solid and is a direct consequence of the emission of thermal radiation from the surface. In turn, the surface will intercept and absorb radiation originating from the surroundings.

FIGURE 12.1 Radiation cooling of a hot solid.

We know that radiation originates due to emission by matter and that its subsequent transport does not require the presence of any matter. But what is the nature of this transport? One theory views radiation as the propagation of a collection of particles termed *photons* or *quanta.* Alternatively, radiation may be viewed as the propagation of *electromagnetic waves.* In any case we wish to attribute to radiation the standard wave properties of frequency ν and wavelength λ. For radiation propagating in a particular medium, the two properties are related by

$$\lambda = \frac{c}{\nu} \tag{12.1}$$

where c is the speed of light in the medium. For propagation in a vacuum, $c_o = 2.998 \times 10^8$ m/s. The unit of wavelength is commonly the micrometer (μm), where $1\ \mu\text{m} = 10^{-6}$ m.

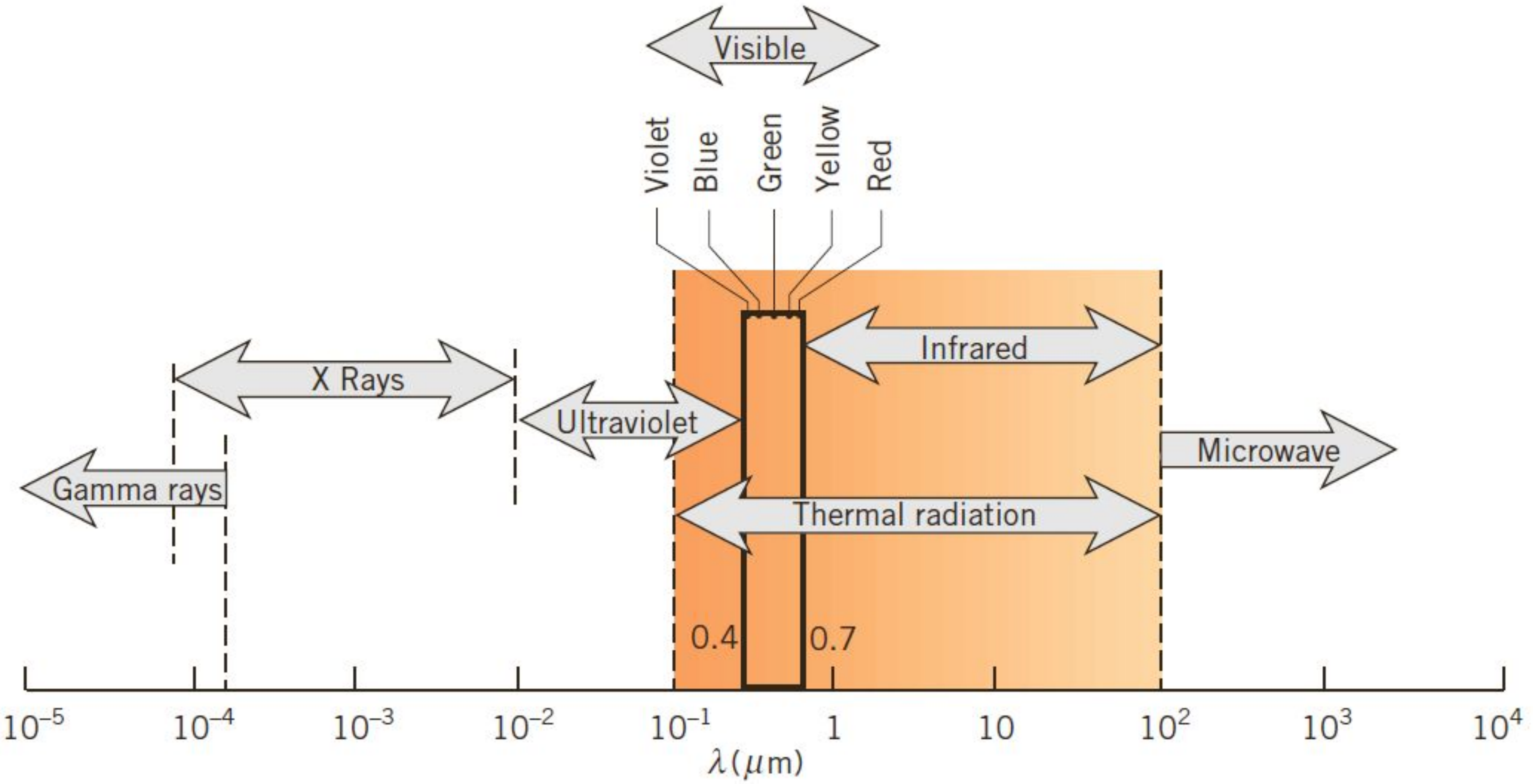


FIGURE 12.3 Spectrum of electromagnetic radiation.

The complete electromagnetic spectrum is delineated in Figure 12.3. The short wavelength gamma rays, X rays, and ultraviolet (UV) radiation are primarily of interest to the high-energy physicist and the nuclear engineer, while the long wavelength microwaves and radio waves ($\lambda > 10^5\ \mu m$) are of concern to the electrical engineer. It is the intermediate portion of the spectrum, which extends from approximately 0.1 to 100 μm and includes a portion of the UV and all of the visible and infrared (IR), that is termed *thermal radiation* because it is both caused by and affects the thermal state or temperature of matter. For this reason, thermal radiation is pertinent to heat transfer.

Thermal radiation emitted by a surface encompasses a range of wavelengths. As shown in Figure 12.4*a*, the magnitude of the radiation varies with wavelength, and the term *spectral* is used to refer to the nature of this dependence. As we will find, both the magnitude of the radiation at any wavelength and the *spectral distribution* vary with the nature and temperature of the emitting surface.

TABLE 12.1 Radiative fluxes (over all wavelengths and in all directions)

Flux (W/m^2)	Description	Comment
Emissive power, E	Rate at which radiation is emitted from a surface per unit area	$E = \varepsilon\sigma T_s^4$
Irradiation, G	Rate at which radiation is incident upon a surface per unit area	Irradiation can be reflected, absorbed, or transmitted
Radiosity, J	Rate at which radiation leaves a surface per unit area	For an opaque surface $J = E + \rho G$
Net radiative flux, $q''_{\text{rad}} = J - G$	Net rate of radiation leaving a surface per unit area	For an opaque surface $q''_{\text{rad}} = \varepsilon\sigma T_s^4 - \alpha G$

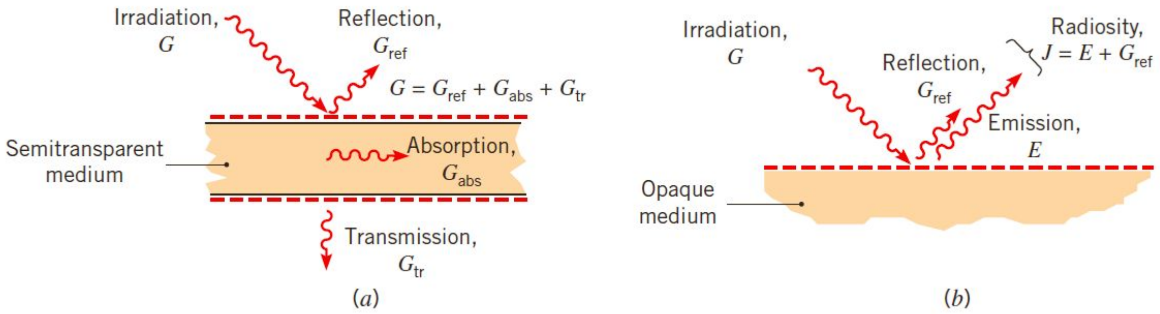


FIGURE 12.5 Radiation at a surface. (*a*) Reflection, absorption, and transmission of irradiation for a semitransparent medium. (*b*) The radiosity for an opaque medium.

Various types of heat fluxes are pertinent to the analysis of radiation heat transfer. Table 12.1 lists four distinct radiation fluxes that can be defined at a surface such as the one in Figure 12.2*b*. The *emissive power*, E (W/m^2), is the rate at which radiation is emitted from a surface per unit surface area, over all wavelengths and in all directions. In Chapter 1, this emissive power was related to the behavior of a *blackbody* through the relation $E = \varepsilon \sigma T_s^4$ (Equation 1.5), where ε is a surface property known as the *emissivity*.

Radiation from the surroundings, which may consist of multiple surfaces at various temperatures, is incident upon the surface. The surface might also be irradiated by the sun or by a laser. In any case, we define the *irradiation,* G (W/m^2), as the rate at which radiation is incident upon the surface per unit surface area, over all wavelengths and from all directions. The two remaining heat fluxes of Table 12.1 are readily described once we consider the fate of the irradiation arriving at the surface.

When radiation is incident upon a *semitransparent medium*, portions of the irradiation may be reflected, absorbed, and transmitted, as discussed in Section 1.2.3 and illustrated in Figure 12.5*a*. Transmission refers to radiation passing through the medium, as

Absorption occurs when radiation interacts with the medium, causing an increase in the internal thermal energy of the medium. Reflection is the process of incident radiation being redirected away from the surface, with no effect on the medium. We define reflectivity ρ as the fraction of the irradiation that is reflected, absorptivity α as the fraction of the irradiation that is absorbed, and transmissivity τ as the fraction of the irradiation that is transmitted. Because all of the irradiation must be reflected, absorbed, or transmitted, it follows that

$$\rho + \alpha + \tau = 1 \tag{12.2}$$

A medium that experiences no transmission ($\tau = 0$) is *opaque*, in which case

$$\rho + \alpha = 1 \tag{12.3}$$

With this understanding of the partitioning of the irradiation into reflected, absorbed, and transmitted components, two additional and useful radiation fluxes can be defined. The *radiosity*, J (W/m^2), of a surface accounts for *all* the radiant energy leaving the surface. For an opaque surface, it includes emission and the reflected portion of the irradiation, as illustrated in Figure 12.5*b*. It is therefore expressed as

$$J = E + G_{\text{ref}} = E + \rho G \tag{12.4}$$

Blackbody Radiation

To evaluate the emissive power, irradiation, radiosity, or net radiative heat flux of a real opaque surface, we must quantify the spectral intensities used in Equations 12.13, 12.18, 12.23, and 12.28. To do so, it is useful to first introduce the concept of a *blackbody*.

1. *A blackbody absorbs all incident radiation, regardless of wavelength and direction.*
2. *For a prescribed temperature and wavelength, no surface can emit more energy than a blackbody.*
3. *Although the radiation emitted by a blackbody is a function of wavelength and temperature, it is independent of direction. That is, the blackbody is a diffuse emitter.*

The Planck Distribution

$$E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}$$

[W/m$^2\mu$m]

where the first and second radiation constants are $C_1 = 2\pi h c_o^2 = 3.742 \times 10^8\ \mathrm{W \cdot \mu m^4/m^2}$ and $C_2 = (hc_o/k_B) = 1.439 \times 10^4\ \mu\mathrm{m \cdot K}$.

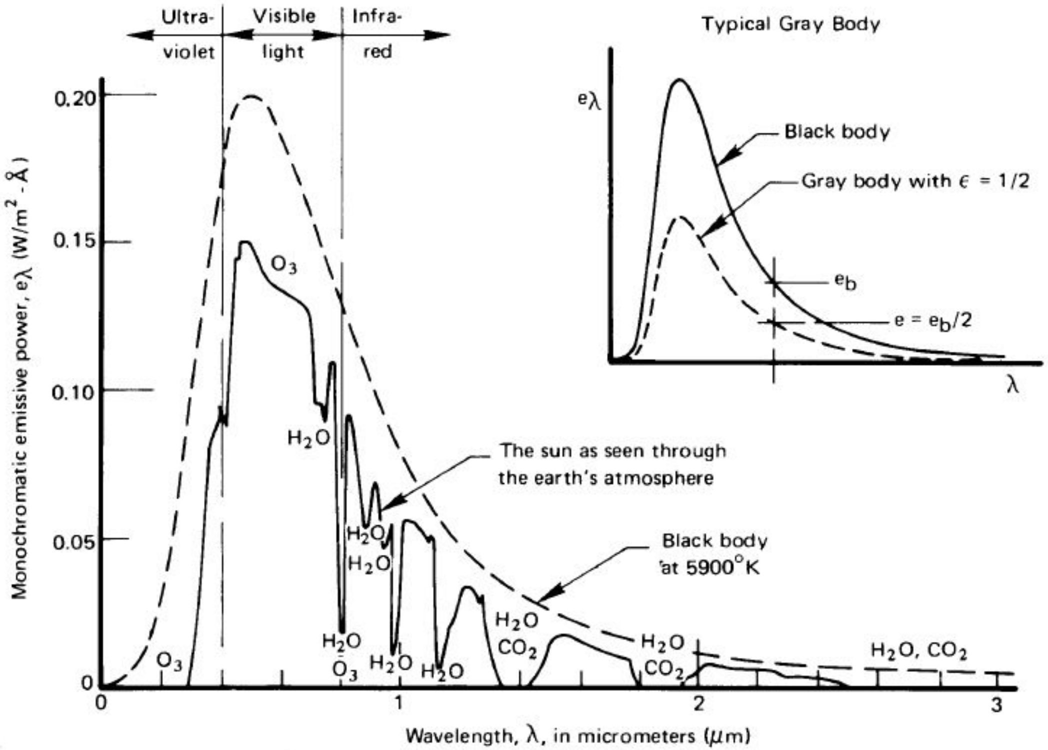


Figure 10.2 Comparison of the sun's energy as typically seen through the earth's atmosphere with that of a black body having the same mean temperature, size, and distance from the earth. (Notice that e_λ, just outside the earth's atmosphere, is far less than on the surface of the sun because the radiation has spread out over a much greater area.)

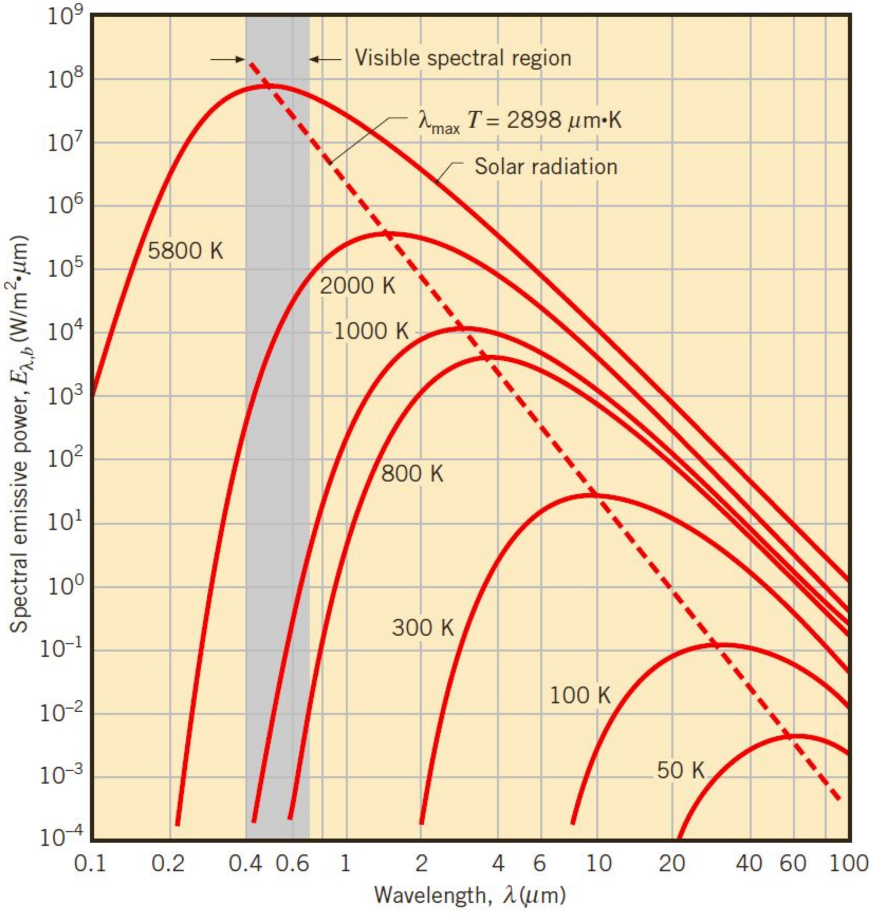


FIGURE 12.12 Spectral blackbody emissive power.

Equation 12.30, known as the *Planck distribution,* or *Plancks law* , is plotted in Figure 12.12 for selected temperatures. Several important features should be noted.

1. The emitted radiation varies *continuously* with wavelength.[1]
2. At any wavelength the magnitude of the emitted radiation increases with increasing temperature.
3. The spectral region in which the radiation is concentrated depends on temperature, with *comparatively* more radiation appearing at shorter wavelengths as the temperature increases.
4. A significant fraction of the radiation emitted by the sun, which may be approximated as a blackbody at 5800 K, is in the visible region of the spectrum. In contrast, for $T \lesssim 800$ K, emission is predominantly in the infrared region of the spectrum and is not visible to the eye.

The Stefan–Boltzmann Law

Substituting the Planck distribution, Equation 12.30, into Equation 12.14, the total emissive power of a blackbody E_b may be expressed as

$$E_b = \int_0^\infty \frac{C_1}{\lambda^5[\exp(C_2/\lambda T) - 1]}\, d\lambda$$

Performing the integration, it may be shown that

$$E_b = \sigma T^4$$

[W/m^2]

where the *StefanBoltzmann* constant, which depends on C_1 and C_2, has the numerical value

$$\sigma = 5.670 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4}$$

Band Emission

To account for spectral effects, it is often necessary to know the fraction of the total emission from a blackbody that is in a certain wavelength interval or *band*. For a prescribed temperature and the interval from 0 to λ, this fraction is determined by the ratio of the shaded section to the total area under the curve of Figure 12.13. Hence

$$F_{(0\rightarrow\lambda)} \equiv \frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\int_0^\infty E_{\lambda,b}\,d\lambda} = \frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5}\,d(\lambda T) = f(\lambda T) \qquad (12.34)$$

Since the integrand $(E_{\lambda,b}/\sigma T^5)$ is exclusively a function of the wavelength–temperature product λT, the integral of Equation 12.34 may be evaluated to obtain $F_{(0\rightarrow\lambda)}$ as a function of only λT. The results are presented in Table 12.2 and Figure 12.14. They may also be used to obtain the fraction of the radiation between any two wavelengths λ_1 and λ_2, since

$$F_{(\lambda_1\rightarrow\lambda_2)} = \frac{\int_0^{\lambda_2} E_{\lambda,b}\,d\lambda - \int_0^{\lambda_1} E_{\lambda,b}\,d\lambda}{\sigma T^4} = F_{(0\rightarrow\lambda_2)} - F_{(0\rightarrow\lambda_1)} \qquad (12.35)$$

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
200	0.000000
400	0.000000
600	0.000000
800	0.000016
1,000	0.000321
1,200	0.002134
1,400	0.007790
1,600	0.019718
1,800	0.039341
2,000	0.066728
2,200	0.100888
2,400	0.140256
2,600	0.183120
2,800	0.227897
2,898	0.250108

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
3,000	0.273232
3,200	0.318102
3,400	0.361735
3,600	0.403607
3,800	0.443382
4,000	0.480877
4,200	0.516014
4,400	0.548796
4,600	0.579280
4,800	0.607559
5,000	0.633747

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
5,200	0.658970
5,400	0.680360
5,600	0.701046
5,800	0.720158
6,000	0.737818
6,200	0.754140
6,400	0.769234
6,600	0.783199
6,800	0.796129
7,000	0.808109
7,200	0.819217

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
7,400	0.829527
7,600	0.839102
7,800	0.848005
8,000	0.856288
8,500	0.874608
9,000	0.890029
9,500	0.903085
10,000	0.914199
10,500	0.923710
11,000	0.931890
11,500	0.939959
12,000	0.945098

λT ($\mu m \cdot K$)	$F_{(0 \to \lambda)}$
13,000	0.955139
14,000	0.962898
15,000	0.969981
16,000	0.973814
18,000	0.980860
20,000	0.985602
25,000	0.992215
30,000	0.995340
40,000	0.997967
50,000	0.998953
75,000	0.999713
100,000	0.999905

Emission from Real Surfaces

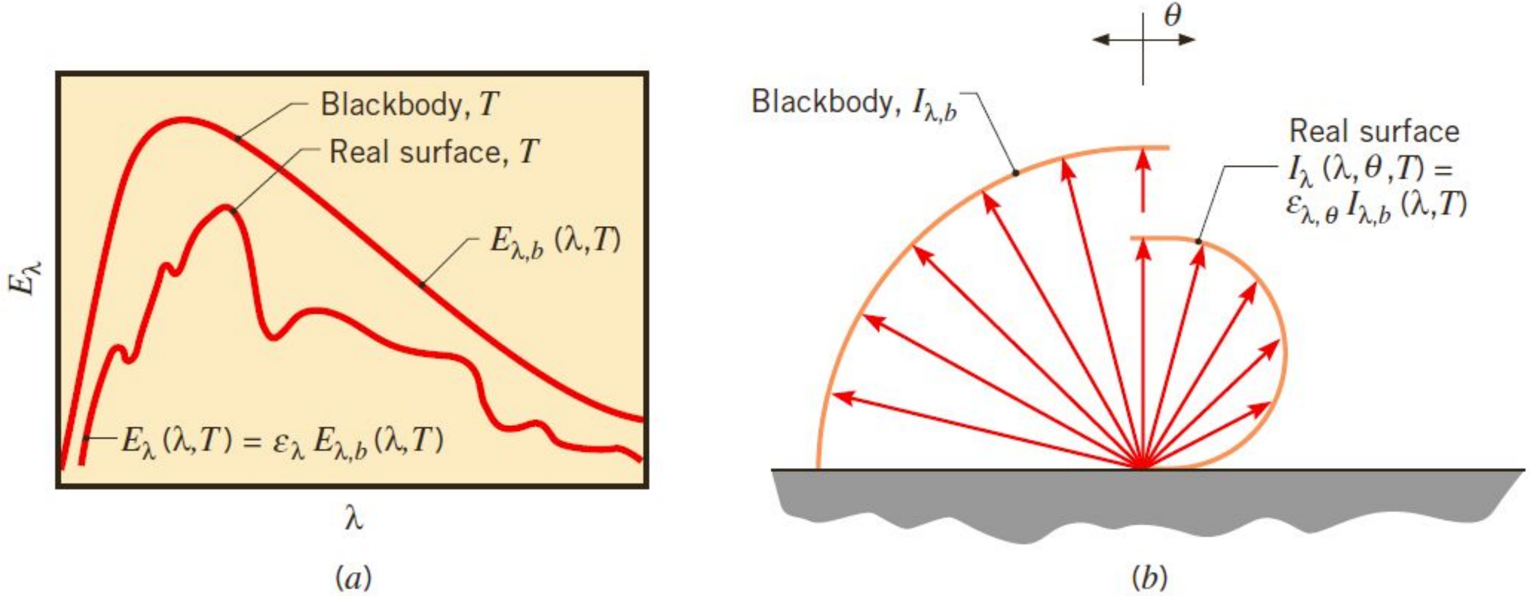


FIGURE 12.15 Comparison of blackbody and real surface emission. (a) Spectral distribution. (b) Directional distribution.

The emissivity that accounts for emission over all wavelengths and in all directions is the *total, hemispherical* emissivity, which is the ratio of the total emissive power of a real surface, $E(T)$, to the total emissive power of a blackbody at the same temperature, $E_b(T)$. That is,

$$\varepsilon(T) \equiv \frac{E(T)}{E_b(T)} \tag{12.36}$$

If the total, hemispherical emissivity of a surface is known, it is a simple matter to express its emissive power in terms of the emissive power of a blackbody by combining Equation 12.36 with Equation 12.32, namely

$$E(T) = \varepsilon(T)E_b(T) = \varepsilon(T)\sigma T^4 \tag{12.37}$$

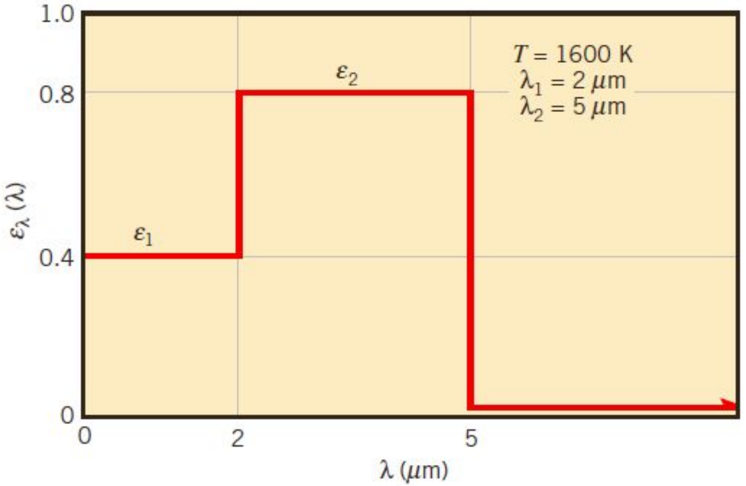
For most engineering calculations, it is desirable to work with surface properties that represent directional averages. A *spectral, hemispherical emissivity* is therefore defined as

$$\varepsilon_\lambda(\lambda, T) \equiv \frac{E_\lambda(\lambda, T)}{E_{\lambda,b}(\lambda, T)} \tag{12.40}$$

The *total, hemispherical emissivity,* which represents an average over all possible directions and wavelengths, is defined in Equation 12.36. Substituting Equations 12.14 and 12.40 into Equation 12.36, it follows that

$$\varepsilon(T) = \frac{\int_0^\infty \varepsilon_\lambda(\lambda, T) E_{\lambda,b}(\lambda, T)\, d\lambda}{E_b(T)} \tag{12.43}$$

A diffuse surface at 1600 K has the spectral, hemispherical emissivity shown as follows.



Determine the total, hemispherical emissivity and the total emissive power. At what wavelength will the spectral emissive power be a maximum?

Known: Spectral, hemispherical emissivity of a diffuse surface at 1600 K.

Find:

1. Total, hemispherical emissivity.
2. Total emissive power.

Assumptions: Surface is a diffuse emitter.

Analysis:

1. The total, hemispherical emissivity is given by Equation 12.43, where the integration may be performed in parts as follows:

$$\varepsilon = \frac{\int_0^{\infty} \varepsilon_\lambda E_{\lambda,b}\, d\lambda}{E_b} = \frac{\varepsilon_1 \int_0^{2} E_{\lambda,b}\, d\lambda}{E_b} + \frac{\varepsilon_2 \int_2^{5} E_{\lambda,b}\, d\lambda}{E_b}$$

or

$$\varepsilon = \varepsilon_1 F_{(0\to 2\,\mu\text{m})} + \varepsilon_2 [F_{(0\to 5\,\mu\text{m})} - F_{(0\to 2\,\mu\text{m})}]$$

From Table 12.2 we obtain

$$\lambda_1 T = 2\,\mu\text{m} \times 1600\,\text{K} = 3200\,\mu\text{m}\cdot\text{K}: \qquad F_{(0\to 2\,\mu\text{m})} = 0.318$$

$$\lambda_2 T = 5\,\mu\text{m} \times 1600\,\text{K} = 8000\,\mu\text{m}\cdot\text{K}: \qquad F_{(0\to 5\,\mu\text{m})} = 0.856$$

Hence

$$\varepsilon = 0.4 \times 0.318 + 0.8[0.856 - 0.318] = 0.558$$

2. From Equation 12.36 the total emissive power is

$$E = \varepsilon E_b = \varepsilon \sigma T^4$$

$$E = 0.558(5.67 \times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4)(1600\,\text{K})^4 = 207\,\text{kW/m}^2$$

Absorption, Reflection, and Transmission by Real Surfaces

In the preceding section, we learned that emission from a real surface is associated with a surface property termed the emissivity ε. To determine the net radiative heat flux from the surface, it is also necessary to consider properties that determine the absorption, reflection, and transmission of the irradiation. In Section 12.3.3 we defined the *spectral irradiation* G_λ $(\mathrm{W/m^2 \cdot \mu m})$ as the rate at which radiation of wavelength λ is incident on a surface per unit area of the surface and per unit wavelength interval $d\lambda$ about λ. It may be incident from all possible directions, and it may originate from several different sources. The *total irradiation* G $(\mathrm{W/m^2})$ encompasses all spectral contributions and may be evaluated from Equation 12.19.

In the most general situation the irradiation interacts with a *semitransparent medium,* such as a layer of water or a glass plate. As shown in Figure 12.20 for a spectral component of the irradiation, portions of this radiation may be *reected, absorbed,* and *transmitted.* From a radiation balance on the medium, it follows that

$$G_\lambda = G_{\lambda,\mathrm{ref}} + G_{\lambda,\mathrm{abs}} + G_{\lambda,\mathrm{tr}} \tag{12.46}$$

Absorptivity

It is implicit in the foregoing result that surfaces may exhibit selective absorption with respect to the wavelength and direction of the incident radiation. For most engineering calculations, however, it is desirable to work with surface properties that represent directional averages. We therefore define a *spectral, hemispherical absorptivity* $\alpha_\lambda(\lambda)$ as

$$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,\text{abs}}(\lambda)}{G_\lambda(\lambda)} \tag{12.48}$$

The *total, hemispherical absorptivity,* α, represents an integrated average over both direction and wavelength. It is defined as the fraction of the total irradiation absorbed by a surface

$$\alpha \equiv \frac{G_{\text{abs}}}{G} \tag{12.51}$$

and, from Equations 12.19 and 12.48, it may be expressed as

$$\alpha = \frac{\int_0^\infty \alpha_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} \tag{12.52}$$

Reflectivity

The *spectral, hemispherical reectivity* $\rho_\lambda(\lambda)$ is then defined as the fraction of the spectral irradiation that is reflected by the surface. Accordingly,

$$\rho_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{ref}}(\lambda)}{G_\lambda(\lambda)} \tag{12.55}$$

The *total, hemispherical reectivity* ρ is then defined as

$$\rho \equiv \frac{G_{\mathrm{ref}}}{G} \tag{12.57}$$

in which case

$$\rho = \frac{\int_0^\infty \rho_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} \tag{12.58}$$

Transmissivity

Although treatment of the response of a semitransparent material to incident radiation is a complicated problem [7], reasonable results may often be obtained through the use of hemispherical transmissivities defined as

$$\tau_\lambda = \frac{G_{\lambda,\mathrm{tr}}(\lambda)}{G_\lambda(\lambda)} \tag{12.59}$$

and

$$\tau = \frac{G_{\mathrm{tr}}}{G} \tag{12.60}$$

The total transmissivity τ is related to the spectral component τ_λ by

$$\tau = \frac{\int_0^\infty G_{\lambda,\mathrm{tr}}(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} = \frac{\int_0^\infty \tau_\lambda(\lambda) G_\lambda(\lambda)\, d\lambda}{\int_0^\infty G_\lambda(\lambda)\, d\lambda} \tag{12.61}$$

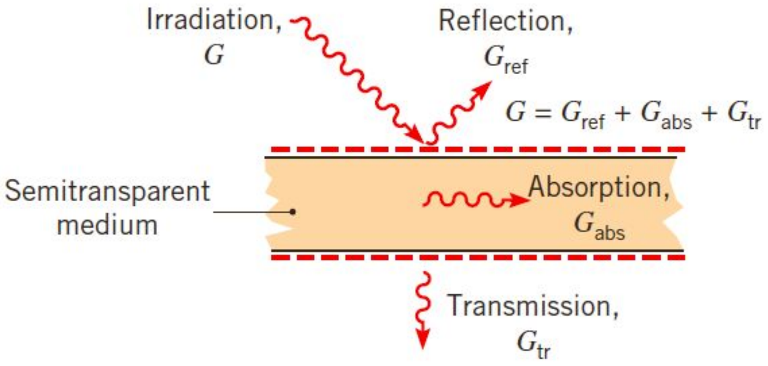
From the radiation balance of Equation 12.46 and the foregoing definitions,

$$\rho_\lambda + \alpha_\lambda + \tau_\lambda = 1 \tag{12.62}$$

for a *semitransparent* medium. This is analogous to Equation 12.2 but on a spectral basis. Of course, if the medium is *opaque*, there is no transmission, and absorption and reflection are surface processes for which

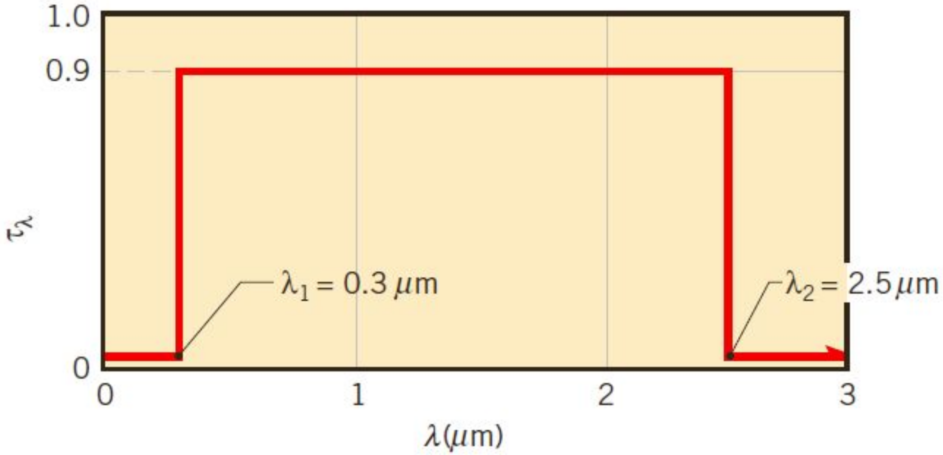
$$\alpha_\lambda + \rho_\lambda = 1 \tag{12.63}$$

which is analogous to Equation 12.3. Hence knowledge of one property implies knowledge of the other.



EXAMPLE 12.9

The cover glass on a flat-plate solar collector has a low iron content, and its spectral transmissivity may be approximated by the following distribution.



What is the total transmissivity of the cover glass to solar radiation?

SOLUTION

Known: Spectral transmissivity of solar collector cover glass.

Find: Total transmissivity of cover glass to solar radiation.

Assumptions: Spectral distribution of solar irradiation is proportional to that of blackbody emission at 5800 K.

Analysis: From Equation 12.61 the total transmissivity of the cover is

$$\tau = \frac{\int_0^\infty \tau_\lambda G_\lambda \, d\lambda}{\int_0^\infty G_\lambda \, d\lambda}$$

where the irradiation G_λ is due to solar emission. Having assumed that the sun emits as a blackbody at 5800 K, it follows that

$$G_\lambda(\lambda) \propto E_{\lambda,b}(5800\ \mathrm{K})$$

With the proportionality constant canceling from the numerator and denominator of the expression for τ, we obtain

$$\tau = \frac{\displaystyle\int_0^\infty \tau_\lambda E_{\lambda,b}(5800\ \mathrm{K})\, d\lambda}{\displaystyle\int_0^\infty E_{\lambda,b}(5800\ \mathrm{K})\, d\lambda}$$

or, for the prescribed spectral distribution of $\tau_\lambda(\lambda)$,

$$\tau = 0.90 \frac{\displaystyle\int_{0.3}^{2.5} E_{\lambda,b}(5800\ \mathrm{K})\, d\lambda}{E_b(5800\ \mathrm{K})}$$

From Table 12.2

$$\lambda_1 = 0.3\ \mu\text{m}, T = 5800\ \text{K}: \qquad \lambda_1 T = 1740\ \mu\text{m}\cdot\text{K}, F_{(0\rightarrow\lambda_1)} = 0.0335$$

$$\lambda_2 = 2.5\ \mu\text{m}, T = 5800\ \text{K}: \qquad \lambda_2 T = 14{,}500\ \mu\text{m}\cdot\text{K}, F_{(0\rightarrow\lambda_2)} = 0.9664$$

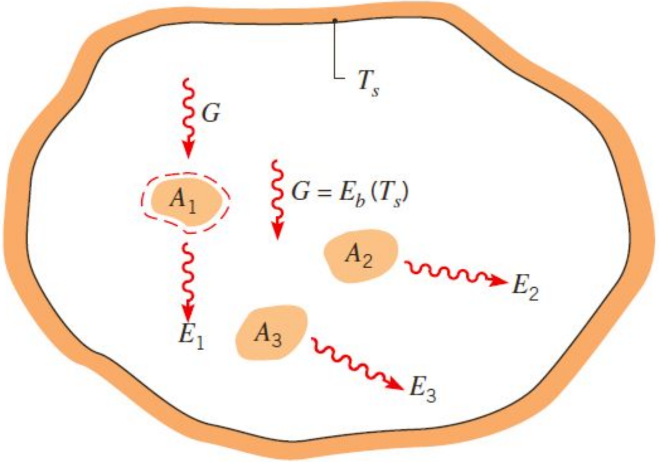
Hence from Equation 12.35

$$\tau = 0.90[F_{(0\rightarrow\lambda_2)} - F_{(0\rightarrow\lambda_1)}] = 0.90(0.9664 - 0.0335) = 0.84 \qquad \triangleleft$$

Comments: It is important to recognize that the irradiation at the cover plate is not equal to the emissive power of a blackbody at 5800 K, $G_\lambda \neq E_{\lambda,b}$ (5800 K). It is simply assumed to be proportional to this emissive power, in which case it is assumed to have a spectral distribution of the same form. With G_λ appearing in both the numerator and denominator of the expression for τ, it is then possible to replace G_λ by $E_{\lambda,b}$.

Kirchhoff's Law

Consider a *large, isothermal enclosure* of surface temperature T_s, within which several small bodies are confined (Figure 12.24). Since these bodies are small relative to the enclosure, they have a negligible influence on the radiation field, which is due to the cumulative effect of emission and reflection by the enclosure surface. Recall that, regardless of its radiative properties, such a surface forms a *blackbody cavity.* Accordingly, regardless of its orientation, the irradiation experienced by any body in the cavity is diffuse and equal to emission from a blackbody at T_s.



$$G = E_b(T_s) \qquad (12.64)$$

$$\alpha_1 G A_1 - E_1(T_s) A_1 = 0$$

$$\boxed{\varepsilon = \alpha}$$

$$\boxed{\varepsilon_\lambda = \alpha_\lambda}$$

Energy balance with a blackbody enclosure: thermal equilibrium

$$\alpha_\lambda(\lambda) \equiv \frac{G_{\lambda,\mathrm{abs}}(\lambda)}{G_\lambda(\lambda)} \Longrightarrow \alpha_\lambda G_\lambda = \varepsilon_\lambda E_{\lambda,b}(T_s) \Longrightarrow \alpha_\lambda = \varepsilon_\lambda$$

$$\alpha \equiv \frac{G_{\mathrm{abs}}}{G} \Longrightarrow \alpha G = \varepsilon E(T_s) \Longrightarrow \alpha = \varepsilon$$

A radiation energy balance can be done either over the entire spectrum of wavelengths or considering only spectral quantities. In the case the spectral energy balance is done, the finding $\varepsilon_\lambda = \alpha_\lambda$ is valid not only for the specific case of the example, but always because ε_λ and α_λ are inherent properties of a body.

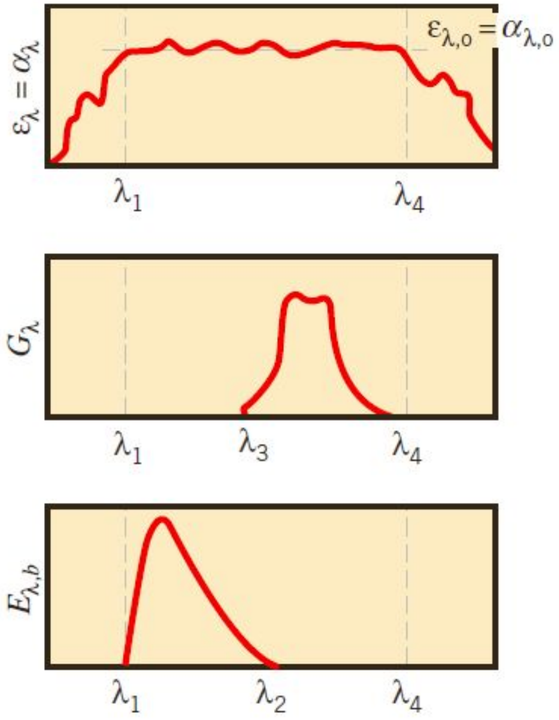
The Gray Surface

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b}(\lambda, T)\, d\lambda}{E_b(T)} \stackrel{?}{=} \frac{\int_0^\infty \alpha_\lambda G_\lambda(\lambda)\, d\lambda}{G} = \alpha \qquad (12.70)$$

Since $\varepsilon_\lambda = \alpha_\lambda$, it follows by inspection of Equation 12.70 that Equation 12.66 applies if *either* of the following conditions is satisfied:

1. The irradiation corresponds to emission from a blackbody at the surface temperature T, in which case $G_\lambda(\lambda) = E_{\lambda,b}(\lambda, T)$ and $G = E_b(T)$.
2. The *surface* is *gray* (α_λ and ε_λ are independent of λ).

To assume gray surface behavior, hence the validity of $\varepsilon = \alpha$, it is not necessary for ε_λ and α_λ to be independent of λ over the entire spectrum. Practically speaking, a gray surface may be defined as one for which ε_λ and α_λ are independent of λ over the spectral regions of the irradiation and the surface emission.



For example, take the sample surface in the figure. The spectral emissivity (and, therefore, the spectral absorptivity) vary with the wavelength. However, in a range that goes from **λ**1 to **λ**4, the spectral emissivity stays uniform.

Since the range **λ**1 - **λ**4 is where emissive power and irradiation are significant, it can be stated that the surface functions as a gray body in the range **λ**1 - **λ**4.

A flat-plate solar collector with no cover plate has a selective absorber surface of emissivity 0.1 and solar absorptivity 0.95. At a given time of day the absorber surface temperature T_s is 120°C when the solar irradiation is 750 W/m^2, the effective sky temperature is -10°C, and the ambient air temperature T_∞ is 30°C. Assume that the heat transfer convection coefficient for the calm day conditions can be estimated from

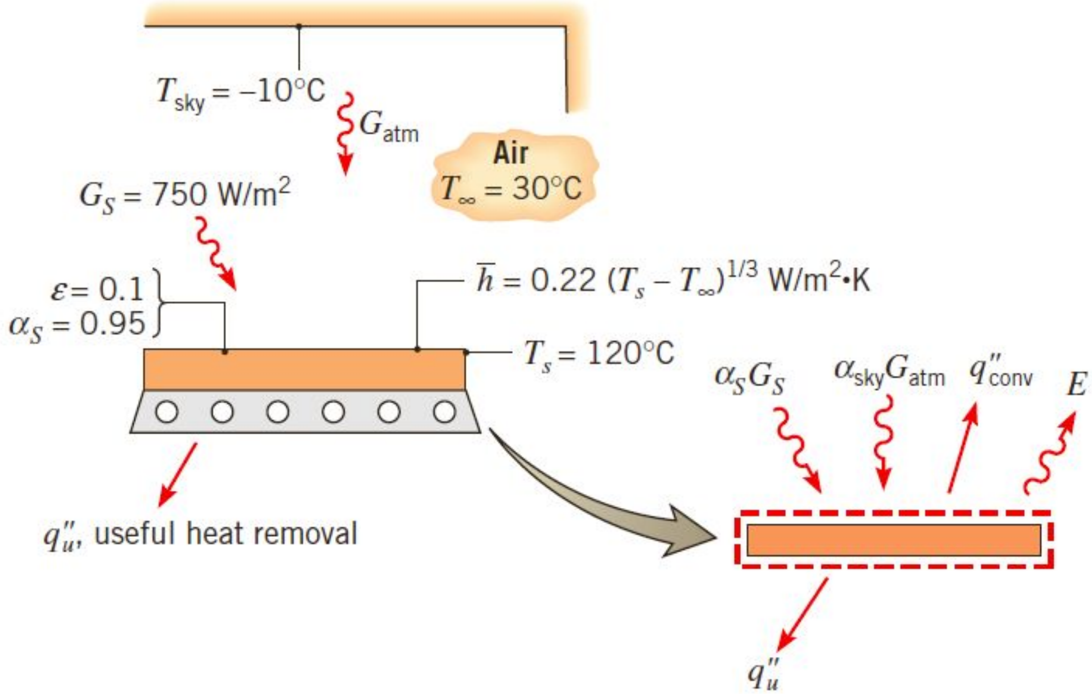
$$\bar{h} = 0.22(T_s - T_\infty)^{1/3}\ \mathrm{W/m^2 \cdot K}$$

Calculate the useful heat removal rate (W/m^2) from the collector for these conditions. What is the corresponding efficiency of the collector?

Known: Operating conditions for a flat-plate solar collector.

Find:

1. Useful heat removal rate per unit area, q''_u (W/m^2).
2. Efficiency η of the collector.



$$\alpha_S G_S + \alpha_{sky} G_{atm} - q''_{conv} - E - q''_u = 0$$

$$G_{\text{atm}} = \sigma T_{\text{sky}}^4$$

Since the atmospheric irradiation is concentrated in approximately the same spectral region as that of surface emission, it is reasonable to assume that

$$\alpha_{\text{sky}} \approx \varepsilon = 0.1$$

With

$$q''_{\text{conv}} = \bar{h}(T_s - T_\infty) = 0.22(T_s - T_\infty)^{4/3} \qquad \text{and} \qquad E = \varepsilon\sigma T_s^4$$

it follows that

$$q''_u = \alpha_S G_S + \varepsilon\sigma T_{\text{sky}}^4 - 0.22(T_s - T_\infty)^{4/3} - \varepsilon\sigma T_s^4$$

$$q''_u = \alpha_S G_S - 0.22(T_s - T_\infty)^{4/3} - \varepsilon\sigma(T_s^4 - T_{\text{sky}}^4)$$

$$q''_u = 0.95 \times 750\ \text{W/m}^2 - 0.22(120 - 30)^{4/3}\ \text{W/m}^2$$

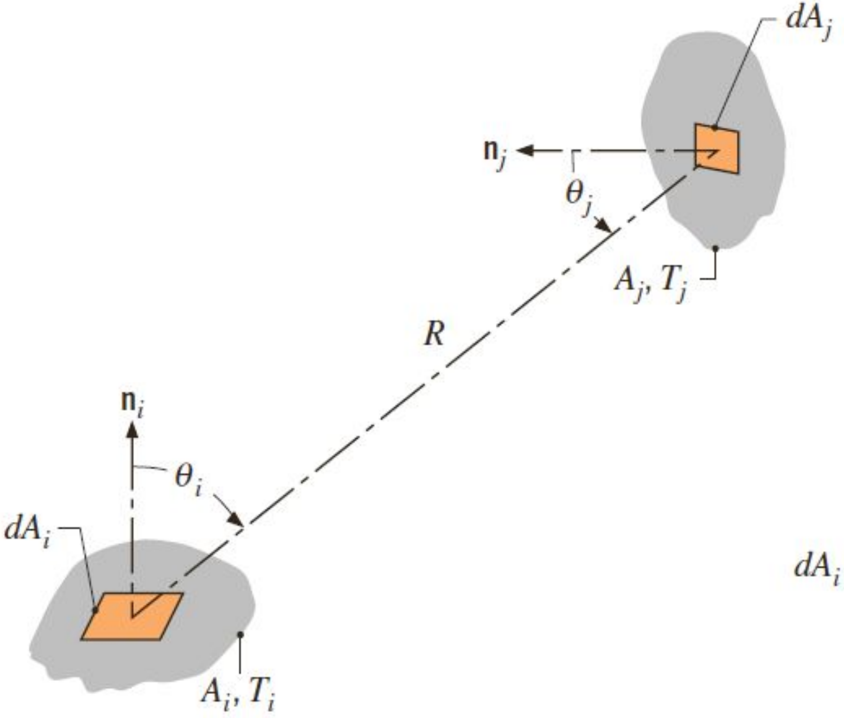
$$-0.1 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4(393^4 - 263^4)\ \text{K}^4$$

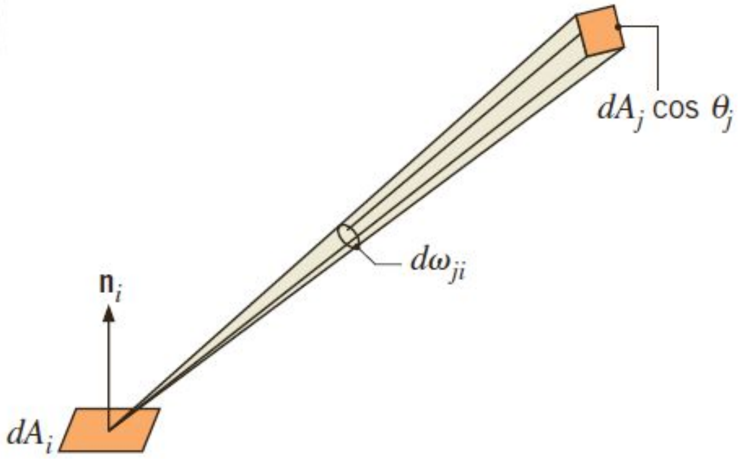
$$q''_u = (712.5 - 88.7 - 108.1)\ \text{W/m}^2 = 516\ \text{W/m}^2 \qquad \triangleleft$$

The collector efficiency, defined as the fraction of the solar irradiation extracted as useful energy, is then

$$\eta = \frac{q''_u}{G_S} = \frac{516\ \text{W/m}^2}{750\ \text{W/m}^2} = 0.69 \qquad \triangleleft$$

The View Factor





From the definition of the radiation intensity, Section 12.3.2, and Equation 12.11, the rate at which radiation *leaves* dA_i and is *intercepted* by dA_j may be expressed as

$$dq_{i\rightarrow j} = I_{e+r,i} \cos \theta_i \, dA_i \, d\omega_{j-i}$$

where $I_{e+r,i}$ is the intensity of radiation leaving surface i by emission and reflection and $d\omega_{j-i}$ is the solid angle subtended by dA_j when viewed from dA_i. With $d\omega_{j-i} = (\cos \theta_j \, dA_j)/R^2$ from Equation 12.7, it follows that

$$dq_{i\rightarrow j} = I_{e+r,i} \frac{\cos \theta_i \cos \theta_j}{R^2} dA_i \, dA_j$$

Assuming that surface i *emits* and *reects diffusely* and substituting from Equation 12.27, we then obtain

$$dq_{i\rightarrow j} = J_i \frac{\cos \theta_i \cos \theta_j}{\pi R^2} dA_i \, dA_j$$

The total rate at which radiation leaves surface i and is intercepted by j may then be obtained by integrating over the two surfaces. That is,

$$q_{i \to j} = J_i \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_i \, dA_j$$

where it is assumed that the radiosity J_i is uniform over the surface A_i. From the definition of the view factor as the fraction of the radiation that leaves A_i and is intercepted by A_j,

$$F_{ij} = \frac{q_{i \to j}}{A_i J_i}$$

it follows that

$$F_{ij} = \frac{1}{A_i} \int_{A_i} \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi R^2} \, dA_i \, dA_j \tag{13.1}$$

View Factor Relations

An important view factor relation is suggested by Equations 13.1 and 13.2. In particular, equating the integrals appearing in these equations, it follows that

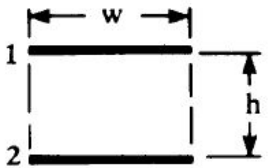
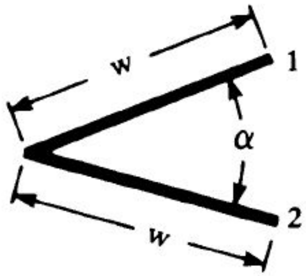
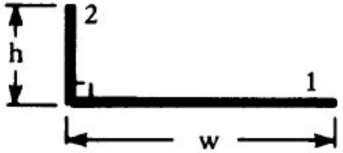
$$A_i F_{ij} = A_j F_{ji} \tag{13.3}$$

This expression, termed the *reciprocity relation*, is useful in determining one view factor from knowledge of the other.

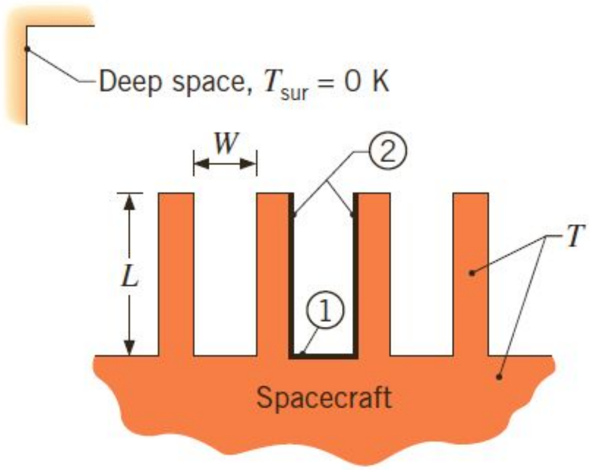
Another important view factor relation pertains to the surfaces of an *enclosure* (Figure 13.2). From the definition of the view factor, the *summation rule*

$$\sum_{j=1}^{N} F_{ij} = 1 \tag{13.4}$$

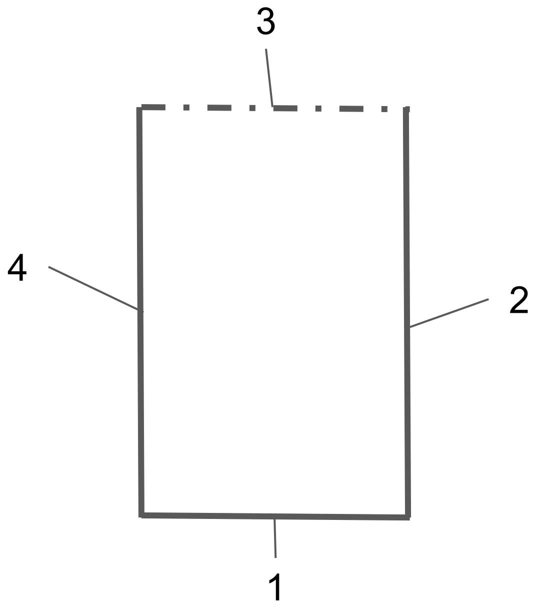
Table 10.2 View factors for a variety of two-dimensional configurations (infinite in extent normal to the paper)

Configuration		*Equation*
1.		$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$
2.		$F_{1\text{-}2} = F_{2\text{-}1} = 1 - \sin(\alpha/2)$
3.		$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$

To enhance heat rejection from a spacecraft, an engineer proposes to attach an array of rectangular fins to the outer surface of the spacecraft and to coat all surfaces with a material that approximates blackbody behavior.



Consider the U-shaped region between adjoining fins and subdivide the surface into components associated with the base (1) and the side (2). Obtain an expression for the rate per unit length at which radiation is transferred from the surfaces to deep space, which may be approximated as a blackbody at absolute zero temperature. The fins and the base may be assumed to be isothermal at a temperature T. Comment on your result. Does the engineer's proposal have merit?

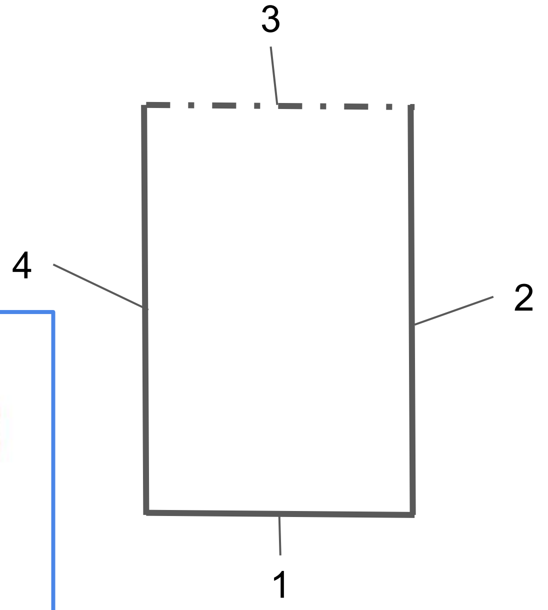


$$Q_{1,net} = F_{1\text{-}3} * E_1$$

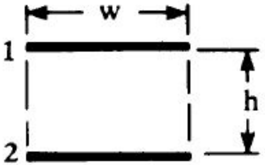
$$Q_{2,net} = F_{2\text{-}3} * E_1 = Q_{3,net}$$

$$Q_{tot} = Q_{1,net} + Q_{2,net} + Q_{3,net} = (F_{1\text{-}3} + 2 * F_{2\text{-}3}) * E1$$

$$\boxed{F_{2\text{-}3} = F_{4\text{-}3}}$$

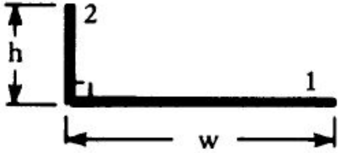


1.



$$F_{1\text{-}2} = F_{2\text{-}1} = \sqrt{1 + \left(\frac{h}{w}\right)^2} - \left(\frac{h}{w}\right)$$

3.



$$F_{1\text{-}2} = \frac{1}{2}\left[1 + \frac{h}{w} - \sqrt{1 + \left(\frac{h}{w}\right)^2}\right]$$

$$\boxed{F_{1\text{-}3} + 2 * F_{2\text{-}3} = 1}$$

The fact that the overall view factor fins-space is equal to 1, means that the heat exchange is equivalent to a surface without fins.