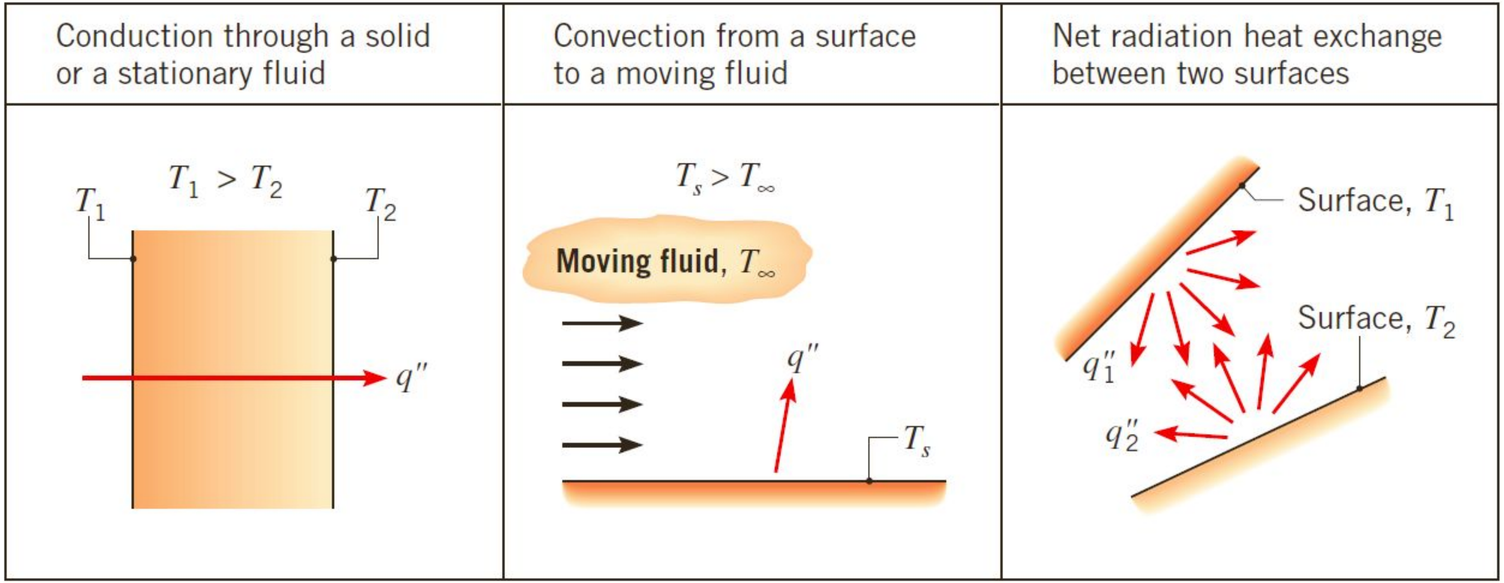
Heat transfer (or heat) is thermal energy in transit due to a spatial temperature difference.



Problem 1.1

A composite wall consists of alternate layers of fir (5 cm thick), aluminum (1 cm thick), lead (1 cm thick), and corkboard (6cm thick). The temperature is 60˚C on the outside of the fir and 10˚C on the outside of the corkboard. Plot the temperature gradient through the wall. Does the temperature profile suggest any simplifying assumptions that might be made in the subsequent analysis of the wall?

Data:
Fir: L=5cm , k=0.12 W/mK

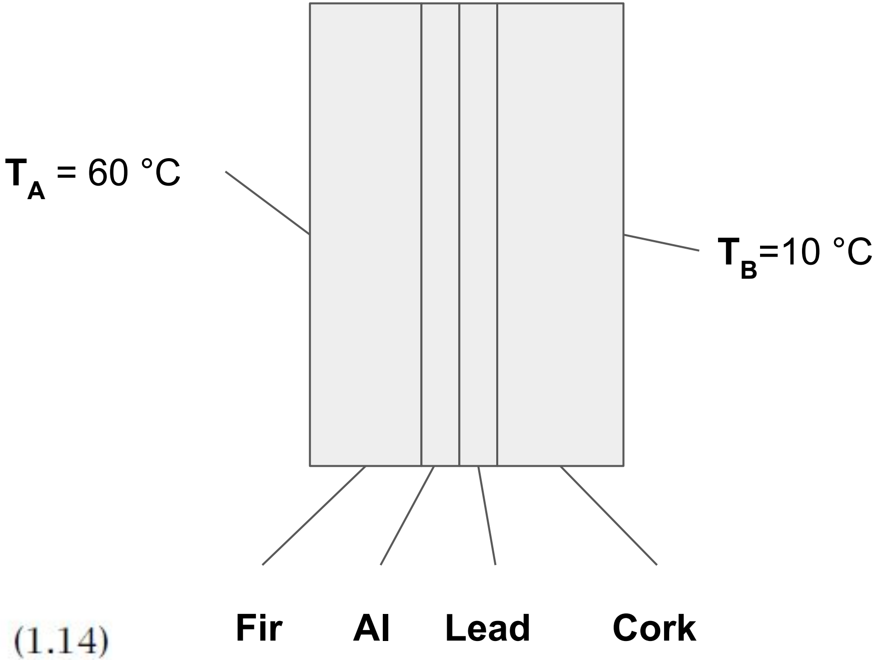
Al: L=1cm, k=237 W/mK

Lead: L=1cm, k=35 W/mK

Corkboard: L=6cm, k=0.04 W/mK

Boundary conditions
T_A = 60 °C , T_B=10 °C

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{\rho c}{k}\frac{\partial T}{\partial t} \equiv \frac{1}{\alpha}\frac{\partial T}{\partial t}} \qquad (1.14)$$



For the thermal conductivity, use the mean temperature:

(60+10)/2 °C = 35 °C

Table A.1 Pro

Metal	ρ (kg/m^3)	c_p (J/kg·K)	k (W/m·K)	α (10^{-5} m^2/s)
	Properties at 20°C			
Stainless steels:				
AISI 304	8,000	400	13.8	0.4
AISI 316	8,000	460	13.5	0.37
AISI 347	8,000	420	15	0.44
AISI 410	7,700	460	25	0.7
AISI 446	7,500	460		
Lead	11,373	130	35	2.34

Table A.2 Properties of nonmetallic solids

Material	*Temperature Range* (°C)	*Density* ρ (kg/m^3)	*Specific Heat* c_p (J/kg·K)	*Thermal Conductivity* k (W/m·K)	*Thermal Diffusivity* α (m^2/s)
Corkboard (medium ρ)	30	170		0.04	
Fir	15	600	2720	0.12	7.4×10^{-8}

1. Integration of the Conduction Equation

$\partial T/\partial t=0$
$\partial^2 T/\partial x^2=0 \Rightarrow$ linear temperature profile

You can integrate the equation in a homogenous domain.

$$\boxed{\begin{array}{l} T=Ax+B \\ dT/dx=\Delta T/L \\ \\ q=-k\,dT/dx=-k\cdot\Delta T/L \end{array}}$$

$q_i=-k_i\cdot\Delta T_i/L_i$

$L_i/k_i\cdot q_i=-\Delta T_i$

Where i is a single layer of a defined material

2. Application of the Energy Conservation

Conservation of energy requires that the steady heat fluxes through all the slabs must the same.

$q_{i-1}=q_i=q_{i+1}$

Doing the sum of the Fourier equation for all the elements:

$\sum L_i/k_i\cdot q_i=-\sum\Delta T_i$

$q\sum L_i/k_i=-\sum\Delta T_i$

$q=-\sum\Delta T_i/\sum L_i/k_i$

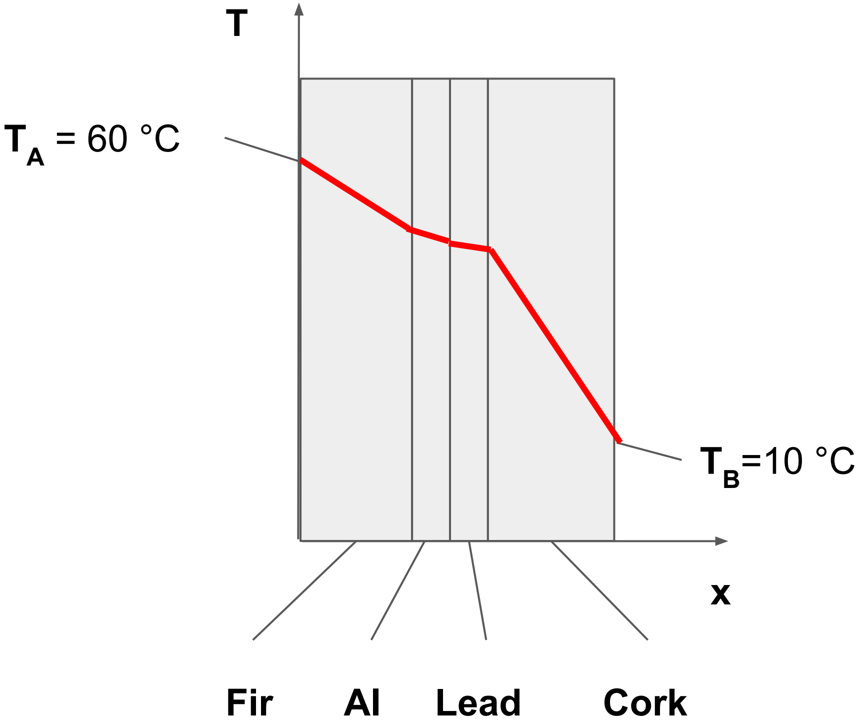
The heat flux is:

$q=26$ W/m^2

3. Calculation of the temperature drops in each layer

Using the following equation for each layer, all the temperature drops are obtained.

$L_i/k_i \cdot q_i = -\Delta T_i$



Thermal Resistance per unit area [m^2K/W] - R

Fir: 0.417; Al: 4.22e-5; Lead: 2.86e-4; Cork: 1.50

Total Specific Thermal Resistance = 1.92

Temperature Drop [K] - ΔT

Fir: 10.8; Al: 1.10e-3; Lead: 7.44e-3; Cork: 39.0

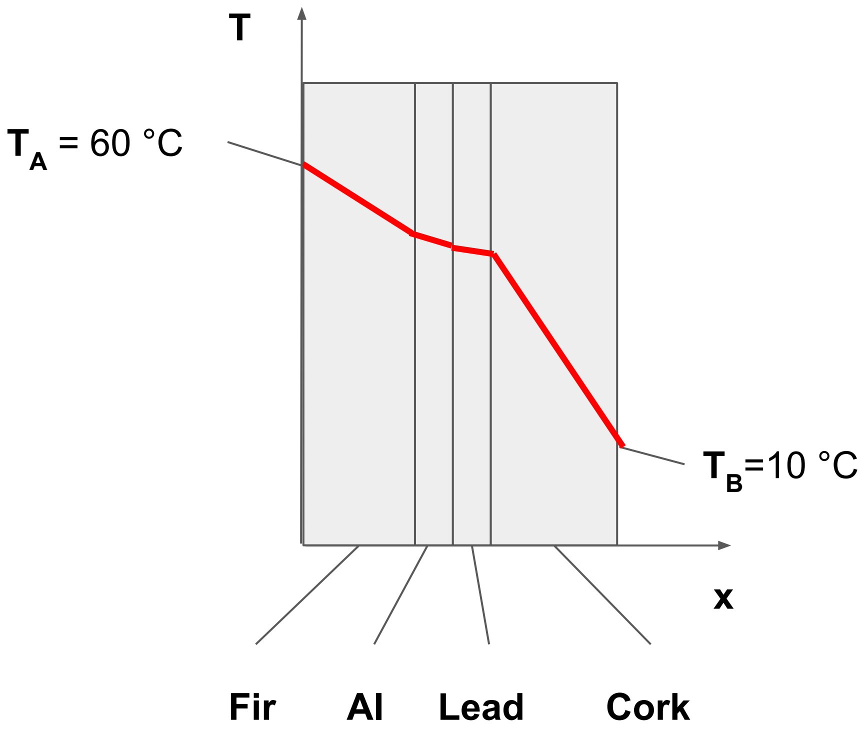
The minus sign in the Fourier equation

The minus sign appearing in the Fourier equation comes from the convention that energy going into a system is positive.

Convention means that it is an arbitrary choice, like it is an arbitrary choice the reference system (spatial and temporal).

The advice proposed here is to check the reference system and the slope of the temperature profile

$$\boxed{q=-k\partial T/\partial x}$$



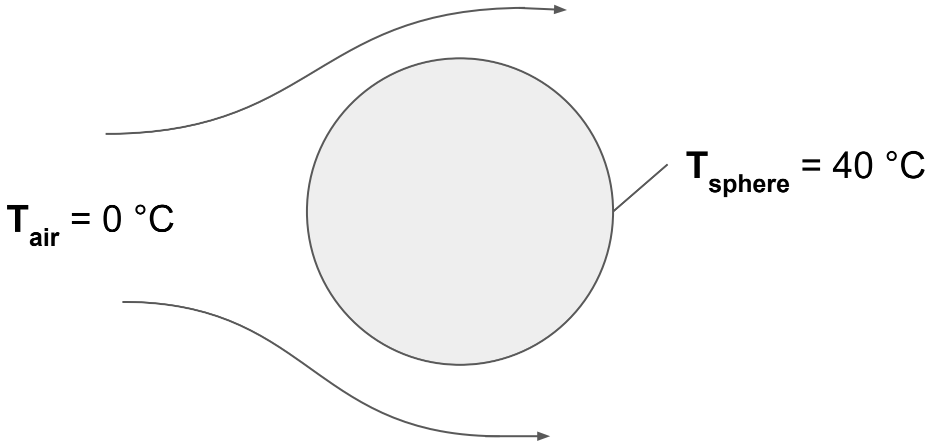
1.8 A copper sphere 2.5 cm in diameter has a uniform temperature of 40°C. The sphere is suspended in a slow-moving air stream at 0°C. The air stream produces a convection heat transfer coefficient of 15 W/m^{2}K. Radiation can be neglected. Since copper is highly conductive, temperature gradients in the sphere will smooth out rapidly, and its temperature can be taken as uniform throughout the cooling process (i.e., Bi < 1). Write the instantaneous energy balance between the sphere and the surrounding air. Solve this equation and plot the resulting temperatures as a function of time between 40°C and 0°C.

Data:
D = 2.5 cm ; T = 40 °C

k = 398 W/mK ;

ρ=8954 kg/m^3; c = 384 J/kgK;

Boundary conditions
Tair= 0 °C ; h = 15 W/m^{2}K



$$\boxed{Q = \frac{dU}{dt} = mc\frac{dT}{dt}} \quad (1.3)$$

$$\boxed{q = \overline{h}\left(T_{\text{body}} - T_{\infty}\right)} \quad (1.17)$$

Table A.1

Metal	Properties at 20°C ρ (kg/m^3)	c_p (J/kg·K)	k (W/m·K)	α (10^{-5} m^2/s)
Aluminums				
Pure	2,707	905	237	9.61
99% pure			211	
Duralumin (≈4% Cu, 0.5% Mg)	2,787	883	164	6.66
Alloy 6061-T6	2,700	896	167	6.90
Alloy 7075-T6	2,800	841	130	5.52
Chromium	7,190	453	90	2.77
Cupreous metals				
Pure Copper	8,954	384	398	11.57
DS-C15715*	8,900	≈384	365	≈10.7
Beryllium copper (2.2% Be)	8,250	420	103	2.97
Brass (30% Zn)	8,522	385	109	3.32
Bronze (25% Sn)§	8,666	343	26	0.86
Constantan (40% Ni)	8,922	410	22	0.61
German silver (15% Ni, 22% Zn)	8,618	394	25	0.73
Gold	19,320	129	318	12.76
Ferrous metals				
Pure iron	7,897	447	80	2.26
Cast iron (4% C)	7,272	420	52	1.70
Steels (C ≤ 1.5%)‖				
AISI 1010††	7,830	434	64	1.88
0.5% carbon	7,833	465	54	1.47
1.0% carbon	7,801	473	43	1.17
1.5% carbon	7,753	486	36	0.97

The thermophysical properties, like density, specific heat and thermal conductivity, are a function of the temperature.

For the maximum accuracy, one can solve the equation with a guess value for the temperature, then proceeding with an iterative method.

Even more accurate are the numerical methods.

For the purpose of solving a pen-and-paper problem, a mean value of the temperature is sufficient to determine the thermophysical properties.

If a value of temperature is not provided in the table, a linear interpolation can be used.

1. Calculation of the Biot Number

$$\text{Bi} \equiv \frac{\overline{h}L}{k}$$

Bi = V/A · h_{fluid} / k_{solid}

V=4/3πR3

A=4πR2

Bi = R/3 · h_{fluid} / k_{solid} = 1.57e-4 << 0.01

The Biot Number is the ratio between the conductive and convective resistances. When the convective resistance is much greater than the conductive one, the temperature gradients of the solid body can be neglected.

2. Integration of the Energy Balance

$$\underbrace{Q}_{-\overline{h}A(T-T_\infty)} = \underbrace{\frac{dU}{dt}}_{\frac{d}{dt}[\rho c V(T-T_{\text{ref}})]} \tag{1.19}$$

where A and V are the surface area and volume of the body, T is the temperature of the body, $T = T(t)$, and T_{ref} is the arbitrary temperature at which U is defined equal to zero. Thus[5]

$$\frac{d(T-T_\infty)}{dt} = -\frac{\overline{h}A}{\rho c V}(T-T_\infty) \tag{1.20}$$

The general solution to this equation is

$$\ln(T-T_\infty) = -\frac{t}{(\rho c V/\overline{h}A)} + C \tag{1.21}$$

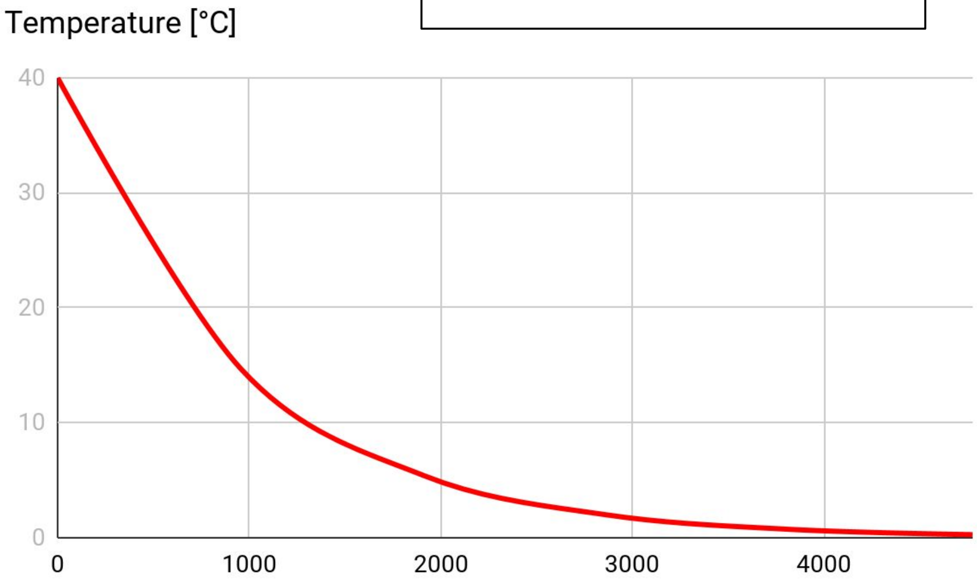
The group $\rho c V/\overline{h}A$ is the *time constant*, $\boldsymbol{T}$. If the initial temperature is $T(t=0) \equiv T_i$, then $C = \ln(T_i - T_\infty)$, and the cooling of the body is given by

$$\boxed{\frac{T-T_\infty}{T_i-T_\infty} = e^{-t/\boldsymbol{T}}} \tag{1.22}$$

Time Constant [s] - τ

$\tau = \rho c R/3h = 955$

$$T(t) = T_{air}+(T_i-T_{air})\exp(-t/\tau)$$

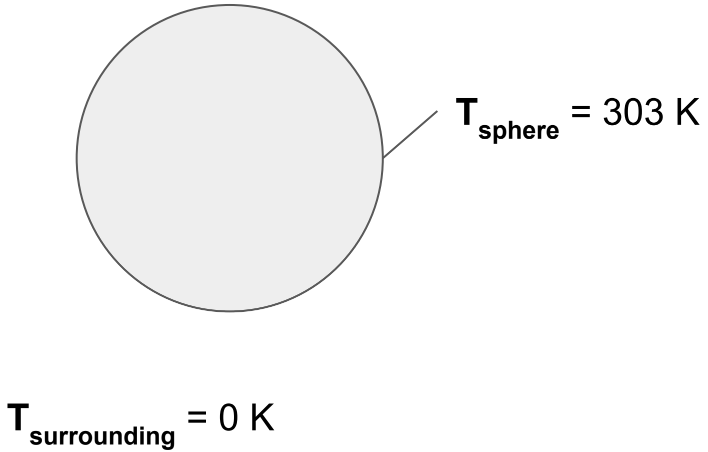


Time [s]	Temperature [°C]
τ	14.7
2τ	5.41
3τ	1.99
4τ	0.733
5τ	0.270

1.18 As part of a space experiment, a small instrumentation package is released from a space vehicle. It can be approximated as a solid aluminum sphere, 4 cm in diameter. The sphere is initially at 30°C and it contains a pressurized hydrogen component that will condense and malfunction at 30 K. If we take the surrounding space to be at 0 K, how long may we expect the implementation package to function properly? Is it legitimate to use the lumped-capacity method in solving the problem? (*Hint:* See the directions for Problem 1.17.) [Time = 5.8 weeks.]

Data:
D = 4 cm ; k = 242 W/mK
ρ=2707 kg/m^3; c = 905 J/kgK;
Ts = 30 °C (Initial Condition)
Sphere and surrounding = black bodies

Boundary conditions
T_{final}= 30 K
$T_{surrounding}$ = 0 K



Radiant heat exchange. Suppose that a heated object (1 in Fig. 1.16a) radiates only to some other object (2) and that both objects are thermally black. All heat leaving object 1 arrives at object 2, and all heat arriving at object 1 comes from object 2. Thus, the net heat transferred from object 1 to object 2, Q_{net}, is the difference between $Q_{1\text{ to }2} = A_1 e_b(T_1)$ and $Q_{2\text{ to }1} = A_1 e_b(T_2)$

$$Q_{net} = A_1 e_b(T_1) - A_1 e_b(T_2) = A_1 \sigma \left(T_1^4 - T_2^4\right) \tag{1.31}$$

σ, is 5.670400×10^{-8} W/m^2·K^4

Table A.1 Properties of metallic solids

	Properties at 20°C				Thermal Conductivity, k (W/m·K)									
Metal	ρ (kg/m^3)	c_p (J/kg·K)	k (W/m·K)	α (10^{-5} m^2/s)	−170°C	−100°C	0°C	100°C	200°C	300°C	400°C	600°C	800°C	1000°C
Aluminums														
Pure	2,707	905	237	9.61	302	242	236	240	238	234	228	215	≈95 (liq.)	

For the density and the specific heat, usually there is no significant variation with the temperature.

For the thermal conductivity, use the mean temperature during the transient

(303+30)/2 K = 166.5 K = -108.5 °C

1. Calculation of the Biot Number

$$\mathrm{Bi} \equiv \frac{\overline{h}L}{k}$$

It is possible to define an radiation heat transfer coefficient (h_r) for the radiation.

$q=\varepsilon\sigma$
$(T^4_{sphere}-T^4_{surrounding})=h(T_{sphere}-T_{surrounding})$

$h_r=\varepsilon\sigma(T^2_{sphere}+T^2_{surrounding})(T_{sphere}+T_{surrounding})$

In this case:

$h_r=\sigma T^3_{sphere}$

So, the Biot number would be:

$Bi=R/3\cdot\sigma T^3_{sphere}/k$

$= 4.35e\text{-}5$

2. Integration of the Energy Balance

$dU/dt = Q_{net}$

$\rho cV\, dT/dt = \sigma A(T^4_{sphere}-T^4_{surrounding})$

$dT/T^4 = -\sigma A/\rho cV\, dt$

Integrating the energy balance with the limits of integration $[T_i, T]$, $[0, t]$:

$\frac{1}{3}(T^{-3} - T_i^{-3}) = \sigma A/\rho cV\cdot t$

t = 3.55e6 s = 987 hours = 41 days
= 5.86 weeks