

The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity $k_i = 0.046 \text{ W/m}\cdot\text{K}$ and thickness $L_i = 50 \text{ mm}$ and steel panels, each of thermal conductivity $k_p = 60 \text{ W/m}\cdot\text{K}$ and thickness $L_p = 3 \text{ mm}$. If the wall separates refrigerated air at $T_{\infty,i} = 4^\circ\text{C}$ from ambient air at $T_{\infty,o} = 25^\circ\text{C}$, what is the heat gain per unit surface area? Coefficients associated with natural convection at the inner and outer surfaces may be approximated as $h_i = h_o = 5 \text{ W/m}^2\cdot\text{K}$.

Steel (2 layers)

$K_{\text{steel}} = 60 \text{ W/mK}$, $L_{\text{steel}} = 3 \text{ mm}$.

Fiberglass

$K_f = 0.046 \text{ W/mK}$, $L_f = 50 \text{ mm}$.

Air (inner)

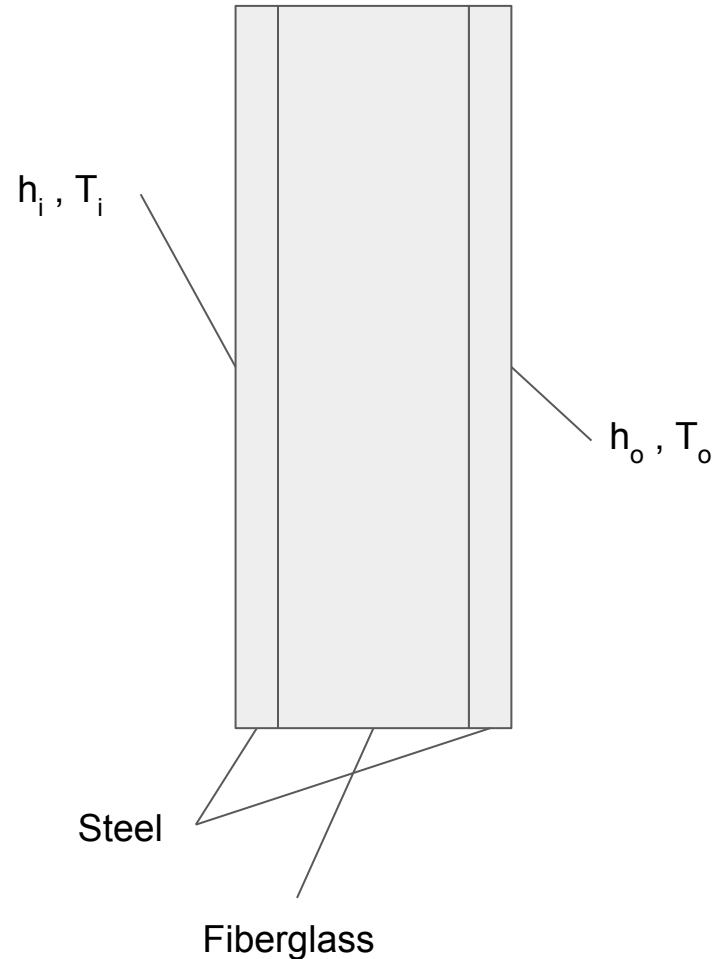
$h_i = 5 \text{ W/m}^2\text{K}$

$T_i = 4^\circ\text{C}$

Air (outer)

$h_o = 5 \text{ W/m}^2\text{K}$

$T_o = 20^\circ\text{C}$

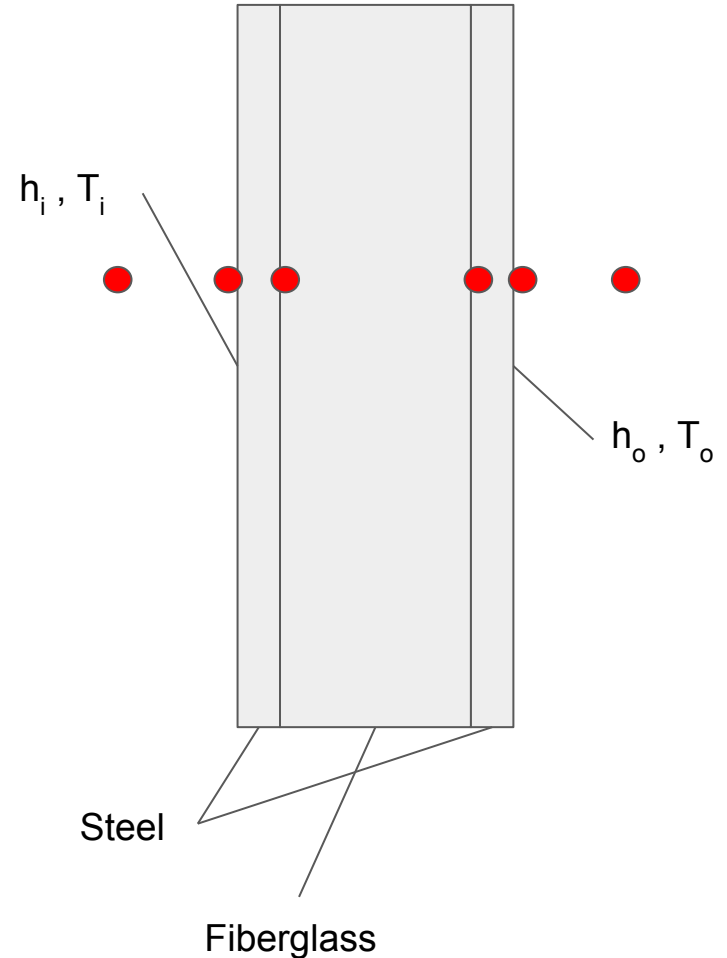


1. Identify the isothermal surfaces

The isothermal surfaces are identified with the help of the boundary conditions. The convection on the two sides of the sandwich defines two isothermal surfaces. Then, between each layer of the sandwich, there are the other four.

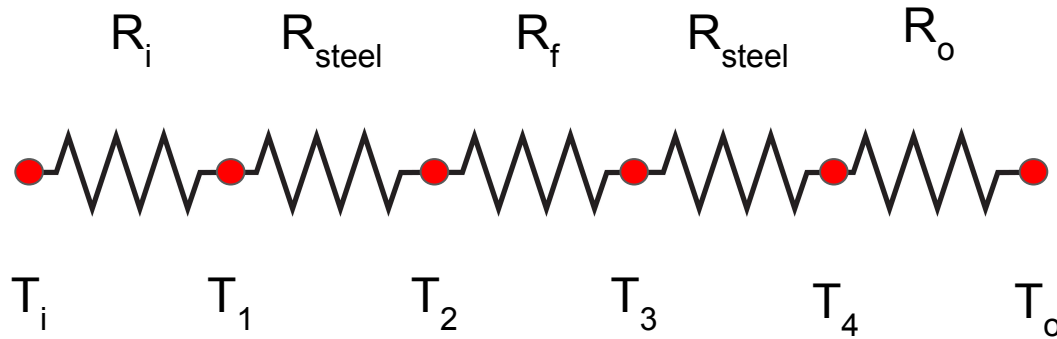
When between two isothermal surfaces there is only one contribution to the thermal resistance, then the scheme is in series.

In a scheme in series, the heat rate (or the flux) is the same for each resistance.



2. Calculate each thermal resistance

In the case of the sandwich, it is convenient to calculate directly the specific thermal resistance.



$$R_i = 1/h_i = 0.2 \text{ m}^2\text{K/W}$$

$$R_o = 1/h_o = 0.2 \text{ m}^2\text{K/W}$$

$$R_{\text{steel}} = L/k_{\text{steel}} = 5e-5 \text{ m}^2\text{K/W}$$

$$R_f = L/k_f = 1.09 \text{ m}^2\text{K/W}$$

3. Calculate the overall thermal resistance (or overall heat transfer coefficient)

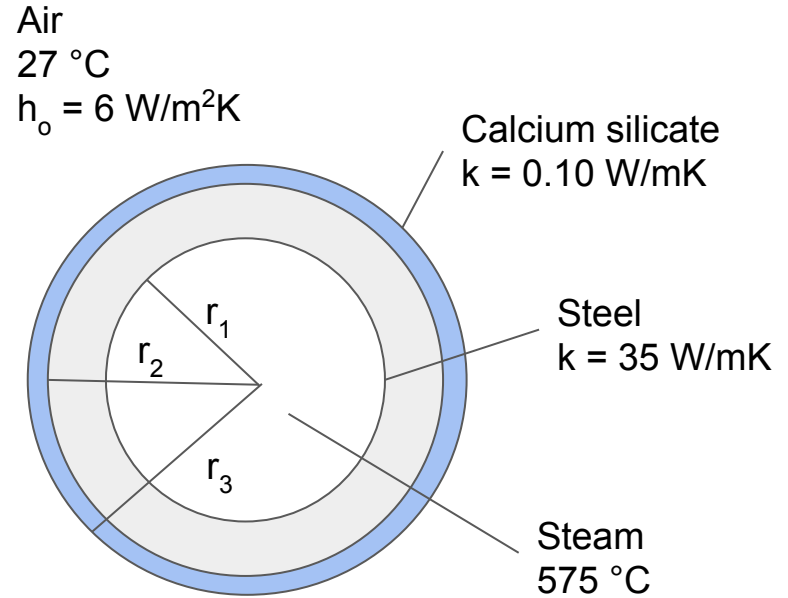
$$R_{\text{tot}} = \sum R = 1.49 \text{ K/m}^2\text{K}$$

$$U = 0.67 \text{ W/m}^2\text{K}$$

$$Q = U\Delta T_{\text{tot}} = 10.7 \text{ W/m}^2$$

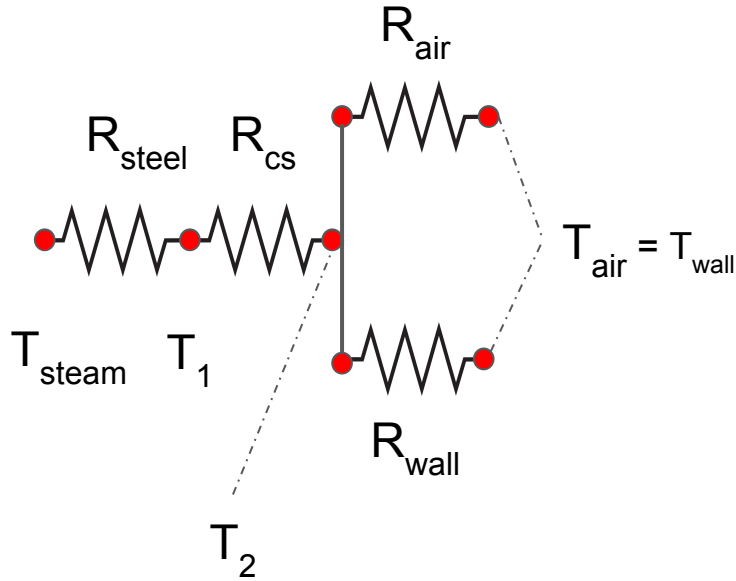
Superheated steam at 575°C is routed from a boiler to the turbine of an electric power plant through steel tubes ($k = 35 \text{ W/m}\cdot\text{K}$) of 300-mm inner diameter and 30-mm wall thickness. To reduce heat loss to the surroundings and to maintain a *safe-to-touch* outer surface temperature, a layer of calcium silicate insulation ($k = 0.10 \text{ W/m}\cdot\text{K}$) is applied to the tubes, while degradation of the insulation is reduced by wrapping it in a thin sheet of aluminum having an emissivity of $\varepsilon = 0.20$. The air and wall temperatures of the power plant are 27°C .

- (a) Assuming that the inner surface temperature of a steel tube corresponds to that of the steam and the convection coefficient outside the aluminum sheet is $6 \text{ W/m}^2\cdot\text{K}$, what is the minimum insulation thickness needed to ensure that the temperature of the aluminum does not exceed 50°C ? What is the corresponding heat loss per meter of tube length?

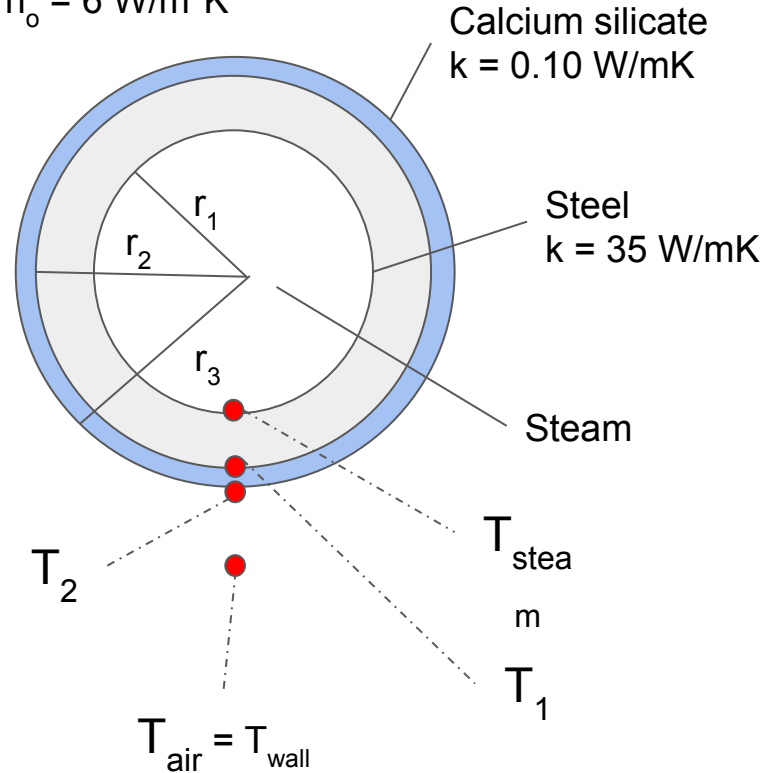


The thin sheet of aluminum can be neglected for the calculation of the conduction thermal resistance, but it can't be neglected for the radiation resistance.

1. Identify the isothermal surfaces



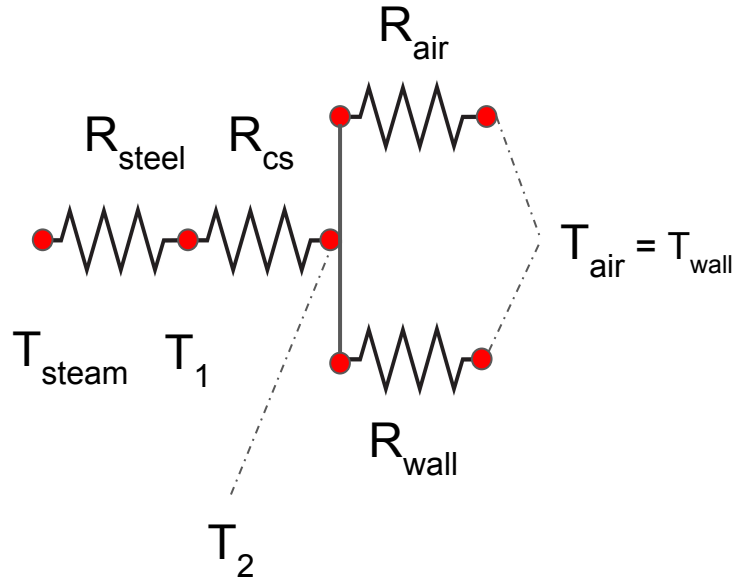
Air
27 °C
 $h_o = 6 \text{ W/m}^2\text{K}$



For the conduction, there is the usual scheme in series.

However, outside the tube there are two different mechanisms of heat transfer (convection and radiation). This scheme is called “parallel”.

2. Conduction thermal resistances and heat rate



$$R_{\text{steel}} = \ln(r_2/r_1)/(2\pi k_{\text{steel}}) = 8.29\text{e-}04 \text{ [mK/W]}$$

$$R_{\text{cs}} = \ln(r_3/r_2)/(2\pi k_{\text{cs}}) = ? \text{ [mK/W]}$$

$$R_{t_{\text{cyl}}} = \frac{\ln(r_o/r_i)}{2\pi l k} \quad (2.22)$$

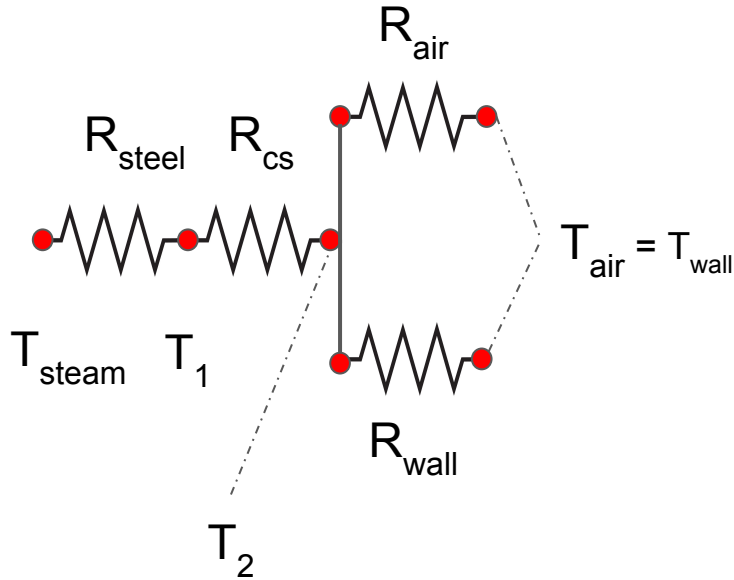
The general definition of conduction thermal resistance [K/W] in a tube is given in 2.22.

It is possible to define a specific conduction thermal resistance [$\text{m}^2\text{K/W}$] by multiplying 2.22 by the inner or outer surface.

Another choice would be defining a thermal resistance per unit tube length [mK/W] by multiplying 2.22 by the tube length.

$$Q_{\text{cond}} = (T_{\text{steam}} - T_2)/(R_{\text{steel}} + R_{\text{cs}})$$

3. Convective and radiative thermal resistances and heat rates



$$R_{\text{air}} = \Delta T/Q = 1/h \cdot 2\pi r_3 = ? \text{ [mK/W]}$$

$$R_{\text{wall}} = 1/(\epsilon\sigma \cdot 2\pi r_3 \cdot (T_2^2 + T_{\text{wall}}^2)(T_2 + T_{\text{wall}})) = ? \text{ [mK/W]}$$

To define a convective thermal resistance [K/W], start from the heat rate by convection.

$$Q = h \cdot A \cdot \Delta T = h \cdot 2\pi r_3 \cdot l \cdot \Delta T$$

$$R = \Delta T/Q = 1/h \cdot 2\pi r_3 \cdot l$$

It is possible to define a convective thermal resistance per unit tube length [mK/W]:

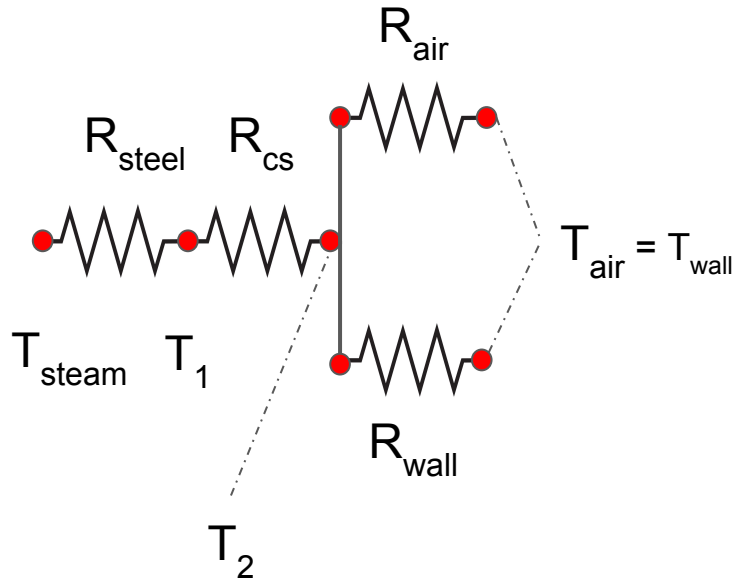
$$R_{\text{air}} = \Delta T/Q = 1/h \cdot 2\pi r_3$$

Same procedure for the radiation thermal resistance.

$$Q = \epsilon\sigma A(T_2^4 - T_{\text{wall}}^4) = \epsilon\sigma \cdot 2\pi r_3 \cdot l \cdot (T_2^4 - T_{\text{wall}}^4)$$

$$R_{\text{wall}} = 1/(\epsilon\sigma \cdot 2\pi r_3 \cdot (T_2^2 + T_{\text{wall}}^2)(T_2 + T_{\text{wall}}))$$

4. Energy balance



$$Q_{cond} = 420.37 \text{ W/m}$$

$$Q_{air} + Q_{wall} = 420.37 \text{ W/m}$$

The energy balance applied to the tube states that:

$$Q_{cond} = Q_{air} + Q_{wall}$$

$$Q_{cond} = (T_{steam} - T_2) / (8.29e-04 + \ln(r_3/r_2) / (2\pi k_{cs})) =$$

$$= 525 / (8.2907e-04 + \ln(r_3/r_2) / 0.6283)$$

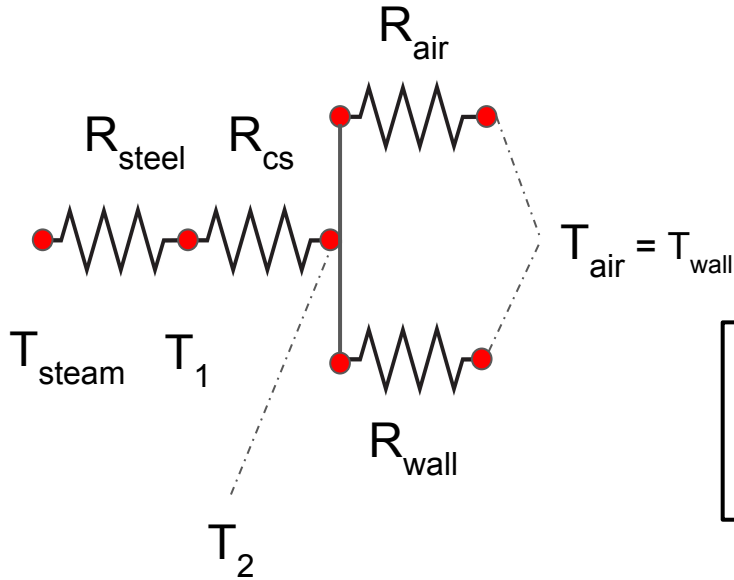
$$Q_{air} = h \cdot A \cdot \Delta T = h \cdot 2\pi r_3 \cdot (T_2 - T_{wall}) = 867.08 r_3$$

$$Q_{wall} = \epsilon \sigma A (T_2^4 - T_{wall}^4) = \epsilon \sigma \cdot 2\pi r_3 \cdot (T_2^4 - T_{wall}^4) =$$

$$= 199.04 \cdot r_3$$

$$r_3 = 0.39430 \text{ m; thickness} = 0.214 \text{ m}$$

5. Total thermal resistance



$$R_{\text{steel}} = \ln(r_2/r_1)/(2\pi k_{\text{steel}}) = 8.29\text{e-}04 \text{ [mK/W]}$$

$$R_{\text{cs}} = \ln(r_3/r_2)/(2\pi k_{\text{cs}}) = 1.2481 \text{ [mK/W]}$$

$$R_{\text{air}} = \Delta T/Q = 1/h \cdot 2\pi r_3 = 0.0672 \text{ [mK/W]}$$

$$R_{\text{wall}} = 1/(\epsilon\sigma \cdot 2\pi r_3 \cdot (T_2^2 + T_{\text{wall}}^2)(T_2 + T_{\text{wall}})) = 0.29033 \text{ [mK/W]}$$

$$R_{\text{tot}} = R_{\text{steel}} + R_{\text{cs}} + (R_{\text{air}}^{-1} + R_{\text{wall}}^{-1})^{-1} = 1.3035 \text{ [mK/W]}$$

Note: While in a series scheme the highest resistance is the dominant, in a parallel scheme the lowest resistance is the dominant.