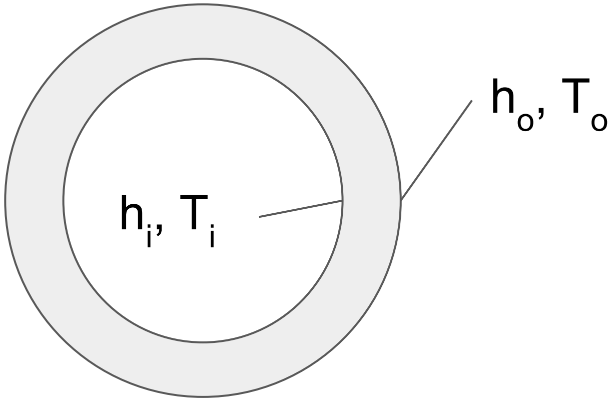
2.5

Solve for the temperature distribution in a thick-walled pipe if the bulk interior temperature and the exterior air temperature, $T_{\infty i}$, and $T_{\infty o}$, are known. The interior and the exterior heat transfer coefficients are h_i and h_o, respectively. Follow the method in Example 2.6 and put your result in the dimensionless form:

$$\frac{T - T_{\infty_i}}{T_{\infty_i} - T_{\infty_o}} = \mathrm{fn}\left(\mathrm{Bi}_i, \mathrm{Bi}_o, r/r_i, r_o/r_i\right)$$



The boundary conditions are uniform against the angle and axial directions (θ and z).

Also, there is no variation of the boundary conditions in time (steady state).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \underbrace{\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\hat{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}_{=0} \Longrightarrow \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

General solution:

$$\boxed{T = C_1 ln(r) + C_2}$$

Boundary Conditions:

$$\begin{cases} -k\frac{\partial T}{\partial r} = -h_i\left(T - T_{\infty,i}\right), & for\ r = r_i \\ -k\frac{\partial T}{\partial r} = h_o\left(T - T_{\infty,0}\right), & for\ r = r_o \end{cases}$$

$$\begin{cases} -k\frac{C_1}{r_i} = -h_i\left(C_1 ln(r_i) + C_2 - T_{\infty,i}\right) \\ -k\frac{C_1}{r_o} = h_o\left(C_1 ln(r_o) + C_2 - T_{\infty,0}\right) \end{cases}$$

$$C_1 = \frac{T_{\infty,o} - T_{\infty,i}}{ln\left(\frac{r_o}{r_i}\right) + \frac{k}{h_o r_o} + \frac{k}{h_i r_i}}$$

$$C_2 = C_1\left(\frac{k}{h_i r_i} - ln(r_i)\right) + T_{\infty,i}$$

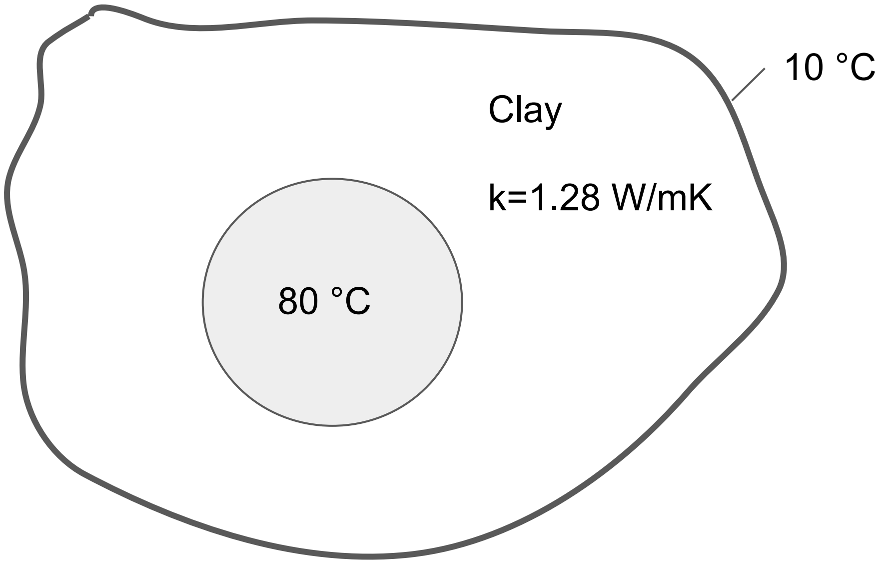
$$\boxed{\frac{T - T_{\infty,i}}{T_{\infty,o} - T_{\infty,i}} = \frac{ln\left(\frac{r}{r_i}\right) + \frac{k}{h_i r_i}}{ln\left(\frac{r_o}{r_i}\right) + \frac{k}{h_o r_o} + \frac{k}{h_i r_i}}}$$

$$Bi_o^{-1} = \frac{k}{h_o r_o} \qquad Bi_i^{-1} = \frac{k}{h_i r_i}$$

$$\boxed{\frac{T - T_{\infty,i}}{T_{\infty,o} - T_{\infty,i}} = \frac{\ln\left(\frac{r}{r_i}\right) + Bi_i^{-1}}{\ln\left(\frac{r_o}{r_i}\right) + Bi_o^{-1} + Bi_i^{-1}}}$$

2.15

An isothermal sphere 3 cm in diameter is kept at 80°C in a large clay region. The temperature of the clay far from the sphere is kept at 10°C. How much heat must be supplied to the sphere to maintain its temperature if $k_{clay} = 1.28$ W/m·K? (*Hint:* You must solve the boundary value problem not in the sphere but in the clay surrounding it.)
[$Q = 16.9$ W.]



The sphere is uniformly at 80 °C.

The boundary conditions are uniform against the two angles directions (θ and Φ).

Also, there is no variation of the boundary conditions in time (steady state).

$$\nabla^2 T + \cancel{\frac{\dot{q}}{k}} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right)} + \cancel{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = 0$$

Integrating twice

General solution:

$$\boxed{T = -\frac{C_1}{r} + C_2}$$

Boundary Conditions:

$$\begin{cases} T(r = r_s) = T_s \\ T(r = \infty) = T_\infty \end{cases} \Rightarrow \begin{cases} C_1 = (T_\infty - T_s) r_s \\ C_2 = T_\infty \end{cases}$$

Temperature function:

$$T = -\frac{(T_\infty - T_s) r_s}{r} + T_\infty$$

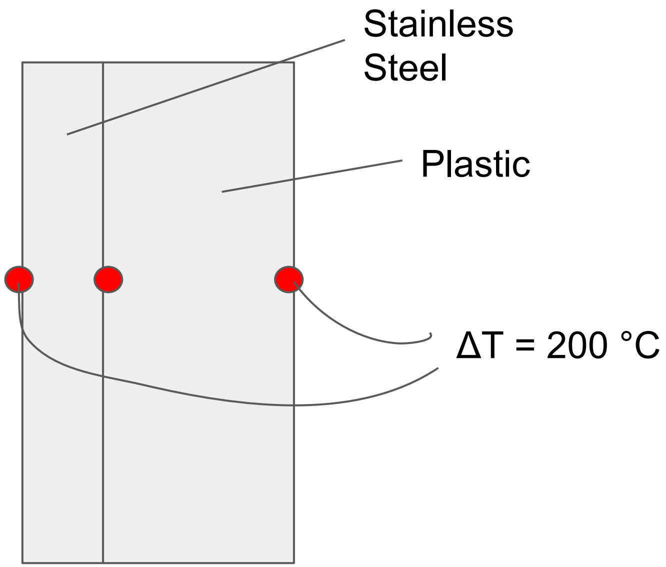
Heat Rate:

$$Q = qA = -4\pi r^2 k \frac{\partial T}{\partial r}$$

$$Q = -4\pi k (T_\infty - T_s) r_s = 16.9\ W$$

2.17

A wall consists of layers of metals and plastic with heat transfer coefficients on either side. U is 255 W/m^{2}K and the overall temperature difference is 200°C. One layer in the wall is stainless steel (k = 18 W/m·K) 3 mm thick. What is ΔT across the stainless steel?



$Q = UA\Delta T_{tot}$

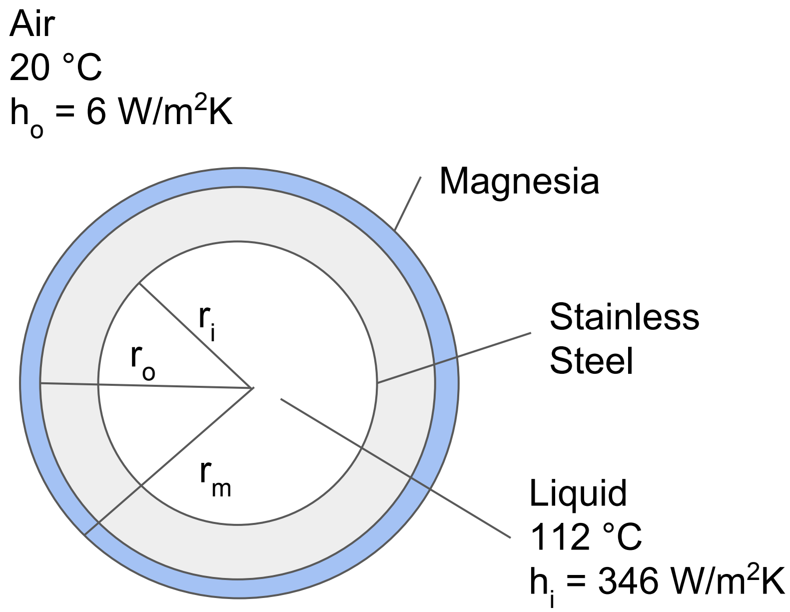
$q = U\Delta T_{tot} = 51e3\ W/m^2$

$q = k_{steel}/L\ \Delta T_{steel}$

$\Delta T_{steel} = qL/k_{steel} = 8.5\ K$

2.35

A type 316 stainless steel pipe has a 6 cm inside diameter and an 8 cm outside diameter with a 2 mm layer of 85% magnesia insulation around it. Liquid at 112°C flows inside, so hi = 346 W/m^{2}K. The air around the pipe is at 20°C, and h_0 = 6 W/m^{2}K. Calculate U based on the inside area. Sketch the equivalent electrical circuit, showing all known temperatures. Discuss the results.

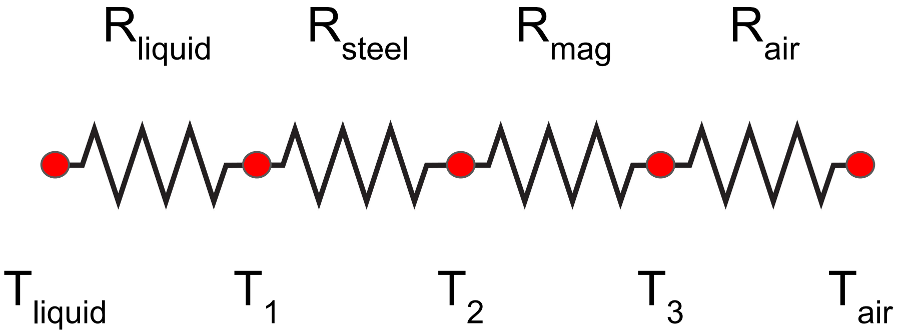


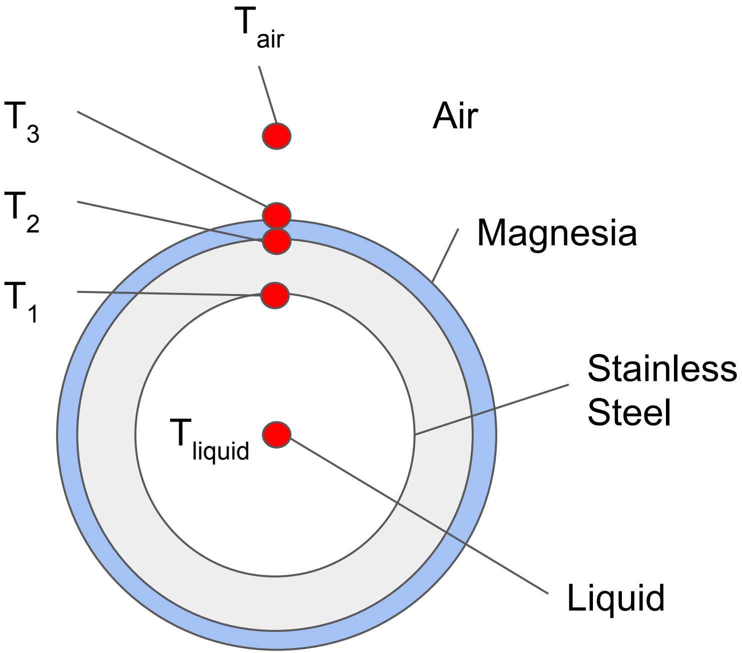
Stainless Steel 316
k = 15 W/mK
r_i = 3 cm
r_o = 4 cm

Magnesia 85%
k = 0.067 W/mK
r_m = 4.2 cm

$$Q=U\cdot A\cdot \Delta T$$

$$U=Q/(A\Delta T)=R_{tot}^{-1}$$





$R_{liquid} = 1/h_{liquid} =$ **2.89e-3** m^2K/W

$R_{steel} = r_i/k_{steel} \ln(r_o/r_i) =$ **5.75e-04** m^2K/W

$R_{mag} = r_i/k_{mag} \ln(r_m/r_o) =$ **0.022** m^2K/W

$R_{air} = r_i/(r_m h_{air}) =$ **0.12** m^2K/W

$R_{tot} = 0.1455\ m^2K/W$

$U = 6.87\ W/m^2K$