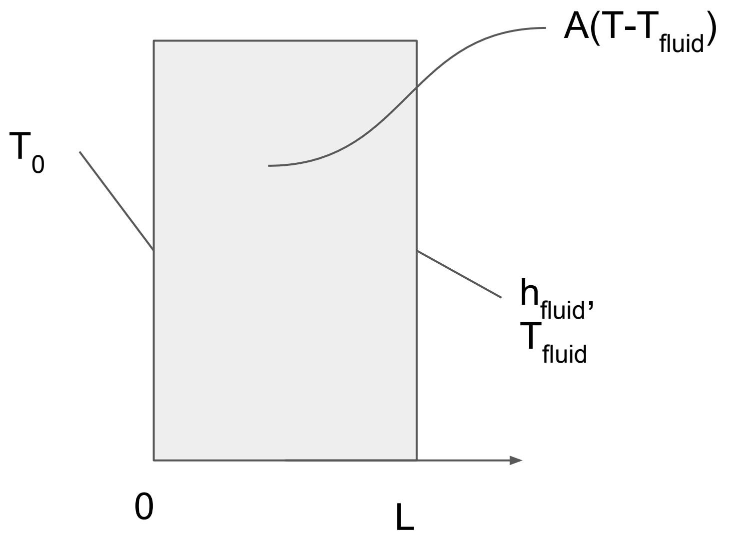
4.2

The left side of a slab of thickness L is kept at $0^{\circ}C$. The right side is cooled by air at $T_{\infty}{}^{\circ}C$ blowing on it. h_{RHS} is known. An exothermic reaction takes place in the slab such that heat is generated at $A(T - T_{\infty})$ W/m^3, where A is a constant. Find a fully dimensionless expression for the temperature distribution in the wall.



$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}_{\substack{= 0, \text{ since} \\ T \neq T(y \text{ or } z)}} + \frac{\dot{q}}{k} = \underbrace{\frac{1}{\alpha}\frac{\partial T}{\partial t}}_{\substack{= 0, \text{ since} \\ \text{steady}}}$$

$$\frac{d^2 T}{dx^2} = -\frac{A(T - T_{fluid})}{k}$$

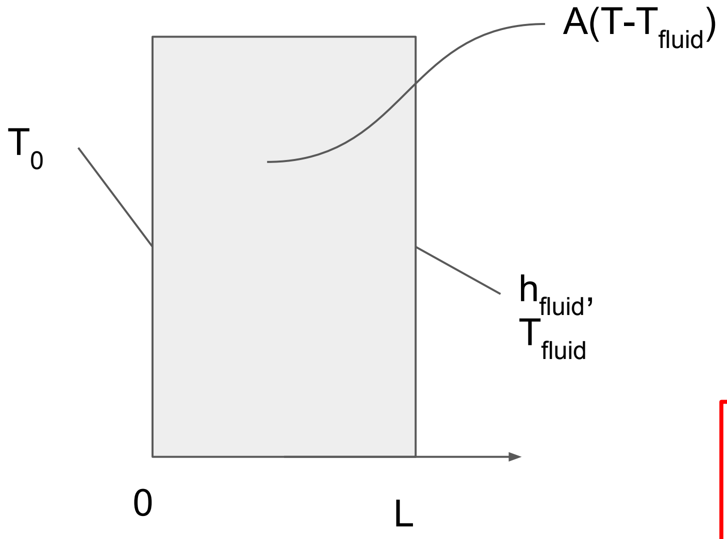
$$X = T - T_{fluid}$$

$$\frac{d^2 (T - T_{fluid})}{dx^2} = -\frac{A(T - T_{fluid})}{k}$$

$$\lambda = \sqrt{\frac{A}{k}}$$

$$X = C_1 sin(\lambda x) + C_2 cos(\lambda x)$$

$$X'' = -\lambda^2 X$$



$$Bi^{-1} = \frac{k\lambda}{h}$$

Boundary conditions

$$T(0) - T_{fluid} = T_0 - T_{fluid}$$

$$-k\frac{d(T-T_{fluid})}{dx}\Big|_L = h\left(T(L) - T_{fluid}\right)$$

Solution

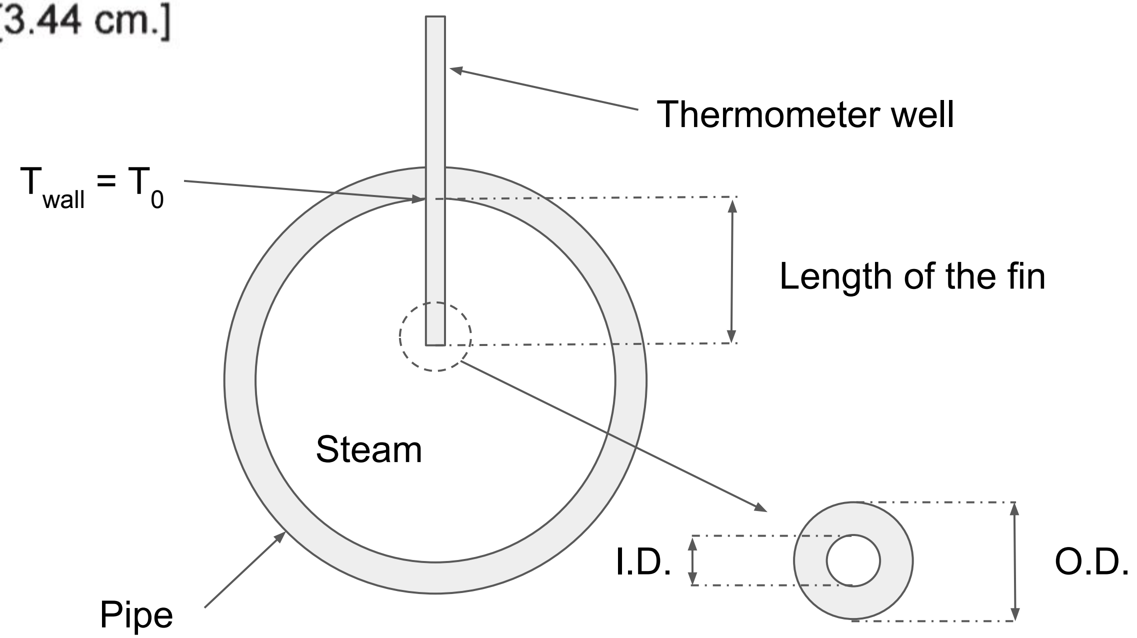
$$C_1 = (T_0 - T_{fluid})\frac{Bi^{-1}sin(\lambda L)-cos(\lambda L)}{Bi^{-1}cos(\lambda L)+sin(\lambda L)}$$

$$C_2 = T_0 - T_{fluid}$$

$$\frac{T-T_{fluid}}{T_0-T_{fluid}} = \frac{Bi^{-1}sin(\lambda L)-cos(\lambda L)}{Bi^{-1}cos(\lambda L)+sin(\lambda L)}sin(\lambda x) + cos(\lambda x)$$

4.13

What is the minimum length, *l*, of a thermometer well necessary to ensure an error less than 0.5% of the difference between the pipe wall temperature and the temperature of fluid flowing in a pipe? The well consists of a tube with the end closed. It has a 2 cm O.D. and a 1.88 cm I.D. The material is type 304 stainless steel. Assume that the fluid is steam at 260°C and that the heat transfer coefficient between the steam and the tube wall is 300 W/m^{2}K. [3.44 cm.]



DATA

T_{steam} = 260 °C
h_{steam} = 300 Wm2K

k_{steel} = 17 W/mK

$$\boxed{\Theta = \frac{\cosh mL(1-\xi)}{\cosh mL}} \qquad (4.41)$$

$$mL = \sqrt{\frac{\overline{h}P}{kA}L^2}$$

$$\Theta = \frac{T - T_\infty}{T_0 - T_\infty}$$

P is the perimeter of the fin in contact with the fluid

$P = \pi OD = 0.0628$ m

A is the cross-section area of the fin

$A = \pi(OD^2 - ID^2)/4 = 3.6568\text{e-}05\ m^2$

$m = \mathrm{sqrt}\ (hP/kA) = 174\ m^{-1}$

$\Theta = 0.005 = \cosh\ (mL)^{-1}$

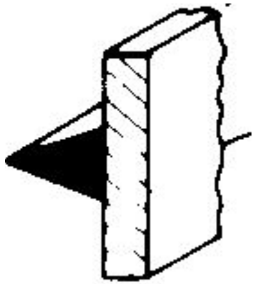
$\cosh\ (mL) = 200$

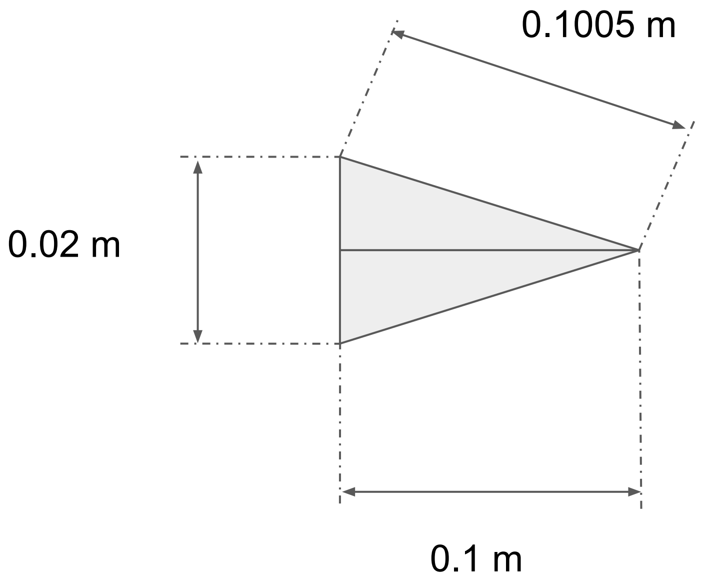
$mL = 5.99$

$L = 3.44$ cm

4.23

A fin of triangular axial section (cf. Fig. 4.12) 0.1 m in length and 0.02 m wide at its base is used to extend the surface area of a 0.5% carbon steel wall. If the wall is at 40°C and heated gas flows past at 200°C (h = 230 W/m^{2}K), compute the heat removed by the fin per meter of breadth, b, of the fin. Neglect temperature distortion at the root.

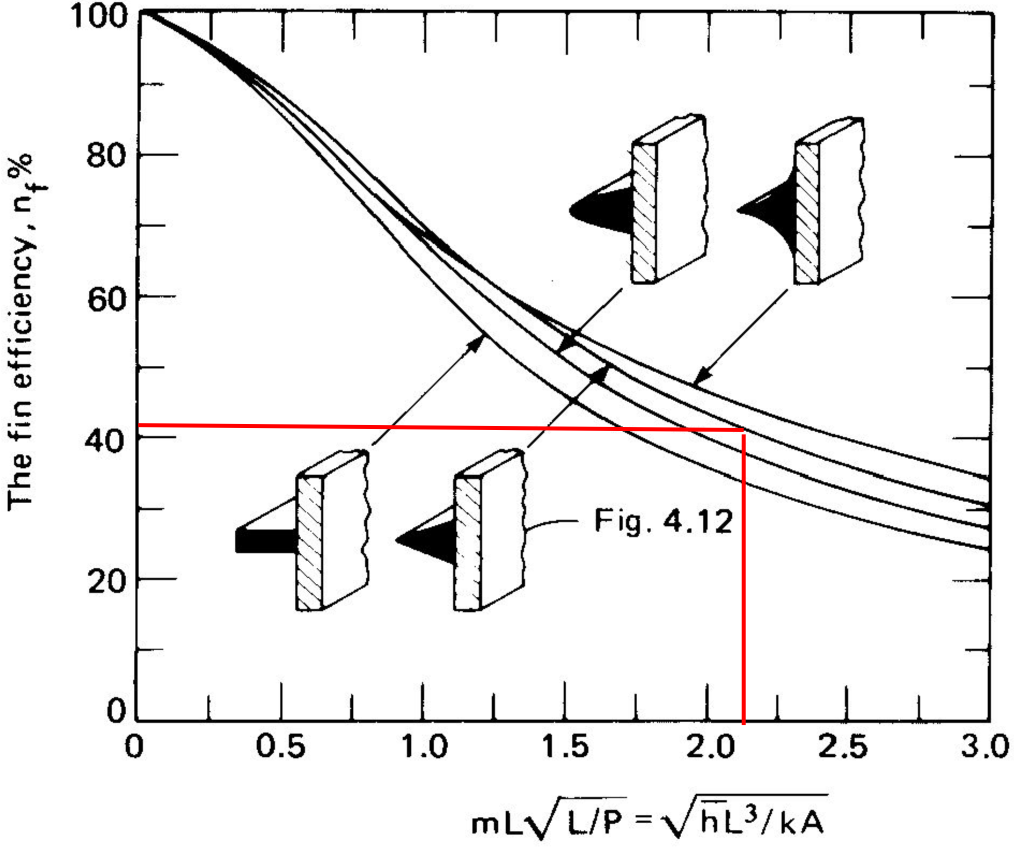




DATA

T_{gas} = 200 °C
h_{gas} = 230 Wm2K

k_{steel} = 52 W/mK



$A = W*L/2 = 0.001\ m^2$

$\mathrm{sqrt}(hL^3/kA) = 2.1$

$\eta = 0.41$

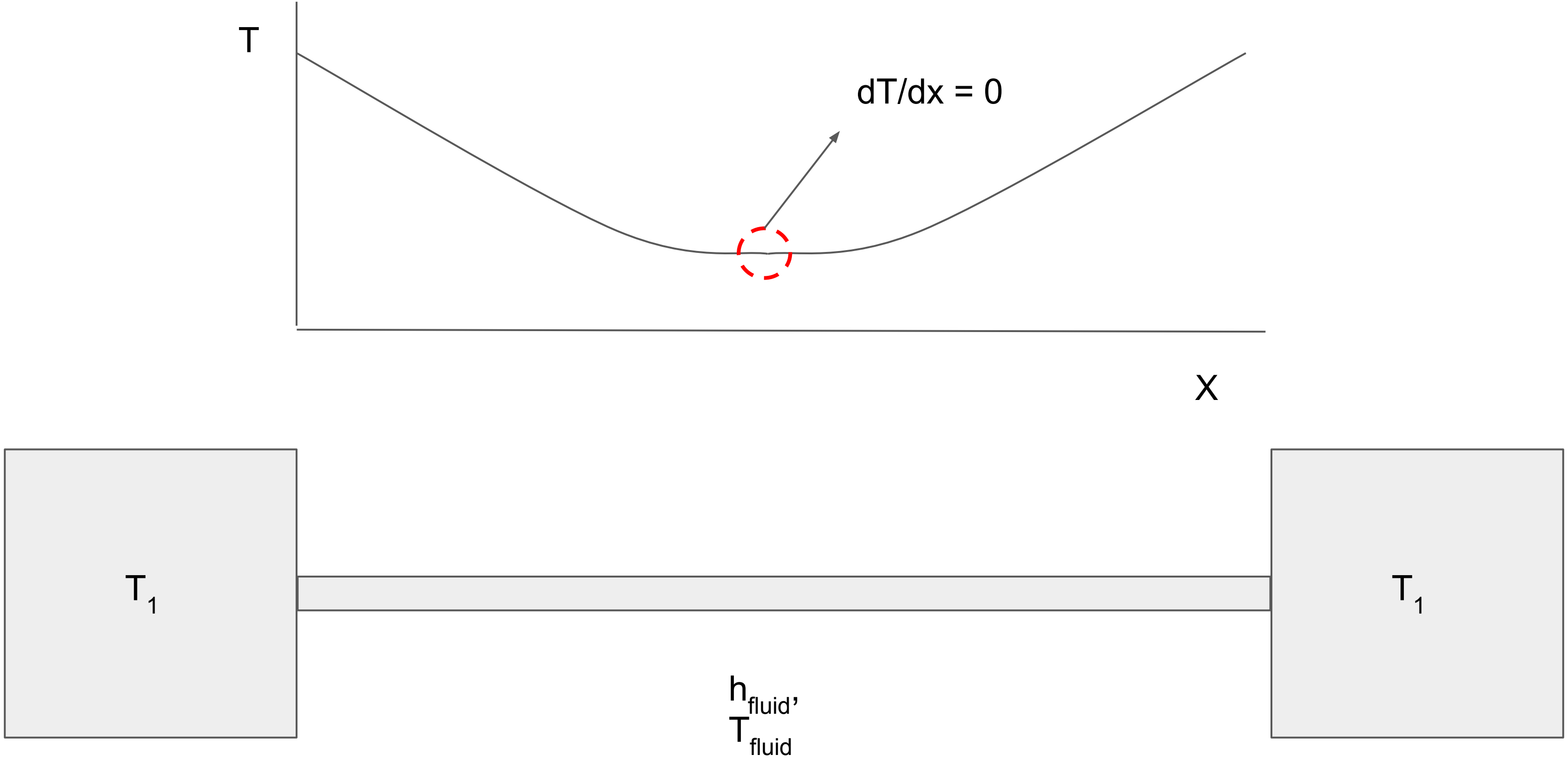
$Q = \eta \cdot h \cdot A_0 \cdot (T_{base} - T_{fluid})$

$A_0 = 2 \cdot 0.1005 \cdot b$

To find the heat rate per unit breadth, b can be assumed equal to 1.

$Q = 3032.7$ W/m

Thermal Symmetry

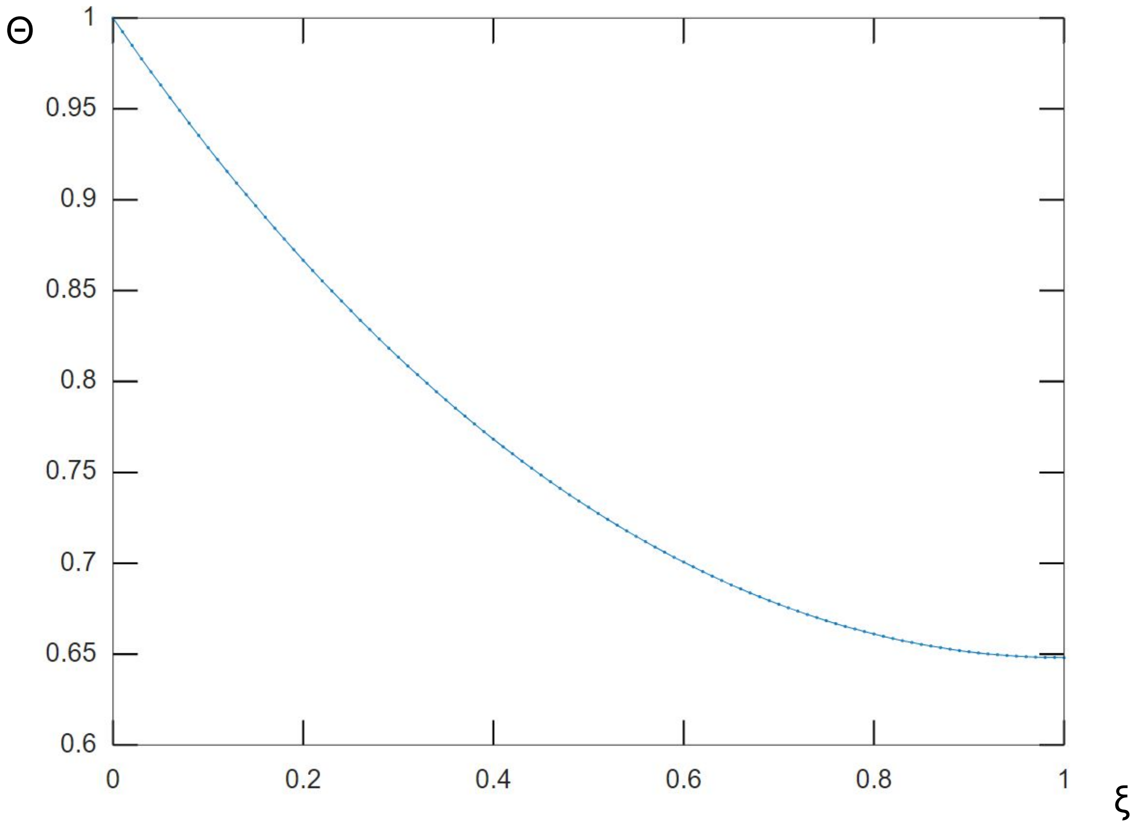


Thermal Symmetry

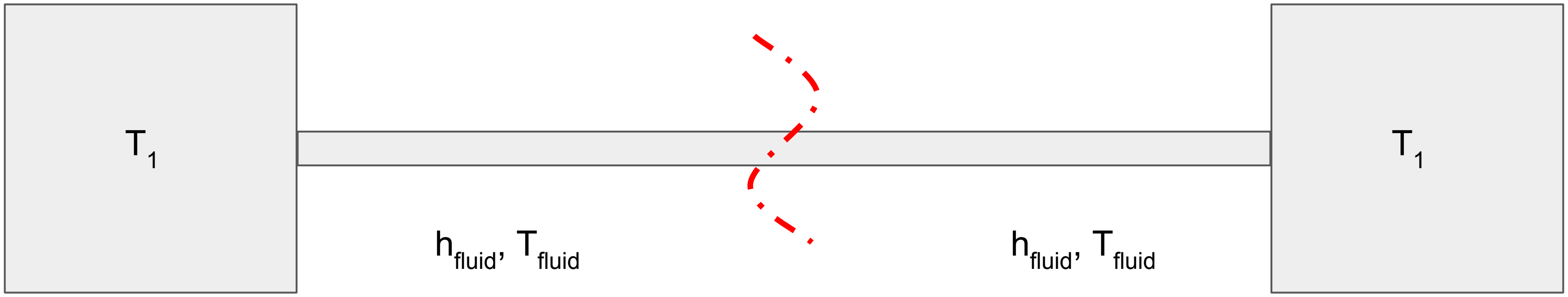
$$\Theta = \frac{\cosh mL(1-\xi)}{\cosh mL}$$

mL = 1

ξ = 1 ⇔ Adiabatic section



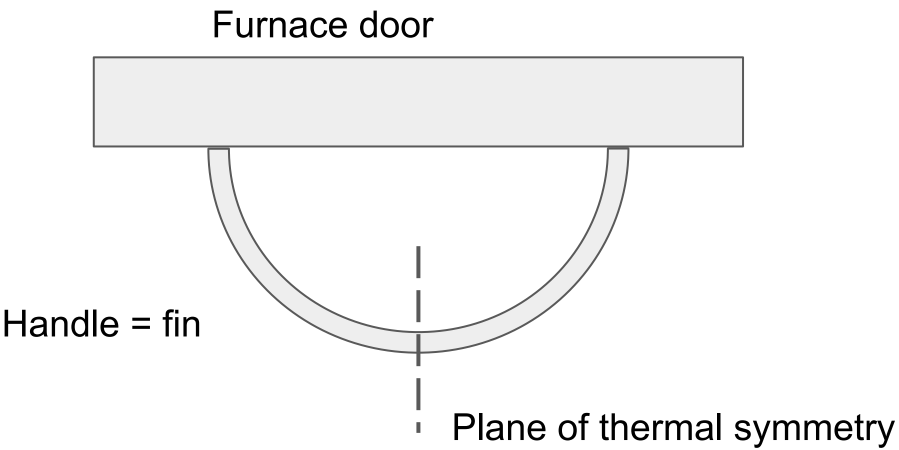
Thermal Symmetry



A plane of thermal symmetry satisfying is region in space where the temperature function is mirrored along its normal direction. Therefore, it is possible to split up the control volume and study only one part of it, applying the adiabatic boundary condition to the plane of thermal symmetry.

4.29

You want to rig a handle for a door in the wall of a furnace. The door is at 160°C. You consider bending a 40 cm length of 6.35 mm diam. 0.5% carbon steel rod into a U-shape and welding the ends to the door. Surrounding air at 24°C will cool the handle (h = 12 W/m^{2}K including both convection and radiation). What is the coolest temperature of the handle? How close to the door can you grasp the handle without getting burned if T_{burn} = 65°C? How might you improve the design?



DATA

T_{door} = 160 °C

T_{air} = 24 °C
h_{air} = 12 Wm^{2}K

k_{steel} = 52 W/mK

1. Temperature at the tip

$$mL = \sqrt{\frac{\overline{h}P}{kA}L^2} = 2.4114$$

$$\Theta = \frac{T - T_\infty}{T_0 - T_\infty} = \cosh (mL)^{-1}$$

T_{tip} = 48.2 °C

2. Length at which T = 65 °C

$$\boxed{\Theta = \frac{\cosh mL(1-\xi)}{\cosh mL}} \qquad (4.41)$$

$$\Theta = \frac{T - T_\infty}{T_0 - T_\infty} = 0.30$$

$x = \xi \cdot L$ = 10.8 cm