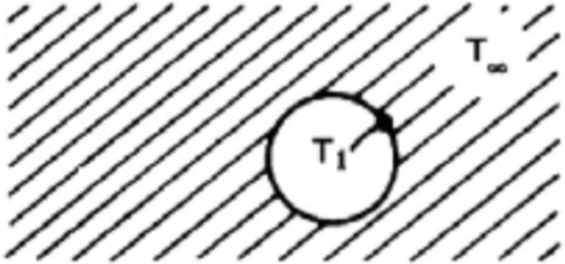
The boundary of a spherical hole of radius R conducting into an infinite medium

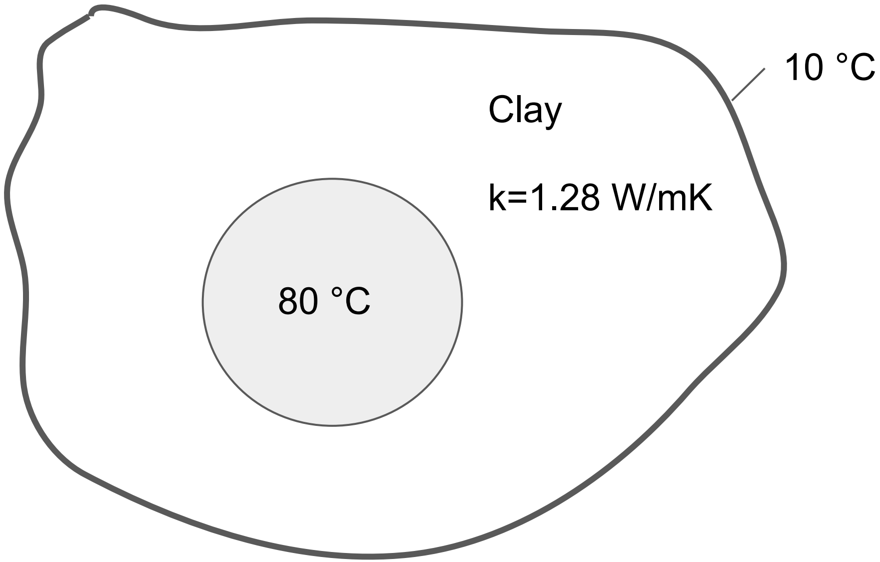


$4\pi R$

$$Q \equiv S\,k\Delta T. \tag{5.66}$$

2.15

An isothermal sphere 3 cm in diameter is kept at 80°C in a large clay region. The temperature of the clay far from the sphere is kept at 10°C. How much heat must be supplied to the sphere to maintain its temperature if k_{clay} = 1.28 W/m·K? (*Hint:* You must solve the boundary value problem not in the sphere but in the clay surrounding it.)
[Q = 16.9 W.]



The sphere is uniformly at 80 °C.

The boundary conditions are uniform against the two angles directions (θ and Φ).

Also, there is no variation of the boundary conditions in time (steady state).

$$\nabla^2 T + \cancel{\frac{\dot{q}}{k}} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right)} + \cancel{\frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2}}$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right) = 0$$

Integrating twice

General solution:

$$\boxed{T = -\frac{C_1}{r} + C_2}$$

Boundary Conditions:

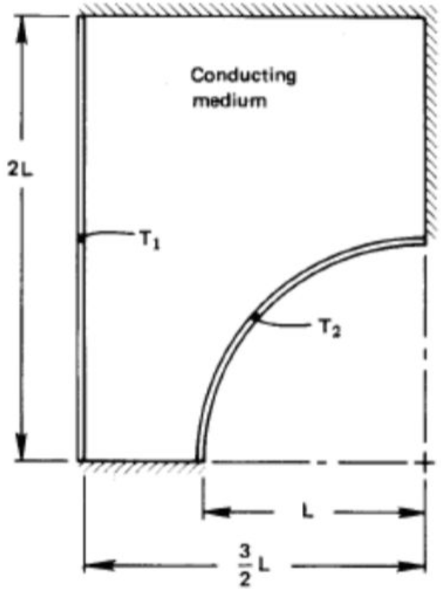
$$\begin{cases} T(r = r_s) = T_s \\ T(r = \infty) = T_\infty \end{cases} \Rightarrow \begin{cases} C_1 = (T_\infty - T_s) r_s \\ C_2 = T_\infty \end{cases}$$

Temperature function:

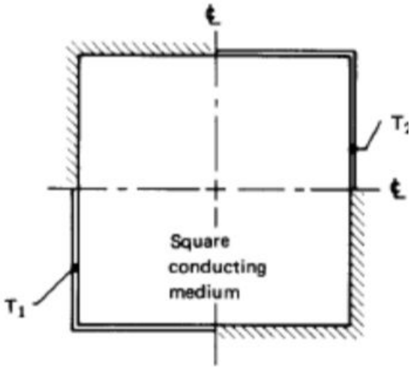
$$T = -\frac{(T_\infty - T_s) r_s}{r} + T_\infty$$

Heat Rate:

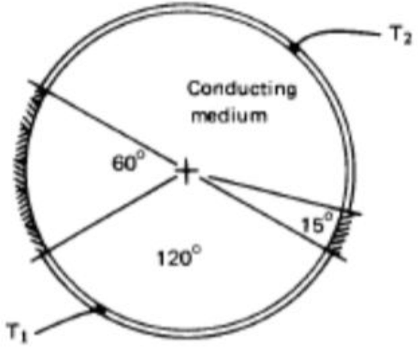
$$\boxed{\begin{aligned} Q &= qA = -4\pi r^2 k \frac{\partial T}{\partial r} \\ Q &= -4\pi k (T_\infty - T_s) r_s = 16.9\ W \end{aligned}}$$



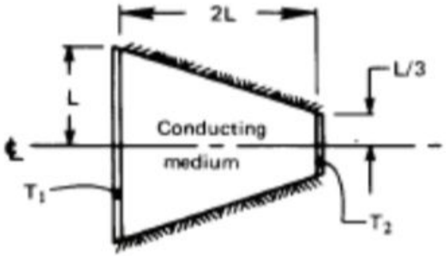
5.22h



5.22j



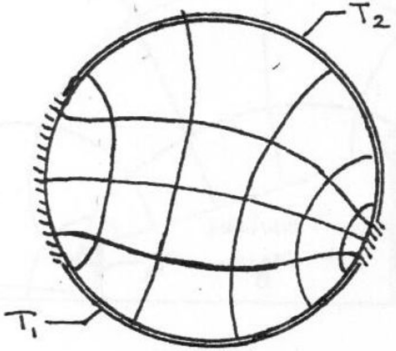
5.22a (Also find T_{center})



5.22c

$$\boxed{Q \text{ W/m} = Nk\,\delta T\,\frac{\delta s}{\delta n} = \frac{N}{I}\,k\Delta T} \tag{5.65}$$

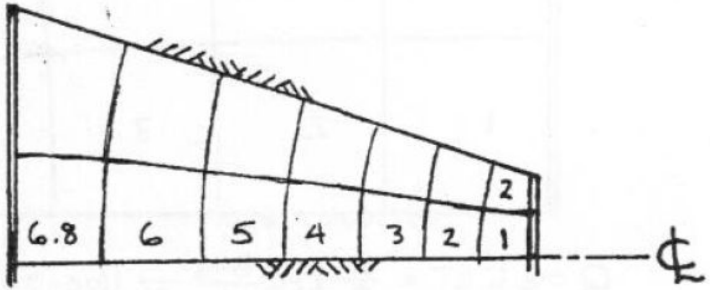
where N is the number of heat flow channels and I is the number of temperature increments, $\Delta T/\delta T$.



$$S = \frac{N}{I} \approx \frac{6}{4} = 1.67$$

5.22c Find S for the inside of the form shown.

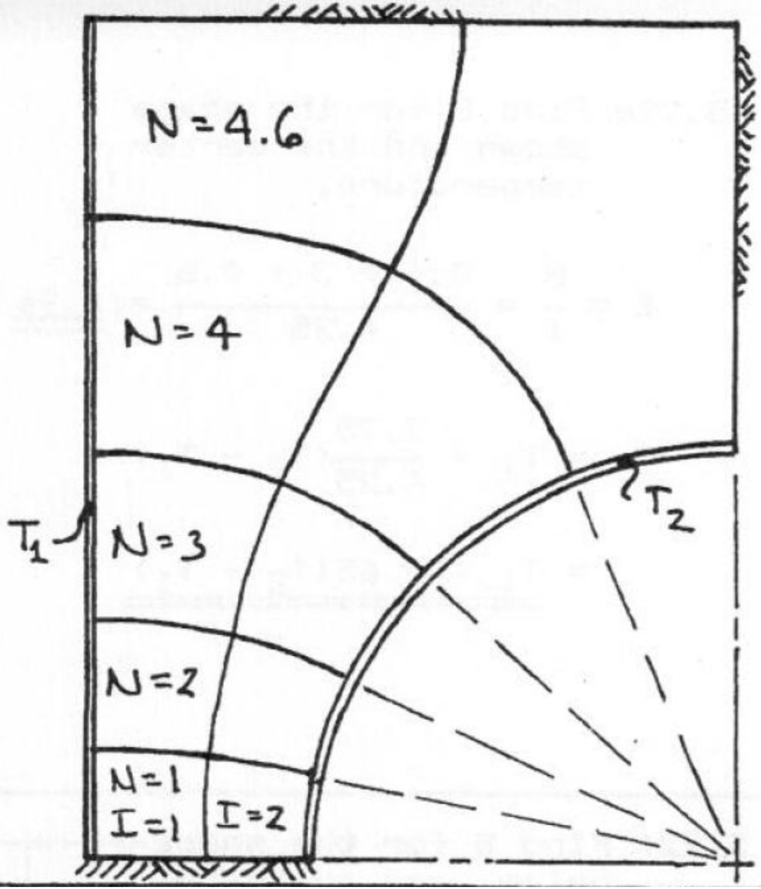
$$S = \frac{2(2)}{6.8} = \underline{\underline{0.59}}$$



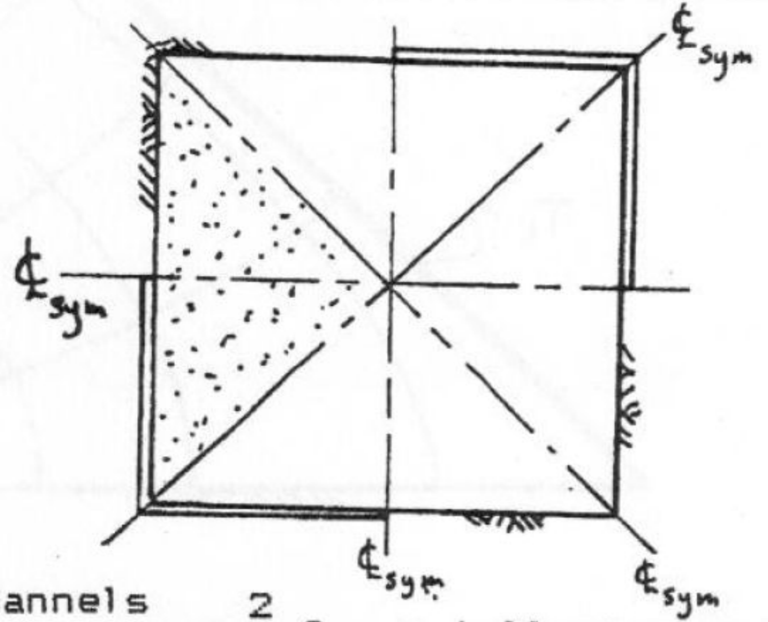
5.22h Find S

$$S = \frac{N}{I} = \frac{4.6}{2}$$

$$\underline{\underline{S = 2.3}}$$



5.22j Find S for the form shown



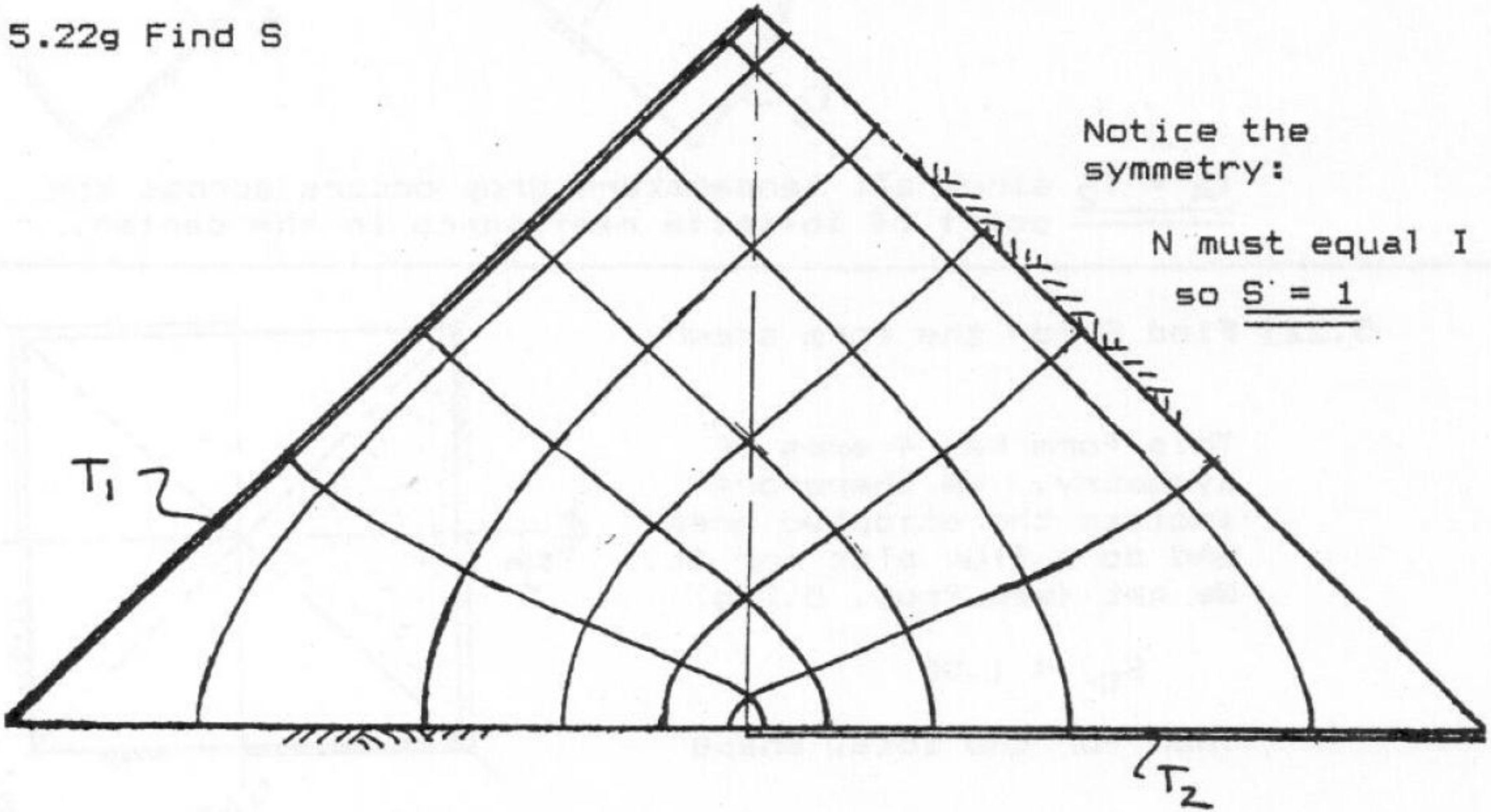
This form has 4 axes of symmetry. We therefore isolate the stippled area and do a flux plot for it. We get (see Prob. 5.22g)

$$S_{\triangleright} = 1.00$$

Then for the total shape

$$S_{\boxtimes} = \frac{\text{twice as many channels}}{\text{twice as many isotherms}} = \frac{2}{2}\, S_{\triangleright} = \underline{\underline{1.00}}$$

5.22g Find S



Notice the symmetry:

N must equal I

so $\underline{\underline{S = 1}}$

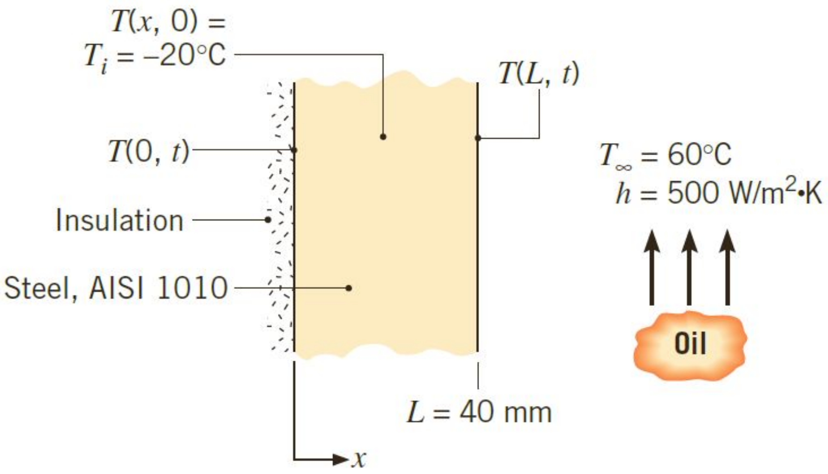
Consider a steel pipeline (AISI 1010) that is 1 m in diameter and has a wall thickness of 40 mm. The pipe is heavily insulated on the outside, and, before the initiation of flow, the walls of the pipe are at a uniform temperature of 20 °C. With the initiation of flow, hot oil at 60 °C is pumped through the pipe, creating a convective condition corresponding to h 500 W/m2 K at the inner surface of the pipe.

1. What are the appropriate Biot and Fourier numbers 2 min after the initiation of flow?

2. At t = 2 min, what is the temperature of the exterior pipe surface covered by the Insulation?

3. What is the heat flux q (W/m2) to the pipe from the oil at t = 2 min?

4. How much energy per meter of pipe length has been transferred from the oil to the pipe at t = 2 min?



Properties: Table A.1, steel type AISI 1010 $[T = (-20 + 60)°C/2 \approx 300\ K]$: $\rho = 7832\ kg/m^3$, $c = 434\ J/kg \cdot K$, $k = 63.9\ W/m \cdot K$, $\alpha = 18.8 \times 10^{-6}\ m^2/s$.

1. What are the appropriate Biot and Fourier numbers 2 min after the initiation of flow?
2. At t = 2 min, what is the temperature of the exterior pipe surface covered by the Insulation?

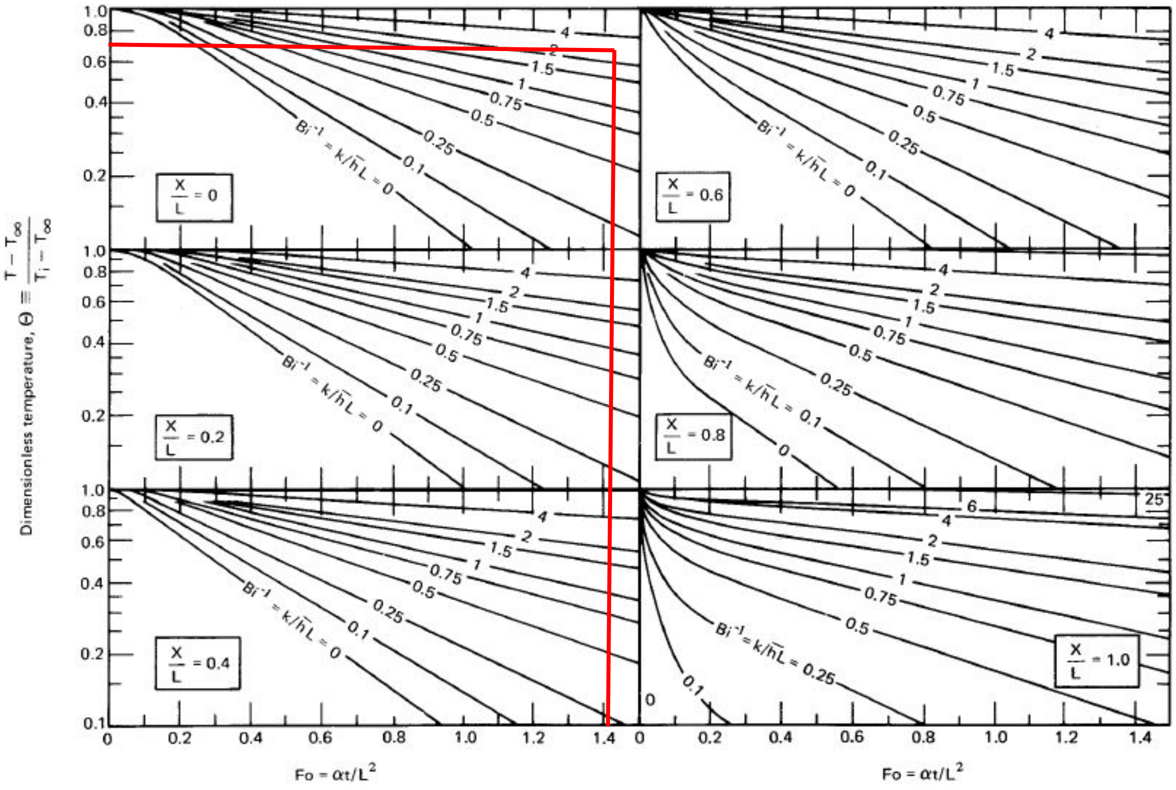


Figure 5.7 The transient temperature distribution in a *slab* at six positions: $x/L = 0$ is the center, $x/L = 1$ is one outside boundary.

Fo = (18.8e-6)*(2*60)/(0.04^2) = 1.4

Bi = 500*0.04/63.9 = 0.313

Θ = 0.7

T(0, 2 min) = 4 °C

3. What is the heat flux q (W/m2) to the pipe from the oil at t = 2 min?

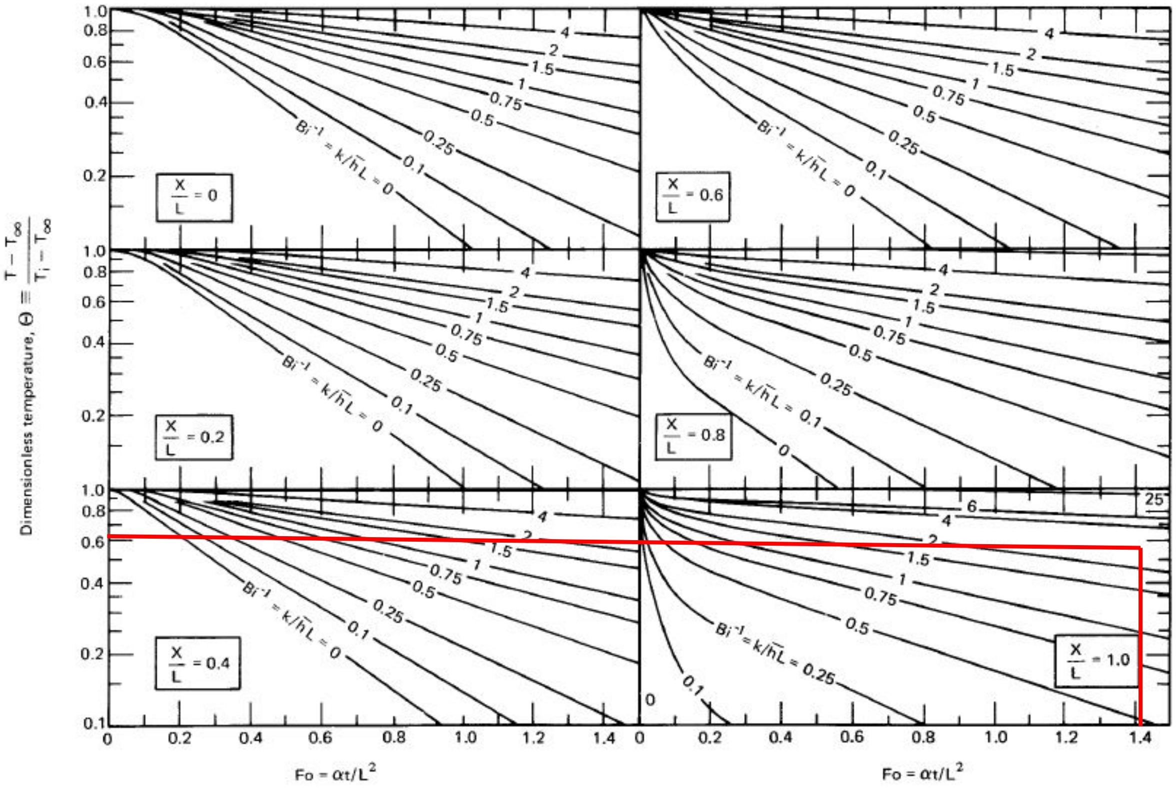


Figure 5.7 The transient temperature distribution in a *slab* at six positions: $x/L = 0$ is the center, $x/L = 1$ is one outside boundary.

$\Theta = 0.6$

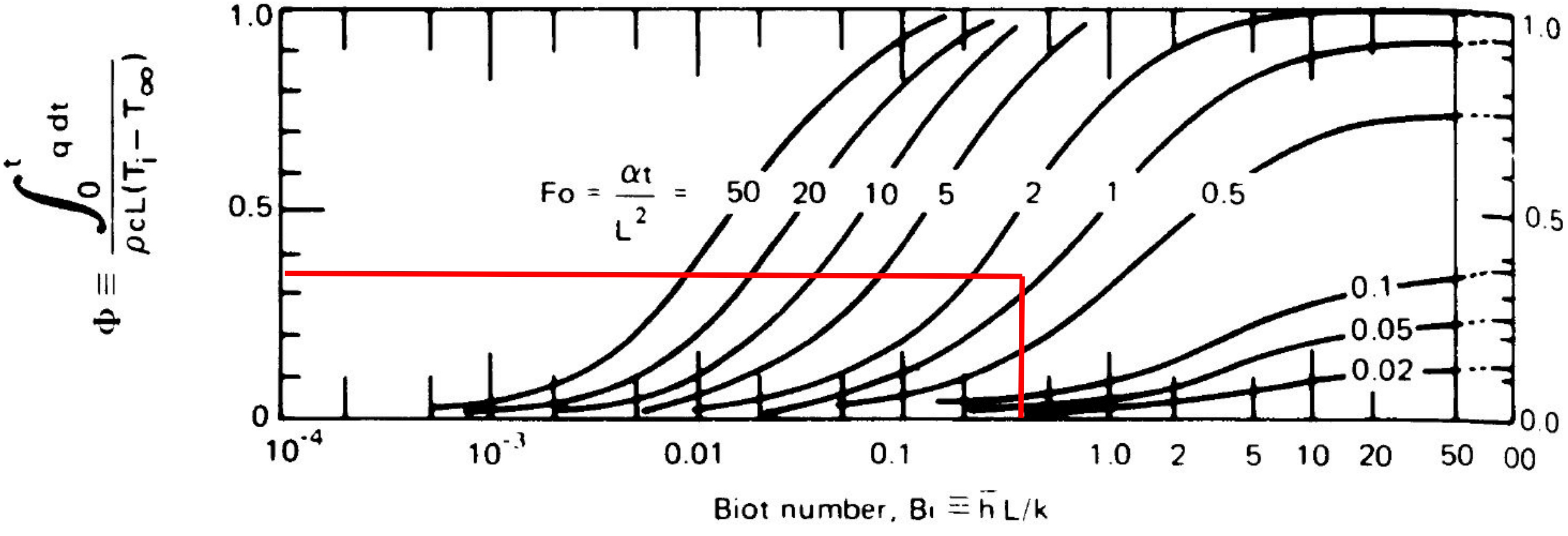
$T(0, 2\text{ min}) = 12\ °C$

$q = h^*(T(L, 2\text{ min}) - T_{fluid}) =$

$= 500^*(12-60) =$

$= -24000\ W/m^2$

4. How much energy per meter of pipe length has been transferred from the oil to the pipe at t = 2 min?



a.) *Slab* of thickness, L, insulated on one side

Figure 5.10 The heat removal from suddenly-cooled bodies as a function of $\overline{h}$ and time.

Q = q□A, where A is the inner surface of the pipe per unit length (πD).

Q = ϕ□ρ□c□L□A□(T_i-T_{fluid}) = - 1.1960e+07 J/m

5.24 Estimate the time required to hard-cook an egg if:

- The minor diameter is 45 mm.
- k for the entire egg is about the same as for egg white. No significant heat release or change of properties occurs during cooking.
- $\overline{h}$ between the egg and the water is 1000 W/m^{2}K.
- The egg has a uniform temperature of 20°C when it is put into simmering water at 85°C.
- The egg is done when the center reaches 75°C.

Material	*Temperature Range* (°C)	*Density* ρ (kg/m^3)	*Specific Heat* c_p (J/kg·K)	*Thermal Conductivity* k (W/m·K)	*Thermal Diffusivity* α (m^2/s)
Egg white	20		3400	0.56	1.37×10^{-7}