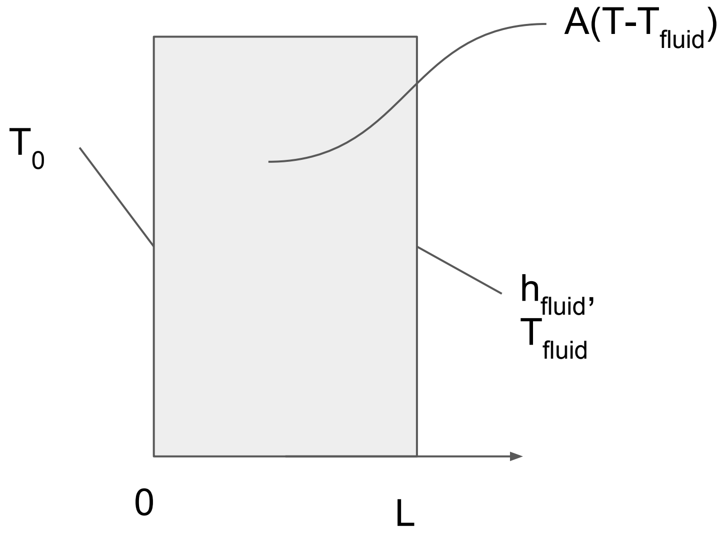
4.2

The left side of a slab of thickness L is kept at $0^{\circ}C$. The right side is cooled by air at $T_{\infty}{}^{\circ}C$ blowing on it. h_{RHS} is known. An exothermic reaction takes place in the slab such that heat is generated at $A(T - T_{\infty})$ W/m^3, where A is a constant. Find a fully dimensionless expression for the temperature distribution in the wall.



$$\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}_{\substack{= 0,\text{ since} \\ T \neq T(y \text{ or } z)}} + \frac{\dot{q}}{k} = \underbrace{\frac{1}{\alpha}\frac{\partial T}{\partial t}}_{\substack{= 0,\text{ since} \\ \text{steady}}}$$

$$\frac{d^2 T}{dx^2} = -\frac{A(T - T_{fluid})}{k}$$

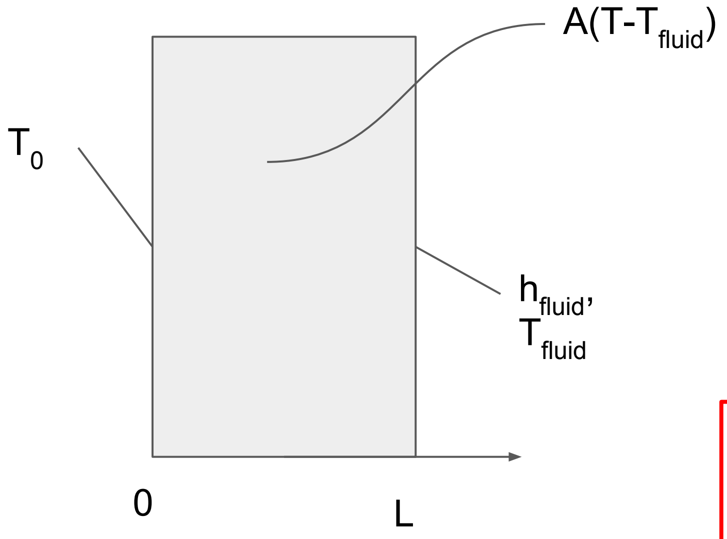
$$X = T - T_{fluid}$$

$$\frac{d^2 (T - T_{fluid})}{dx^2} = -\frac{A(T - T_{fluid})}{k}$$

$$\lambda = \sqrt{\frac{A}{k}}$$

$$X = C_1 sin(\lambda x) + C_2 cos(\lambda x)$$

$$X'' = -\lambda^2 X$$



$$Bi^{-1} = \frac{k\lambda}{h}$$

Boundary conditions

$$T(0) - T_{fluid} = T_0 - T_{fluid}$$

$$-k\frac{d(T-T_{fluid})}{dx}\Big|_L = h\left(T(L) - T_{fluid}\right)$$

Solution

$$C_1 = (T_0 - T_{fluid})\frac{Bi^{-1}sin(\lambda L)-cos(\lambda L)}{Bi^{-1}cos(\lambda L)+sin(\lambda L)}$$

$$C_2 = T_0 - T_{fluid}$$

$$\frac{T-T_{fluid}}{T_0-T_{fluid}} = \frac{Bi^{-1}sin(\lambda L)-cos(\lambda L)}{Bi^{-1}cos(\lambda L)+sin(\lambda L)}sin(\lambda x) + cos(\lambda x)$$