

A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage. Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of $T_i = 25$ °C, and it is heated on both sides by the flow of hot gas. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_{\text{gas}} = 600$ °C and $h = 100$ W/m²K throughout the channel. How long will it take to achieve 75% of the maximum possible energy storage? What is the temperature of the aluminum at this time?

Thermophysical properties (thermal conductivity at $T = 300$ °C)

$$\rho = 2707 \text{ kg/m}^3; \quad c_p = 905; \quad k = 234 \text{ W/mK}$$

Biot number

$$Bi = h \cdot V / (k \cdot A) = h \cdot W / (k \cdot 2) = 0.0107 \quad (\text{lumped-capacity method})$$

Time constant

$$\tau = \rho \cdot c_p \cdot W / (h \cdot 2) = 612.5 \text{ s}$$

Maximum possible energy storage in the aluminum slab

$$Q_{\text{max}} = \rho \cdot c_p \cdot W \cdot (T_i - T_{\text{gas}}) = -7.0433 \times 10^7 \text{ J/m}^2$$

Time at which the heat is 75% of the maximum possible energy storage

$$\text{Heat rate [W/m}^2] = dU/dt = \rho \cdot c_p \cdot W \cdot d(T - T_{\text{gas}})/dt$$

$$Q = 0.75 \cdot Q_{\text{max}} \quad [\text{J/m}^2]$$

Using the temperature solution of the lumped capacity method

$$Q = \rho \cdot c_p \cdot W \cdot (T_i - T_{\text{gas}}) \cdot (1 - \exp(-t/\tau)) = Q_{\text{max}} \cdot (1 - \exp(-t/\tau)) \quad [\text{J/m}^2]$$

$$t = -\tau \cdot \ln(1 - Q/Q_{\text{max}}) = 849 \text{ s}$$

Temperature of the slab at the time t

$$T(t) = T_{\text{gas}} + (T_i - T_{\text{gas}}) \cdot \exp(-t/\tau) = 456 \text{ °C}$$

Saturated water vapor leaves a steam turbine at a flow rate of 1.5 kg/s and a pressure of 5.2 MPa (latent heat of vaporization 1622.9 kJ/kg). The vapor is to be completely condensed to saturated liquid in a shell-and-tube (one shell pass) heat exchanger that uses city water as the cold fluid. The water enters the thin-walled tubes at 17 °C and is to leave at 57 °C. Assuming an overall heat transfer coefficient of 2000 W/m²K, determine the required heat exchanger surface area and the water flow rate. After extended operation, fouling causes the overall heat transfer coefficient to decrease to 1000 W/m²K, and to completely condense the vapor, there must be an attendant reduction in the vapor flow rate. For the same water inlet temperature and flow rate, what is the new vapor flow rate required for complete condensation?

$$\text{Vapor temperature } T_{\text{vapor}} = 266.85 \text{ }^{\circ}\text{C}$$

$$\begin{aligned} \text{Water properties at } T_{\text{water}} &= 37 \text{ }^{\circ}\text{C} \\ c_p &= 4179 \text{ J/kgK} \end{aligned}$$

Heat rate of the heat exchanger without fouling

$$Q = m_{\text{vapor}} * h_{\text{vaporization}} = 2.43435\text{e}+6 \text{ W}$$

Water mass flow rate

$$m_{\text{water}} = Q / (c_p * (T_{\text{water,out}} - T_{\text{water,in}})) = 14.56 \text{ kg/s}$$

Effectiveness - NTU without fouling

$$\epsilon = (T_{\text{water,out}} - T_{\text{water,in}}) / (T_{\text{vapor}} - T_{\text{water,in}}) = 0.16$$

$$\text{NTU} = 0.1745$$

$$A = \text{NTU} * m_{\text{water}} * c_p / U = 5.3 \text{ m}^2$$

Effectiveness - NTU with fouling

$$\text{NTU} = UA / (m_{\text{water}} * c_p) = 0.087$$

$$\epsilon_{\text{fouling}} = 0.0835$$

Heat rate with fouling

$$Q_{\text{fouling}} = \epsilon_{\text{fouling}} * (m_{\text{water}} * c_p) * (T_{\text{vapor}} - T_{\text{water,in}}) = 1.27\text{e}+6 \text{ W}$$

$$m_{\text{vapor,fouling}} = Q_{\text{fouling}} / h_{\text{vaporization}} = 0.783 \text{ kg/s}$$

Consider a concentric tube annulus for which the inner and outer diameters are 25 and 50 mm. Water enters the annular region at 0.4 kg/s and 25 °C. If the inner tube wall is heated electrically at a rate (per unit length) of $q = 400 \text{ W/m}$, while the outer tube wall is insulated, how long must the tubes be for the water to achieve an outlet temperature of 85 °C? What is the inner tube surface temperature at the outlet, where fully developed conditions may be assumed?

$\rho = 985.66 \text{ kg/m}^3$; $c_p = 4183.1 \text{ J/kgK}$; $k = 0.64598 \text{ W/mK}$; $\mu = 503.61 \times 10^{-6} \text{ Pa}\cdot\text{s}$; $Pr = 3.2612$

Energy Balance

$$q \cdot L = m_{\text{water}} \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}})$$

$$L = m_{\text{water}} \cdot c_p \cdot (T_{\text{out}} - T_{\text{in}}) / q = 251 \text{ m}$$

Reynolds and Nusselt numbers

$$u = 4 \cdot m / (\pi \cdot (D_o^2 - D_i^2)) / \rho = 0.276 \text{ m/s}$$

$$D_h = D_o - D_i = 0.025 \text{ m}$$

$$Re = \rho \cdot u \cdot D_h / \mu = 1.3484 \times 10^4 \text{ (turbulent)}$$

$$f = (.79 \cdot \log(Re) - 1.64)^{-2} = 0.029$$

$$Nu = 77 \text{ (Gnielinski correlation)}$$

$$h = Nu \cdot k / D = 1.989 \times 10^3 \text{ W/m}^2\text{K}$$

Heat flux

$$q'' = Q / L / (\pi \cdot D_i) = q / (\pi \cdot D_i) = 8.149 \text{ W/m}^2$$

Wall temperature at the outlet

$$q'' = h \cdot (T_{\text{wall}} - T_{\text{out}})$$

$$T_{\text{wall}} = T_{\text{out}} + q'' / h = 494.6 \text{ }^\circ\text{C}$$

A very long electrical conductor 10 mm in diameter is concentric with a cooled cylindrical tube 50 mm in diameter whose surface is diffuse with an emissivity of 1 and temperature of 27 °C. The electrical conductor has a diffuse, gray surface with an emissivity of 0.6 and is dissipating 6.0 W per meter of length. Assuming that the space between the two surfaces is evacuated, calculate the surface temperature of the conductor.

Energy Balance on the electrical conductor

Surface 1 = Electrical Conductor

Surface 2 = Cooled cylindrical tube

$$Q_1 = A_1 \cdot (E_1 - \alpha_1 G_1), \quad \alpha_1 = \epsilon_1$$

$$G_1 = F_{12} J_2, \quad J_2 = E_{b2} \text{ and } F_{12} = 1$$

$$Q_1/A_1 = \epsilon_1 \cdot \sigma \cdot (T_1^4 - T_2^4)$$

$$T_1 = (Q_1/A_1 / (\epsilon_1 \cdot \sigma) + T_2^4)^{1/4} = 342.3 \text{ K}$$