3. Heat Exchanger Design

John Richard Thome

1er mars 2008

3.1 Function and configuration of heat exchangers

Heat exchanger goal : get energy from one fluid mass to another.

Simple or composite wall of some kind divides the two flows and provides an element of thermal resistance between them.

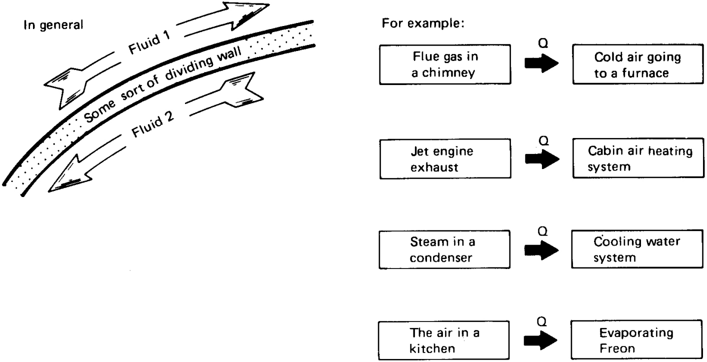


Figure 3.1 Heat exchange.

Exception : Direct-contact form of heat exchanger

Steam is bubbled into water. It condenses and the water is heated at the same time.

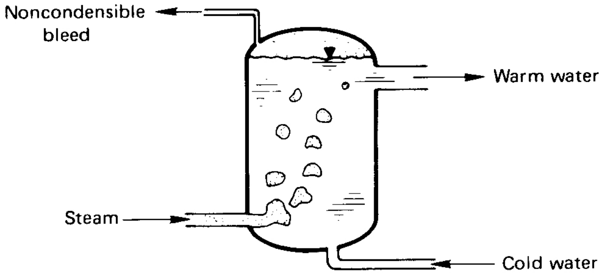


Figure 3.2 A direct-contact heat exchanger.

Three basic types of heat exchanger

- The simple parallel or counterflow configuration

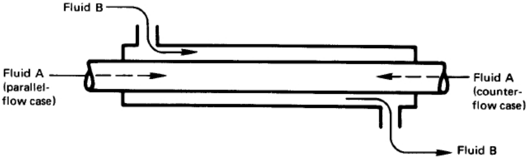


Figure 3.3 Parallel or counterflow heat exchangers.

3.1 Function and configuration of heat exchangers

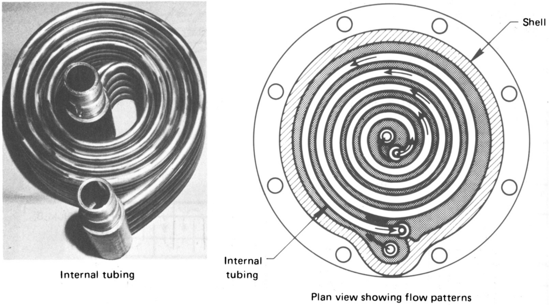


Figure 3.4 Heliflow compact counterflow heat exchanger. (Photograph coutesy of Graham Manufacturing Co., Inc., Batavia, New York.)

- The shell-and-tube configuration

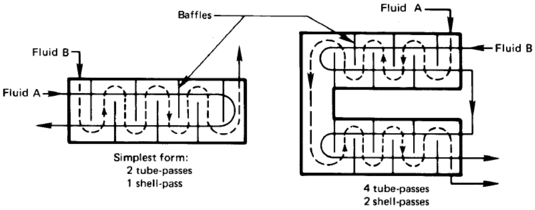


Figure 3.3 Two kinds of shell-and-tube heat exchangers.

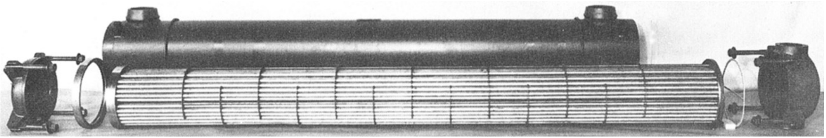


Figure 3.5 Typical commercial one-shell-pass, two-tube-pass heat exchangers.

3.1 Function and configuration of heat exchangers

- The cross-flow configuration

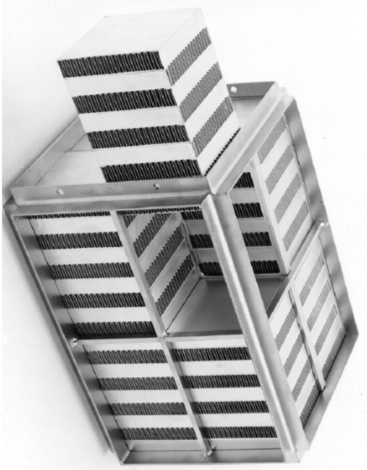
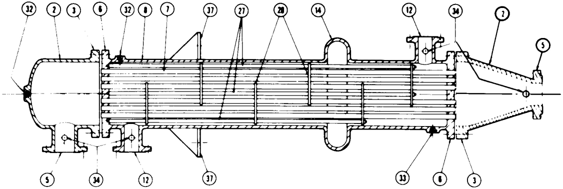


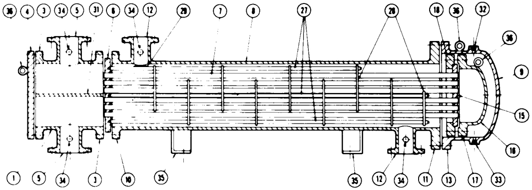
Figure 3.6 c The basic 1 ft/1 ft/2 ft module for a waste heat recuperator. It is a plate-fin, gas-to-air cross-flow heat exchanger with neither flow mixed.

3.1 Function and configuration of heat exchangers

Four typical single-shell-pass heat exchangers (Nomenclature on page 106)

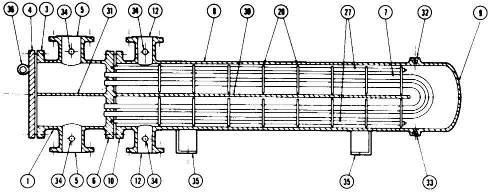


a) Single shell-pass, single tube-pass exchanger



b) One shell-pass, two tube-pass exchanger

3.1 Function and configuration of heat exchangers



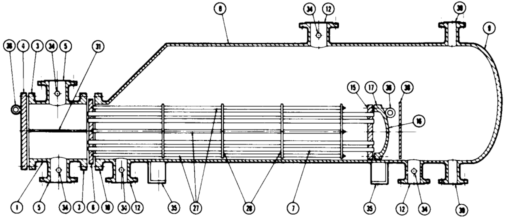


Figure 3.7 Four typical heat exchanger configurations. Drawings courtesy of the Tubular Exchanger Manufacturers' Association (TEMA).

Another variation on the single-pass configuration

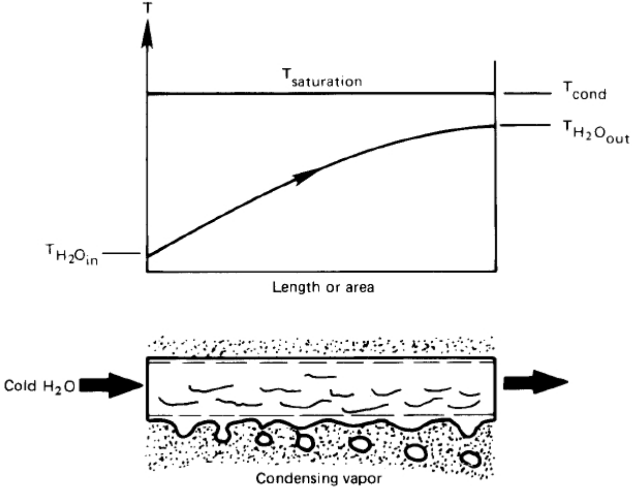


Figure 3.9 The temperature distribution through a condenser.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Overall heat transfer (LMTD)

$$Q = UA\,\Delta T_{mean} \qquad \text{with a constant U} \qquad (3.1)$$

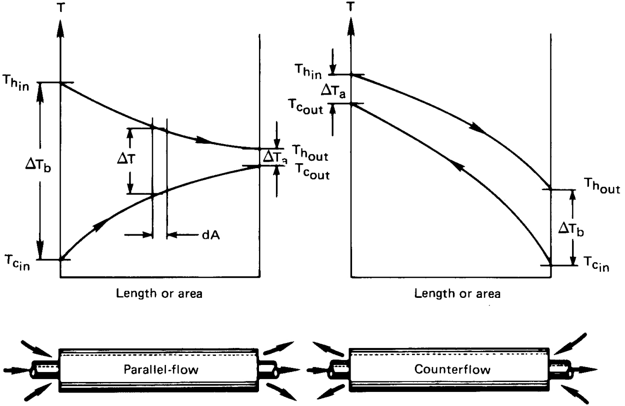


Figure 3.8 The temperature variation through single-pass heat exchangers.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Heat transfer area

$$dQ = U\,\Delta T\,dA \qquad (3.2)$$

where $\Delta T = T_h - T_c$

$$dQ = -(\dot{m}c_p)_h\,dT_h = -C_h\,dT_h \qquad (3.2-3.3)$$

$$dQ = (\dot{m}c_p)_c\,dT_c = C_c\,dT_c \qquad (3.2-3.3)$$

Where C_h and C_c are the hot and the cold fluid heat capacity rates

This equation can be integrated from the lefthand side :

Parallel flow :

$$T_h = T_{h,\,in} \quad T_c = T_{c,\,in}$$

Counterflow :

$$T_h = T_{h,\,in} \quad T_c = T_{c,\,out}$$

"$_{in}$" is for inlet, "$_{out}$" is for outlet, "$_h$" is for the hot fluid and "$_c$" is for the cold fluid.

3.2 Evaluation of the mean temperature difference in a heat exchanger

The temperatures inside are

Parallel flow :

$$T_h = T_{h,\,in} - \frac{C_c}{C_h}\left(T_c - T_{c,\,in}\right) = T_{h,\,in} - \frac{Q}{C_h} \qquad (3.4)$$

Counterflow :

$$T_h = T_{h,\,in} - \frac{C_c}{C_h}\left(T_{c,\,out} - T_c\right) = T_{h,\,in} - \frac{Q}{C_h} \qquad (3.4)$$

Equations (3.4) can be solved for the local temperature differences

$$\Delta T_{\mathrm{parallel}} = T_h - T_c = T_{h,\,in} - \left(1 + \frac{C_c}{C_h}\right) T_c + \frac{C_c}{C_h} T_{c,\,in} \qquad (3.5)$$

$$\Delta T_{\mathrm{counter}} = T_h - T_c = T_{h,\,in} - \left(1 - \frac{C_c}{C_h}\right) T_c + \frac{C_c}{C_h} T_{c,\,out} \qquad (3.5)$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

Substitution of these in equation (3.2)

Parallel flow :

$$\frac{U\,dA}{C_c} = \frac{dT_c}{\left[-\left(1+\frac{C_c}{C_h}\right)T_c + \frac{C_c}{C_h}T_{c,\,in} + T_{h,\,in}\right]} \qquad (3.6)$$

Counterflow :

$$\frac{U\,dA}{C_c} = \frac{dT_c}{\left[-\left(1-\frac{C_c}{C_h}\right)T_c - \frac{C_c}{C_h}T_{c,\,out} + T_{h,\,in}\right]} \qquad (3.6)$$

Equations (3.6) can be integrated across the exchanger

$$\int_0^A \frac{U}{C_c}\,dA = \int_{T_{c,\,in}}^{T_{c,\,out}} \frac{dT_c}{[---]} \qquad (3.7)$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

If U and Cc can be treated as constant

Parallel flow :

$$\ln\left[\frac{-\left(1+\frac{C_c}{C_h}\right)T_{c,\,out}+\frac{C_c}{C_h}T_{c,\,in}+T_{h,\,in}}{-\left(1+\frac{C_c}{C_h}\right)T_{c,\,in}+\frac{C_c}{C_h}T_{c,\,in}+T_{h,\,in}}\right]=-\frac{UA}{C_c}\left(1+\frac{C_c}{C_h}\right) \qquad (3.8)$$

Counterflow :

$$\ln\left[\frac{-\left(1-\frac{C_c}{C_h}\right)T_{c,\,out}-\frac{C_c}{C_h}T_{c,\,out}+T_{h,\,in}}{-\left(1-\frac{C_c}{C_h}\right)T_{c,\,in}-\frac{C_c}{C_h}T_{c,\,out}+T_{h,\,in}}\right]=-\frac{UA}{C_c}\left(1-\frac{C_c}{C_h}\right) \qquad (3.8)$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

with the help of the definitions of ΔT_a and ΔT_b, given in Fig. 3.8

Parallel flow :

$$\ln\left[\frac{\left(1+\frac{C_c}{C_h}\right)(T_{c,\,in}-T_{c,\,out})+\Delta T_b}{\Delta T_b}\right] = -UA\left(\frac{1}{C_c}+\frac{1}{C_h}\right) \qquad (3.9)$$

Counterflow :

$$\ln\frac{\Delta T_a}{\left(-1+\frac{C_c}{C_h}\right)(T_{c,\,in}-T_{c,\,out})+\Delta T_a} = -UA\left(\frac{1}{C_c}-\frac{1}{C_h}\right) \qquad (3.9)$$

Conservation of energy ($Q_c = Q_h$) requires that

$$\frac{C_c}{C_h} = -\frac{T_{h,\,out}-T_{h,\,in}}{T_{c,\,out}-T_{c,\,in}} \qquad (3.10)$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

Then equation (3.9) and equation (3.10) give

Parallel flow :

$$\ln\left[\frac{\overbrace{(T_{c,\,in} - T_{c,\,out}) + (T_{h,\,out} - T_{h,\,in})}^{\Delta T_a - \Delta T_b} + \Delta T_b}{\Delta T_b}\right] = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} + \frac{1}{C_h}\right)$$

Counterflow :

$$\ln\left(\frac{\Delta T_a}{\Delta T_b - \Delta T_a + \Delta T_a}\right) = \ln\left(\frac{\Delta T_a}{\Delta T_b}\right) = -UA\left(\frac{1}{C_c} - \frac{1}{C_h}\right) \qquad (3.11)$$

Finally, we write

$$\frac{1}{C_c} = \frac{T_{c,\,out} - T_{c,\,in}}{Q} \quad \text{and} \quad \frac{1}{C_h} = \frac{T_{h,\,in} - T_{h,\,out}}{Q}$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

in equation (3.11) and we get for either parallel or counterflow

$$Q = UA\left(\frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a/\Delta T_b)}\right) \qquad (3.12)$$

Logarithmic mean temperature difference (LMTD)

$$\Delta T_{\mathrm{mean}} = \mathrm{LMTD} = \frac{\Delta T_a - \Delta T_b}{\ln(\Delta T_a/\Delta T_b)} \qquad (3.13)$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

Example 3.1

Suppose that we had asked, "What mean radius of pipe would have allowed us to compute the conduction through the wall of a pipe as though it were a slab of thickness $L = r_0 - r_i$?" (see Fig. 3.10). To answer this, we compare

$$Q = kA\frac{\Delta T}{L} = 2\pi kl\Delta T\left(\frac{r_{\mathrm{mean}}}{r_0 - r_i}\right)$$

with equation (2.21)

$$Q = 2\pi kl\Delta T\frac{1}{\ln(r_0 - r_i)}$$

3.2 Evaluation of the mean temperature difference in a heat exchanger

Example 3.1 (bis)

It follows that

$$r_{\text{mean}} = \frac{r_0 - r_i}{\ln(r_0/r_i)} = \text{logarithmic mean radius}$$

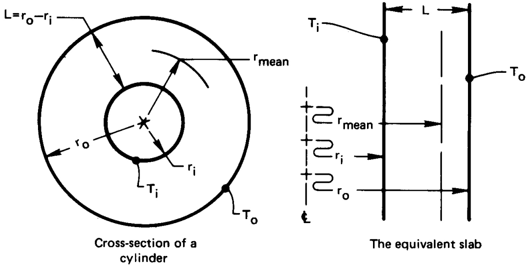


Figure 3.10 Calculation of the mean radius for heat conduction through a pipe.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Example 3.2

Suppose that the temperature difference on either end of a heat exchanger, ΔT_a, and ΔT_b, are equal. Clearly, the effective ΔT must equal ΔT_a and ΔT_b in this case. Does the LMTD reduce to this value ?

SOLUTION. If we substitute $\Delta T_a = \Delta T_b$ in equation (3.13), we get

$$\mathrm{LMTD} = 0$$

Therefore it is necessary to use L'Hospital's rule

$$\mathrm{LMTD} = \Delta T_a = \Delta T_b$$

It follows that the LMTD reduces to the intuitively obvious result in the limit.

Extended use of the LMTD

Limitations :

- LMTD is restricted to the single-pass parallel and counterflow configurations (can be overcome by adjusting the LMTD for other configurations)
- Value of U must be negligibly dependent on T to complete the integration of equation (3.7)

3.2 Evaluation of the mean temperature difference in a heat exchanger

Extended use of the LMTD

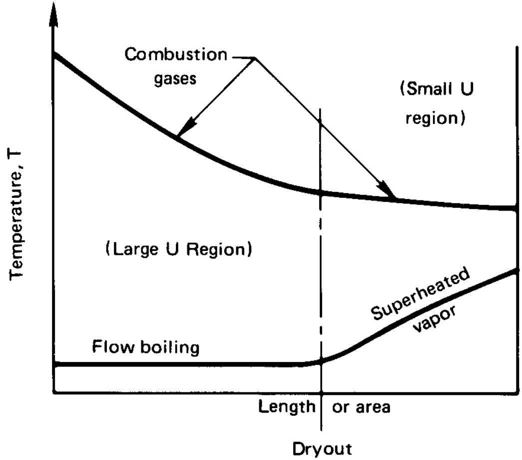


Figure 3.11 A typical case of a heat exchanger in which U varies dramatically.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Extended use of the LMTD

Correction factor, F : is derived analytically from the temperature difference variation with respect to the log mean temperature difference

$$Q = UA(\mathrm{LMTD}) \cdot F \left(\underbrace{\frac{T_{t,\,out} - T_{t,\,in}}{T_{s,\,in} - T_{t,\,in}}}_{P}, \underbrace{\frac{T_{s,\,in} - T_{s,\,out}}{T_{t,\,out} - T_{t,\,in}}}_{R} \right) \tag{3.14}$$

where T_t and T_s are temperatures of tube and shell flows, respectively

- P is the relative influence of the overall temperature difference $(T_{s,\,in} - Tt,\, in)$ on the tube flow temperature.
- R, according to eqn. (3.10), equals the heat capacity ratio C_t/C_s

3.2 Evaluation of the mean temperature difference in a heat exchanger

Extended use of the LMTD

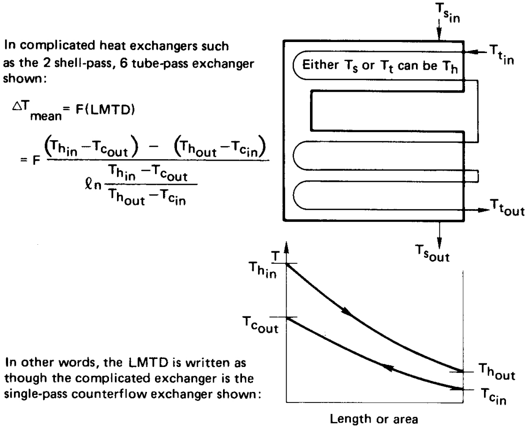


Figure 3.13 The basis of the LMTD in a multipass exchanger, prior to correction.

3.2 Evaluation of the mean temperature difference in a heat exchanger

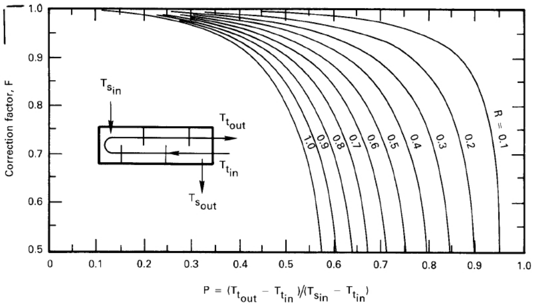


Figure 3.14 LMTD correction factors, F, for multipass shelland- tube heat exchangers and one-pass cross-flow exchangers.

3.2 Evaluation of the mean temperature difference in a heat exchanger

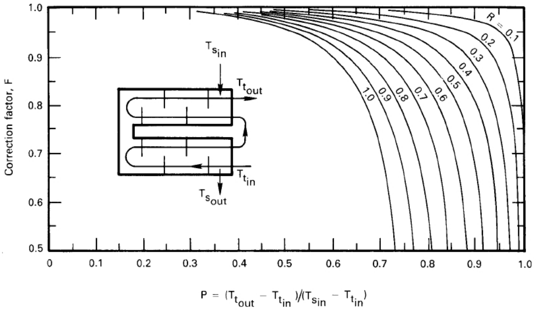


Figure 3.14 LMTD correction factors, F, for multipass shelland- tube heat exchangers and one-pass cross-flow exchangers.

3.2 Evaluation of the mean temperature difference in a heat exchanger

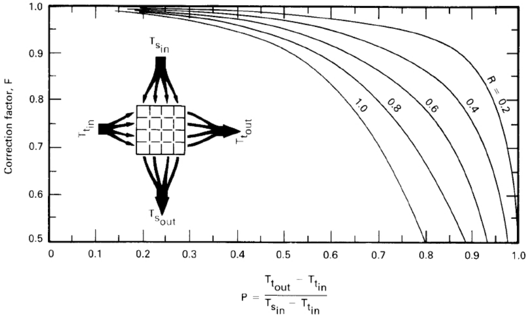


Figure 3.14 LMTD correction factors, F, for multipass shelland- tube heat exchangers and one-pass cross-flow exchangers.

3.2 Evaluation of the mean temperature difference in a heat exchanger

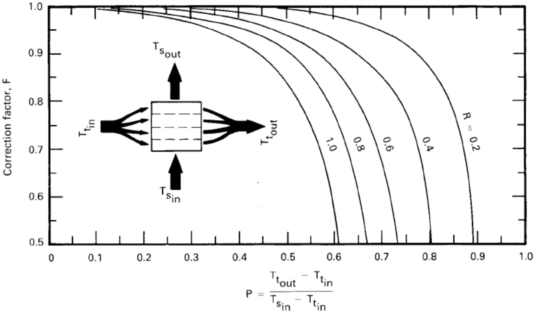


Figure 3.14 LMTD correction factors, F, for multipass shelland- tube heat exchangers and one-pass cross-flow exchangers.

3.2 Evaluation of the mean temperature difference in a heat exchanger

Example 3.4

5.795 kg/s of oil flows through the shell side of a two-shell pass, four tube-pass oil cooler. The oil enters at 181^0C and leaves at 38^0C. Water flows in the tubes, entering at 32.C and leaving at 49^0C. In addition, $c_{p\,\mathrm{poil}} = 2282$ J/kg·K and U = 416 W/m^{2}K. Find how much area the heat exchanger must have. **Solution**

$$\mathrm{LMTD} = 40.76\,K$$

$$R = 8.412 \qquad P = 0.114$$

with figure (3.14) $F = 0.92$ and

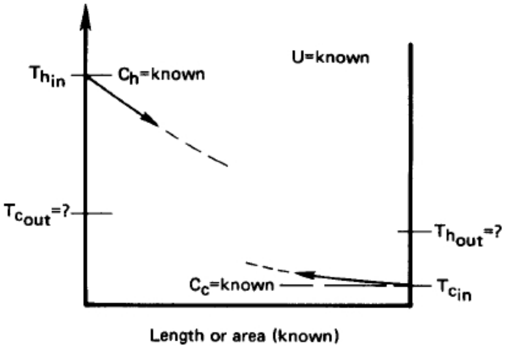
$$Q = UA(\mathrm{LMTD})F$$

we find the area

$$A = 121.2\,m^2$$

3.3 Heat exchanger effectiveness

- LMTD method can only be used if all 4 temperatures are known
- NTU method can be used if only the 2 intlet temperatures are known



Heat exchanger effectiveness

$$\epsilon = \frac{C_h\left(T_{h,\,in} - T_{h,\,out}\right)}{C_{min}\left(T_{h,\,in} - T_{c,\,in}\right)} = \frac{C_c\left(T_{c,\,out} - T_{c,\,in}\right)}{C_{min}\left(T_{h,\,in} - T_{c,\,in}\right)} \qquad (3.16)$$

where C_{min} is smaller of C_h and C_c

if

- $C_h < C_c$, then $Q_{max} = C_h\left(T_{h,\,in} - T_{c,\,in}\right)$
- $C_h > C_c$, then $Q_{max} = C_c\left(T_{h,\,in} - T_{c,\,in}\right)$

ϵ is actual heat transferred divided by the maximum heat that could possibly be transferred from one stream to the other

It follows that

$$Q = \epsilon\, C_{min}\left(T_{h,\,in} - T_{c,\,in}\right) \qquad (3.17)$$

Number of transfer units (NTU)

$$NTU = \frac{UA}{C_{min}} \qquad (3.18)$$

can be viewed as a comparison of the heat capacity of the heat exchanger with the heat capacity of the flow.

Reduce the parallel-flow result from equation (3.9) based on these definitions

$$-\left(\frac{C_{min}}{C_c} + \frac{C_{min}}{C_h}\right) NTU = \ln\left[-\left(1 + \frac{C_c}{C_h}\right)\epsilon\frac{C_{min}}{C_c} + 1\right] \qquad (3.19)$$

For the parallel single-pass heat exchanger

$$\epsilon = \frac{1 - \exp\left\{-NTU\left[1 + \left(\frac{C_{min}}{C_{max}}\right)\right]\right\}}{1 + \left(\frac{C_{min}}{C_{max}}\right)} = f\left(NTU, \frac{C_{min}}{C_{max}}\right) \qquad (3.20)$$

3.3 Heat exchanger effectiveness

The corresponding expression for the counterflow case

$$\epsilon = \frac{1 - \exp\left\{-NTU\left[1 - \left(\frac{C_{min}}{C_{max}}\right)\right]\right\}}{1 - \left(\frac{C_{min}}{C_{max}}\right)\exp\left\{-NTU\left[1 - \left(\frac{C_{min}}{C_{max}}\right)\right]\right\}} \qquad (3.21)$$

Similar calculations give the effectiveness for the other heat exchanger configurations.

Example 3.5

Consider the following parallel-flow heat exchanger specification :
cold flow enters at 40°C : $C_c = 20{,}000$ W/K
hot flow enters at 150°C : $C_h = 10{,}000$ W/K
A = 30 m^2 U = 500 W/m^{2}K.

Determine the heat transfer and the exit temperatures.

Solution

In this case we do not know the exit temperatures, so it is not possible to calculate the LMTD. Instead, we can go either to the parallel-flow effectiveness chart in Figure (3.16) or to equation (3.20), using

$$NTU = \frac{UA}{C_{min}} = 1.5$$

$$\frac{C_{min}}{C_{max}} = 0.5$$

and we obtain $\epsilon = 0.596$. Now from equation (3.17), we find that

$$Q = \epsilon \; C_{min} \left(T_{h,\, in} - T_{c,\, in} \right) = 655.6 \; kW$$

From energy balances such as are expressed in equation (3.4), we get

$$T_{h\,,in} = 84.44^o C$$

$$T_{h\,,out} = 72.78^o C$$

3.3 Heat exchanger effectiveness

For a single fluid stream flowing through an isothermal pipe, the equation for the effectiveness in any configuration must reduce to the same common expression as C_{max} approaches infinity.

In this case ϵ become

$$\lim_{C_{max}\to\infty} \epsilon = 1 - e^{-NTU} \qquad (3.22)$$

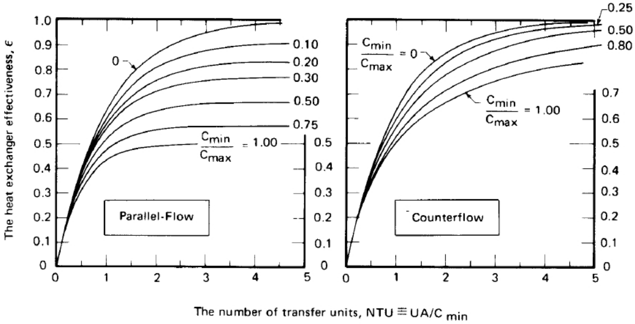
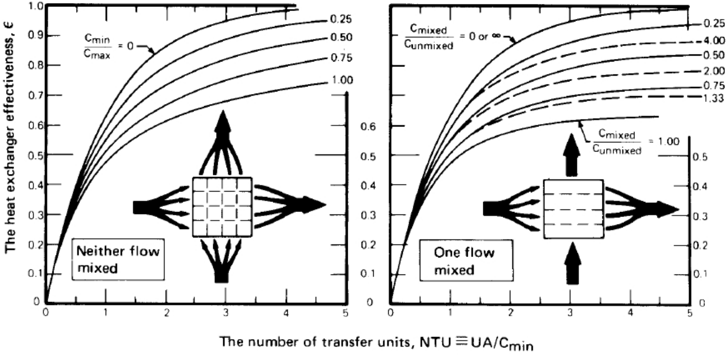


Figure 3.16 The effectiveness of parallel and counterflow heat exchangers.

3.3 Heat exchanger effectiveness



3.3 Heat exchanger effectiveness

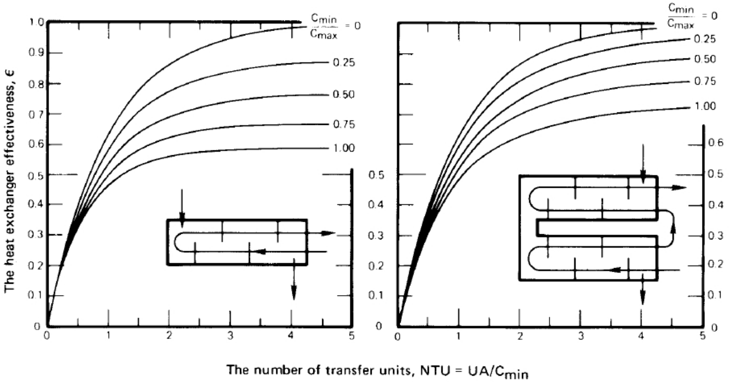


Figure 3.17 The effectiveness of some other heat exchanger configurations.

3.4 Heat exchanger design

Determination of h in a baffled shell remains a problem that cannot be solved analytically.

Apart from predicting heat transfer, a host of additional considerations must be addressed in designing heat exchangers. The primary ones are :

- Minimization of pumping power

$$\text{pumping power} = \frac{\dot{m}\Delta p}{\rho} \qquad (3.23)$$

  where

  $\dot{m}$ *is the mass flow rate of the stream*
  Δp *is the pressure drop of the stream*
  ρ *is the fluid density*

  Determining the pressure drop can be relatively difficult.

- Minimization of fixed costs

3.4 Heat exchanger design

Optimizing the design of an exchanger is not just a matter of making Δp as small as possible.

Augmentation of heat exchange by employing fins or roughening elements :

- will invariably increase the pressure drop
- can also reduce the fixed cost of an exchanger by increasing U and by reducing the required area
- can reduce the required flow rate by increasing the effectiveness and thus balance the increase of Δp

Taborek's list of design considerations for a large shell-and-tube exchanger :

- Choise of fluids
- Cost of the calculation
- Rough estimate of the size of the heat exchanger
- Evaluate the heat transfer, pressure drop, and cost of various exchanger configurations

3.4 Heat exchanger design

Once the exchanger configuration is set :

- U will be approximately set
- area becomes the basic design variabl
- proceed along the lines of Section 3.2 or 3.3.
- If it is possible to begin with a complete specification of inlet and outlet temperatures,

$$\underbrace{Q}_{C\,\Delta T} = \underbrace{U}_{\text{know}} \underbrace{AF(\mathrm{LMTD})}_{\text{calculable}}$$

- A can be calculated and the design completed

Usually, a reevaluation of U and some iteration of the calculation is needed.

More often, we begin without full knowledge of the outlet temperatures :

- invent an appropriate trial-anderror method to get the area
- more complicated sequence of trials for pressure drop and cost optimization