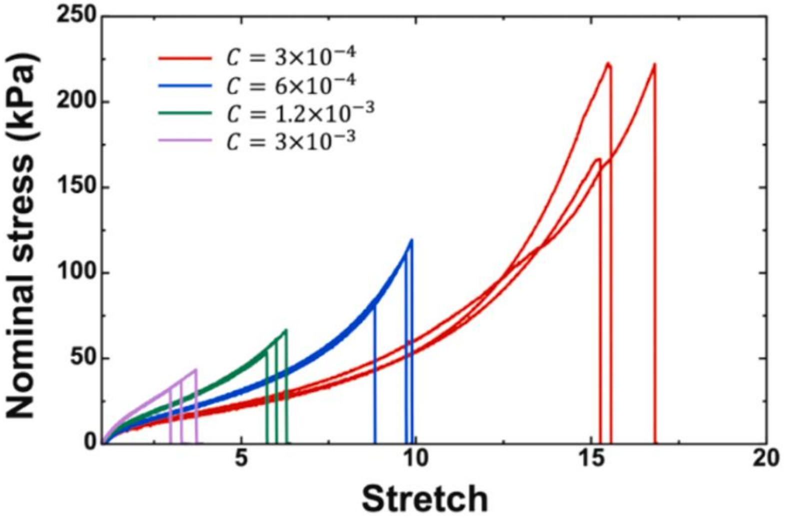
ME 412:

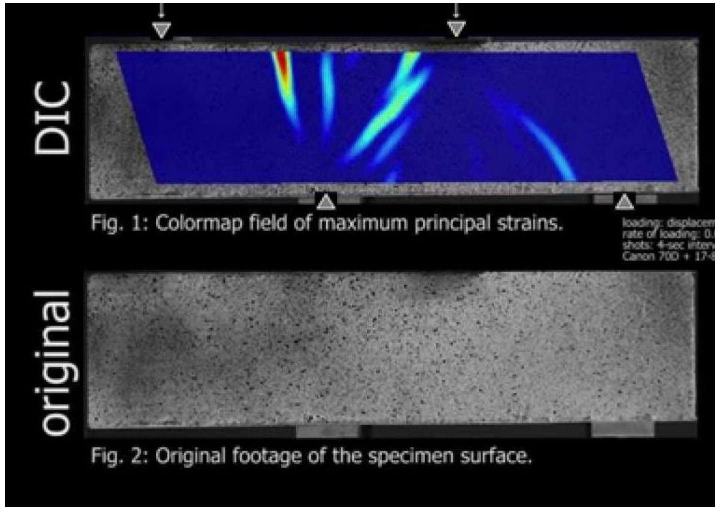
Experimental Methods in Engineering Mechanics

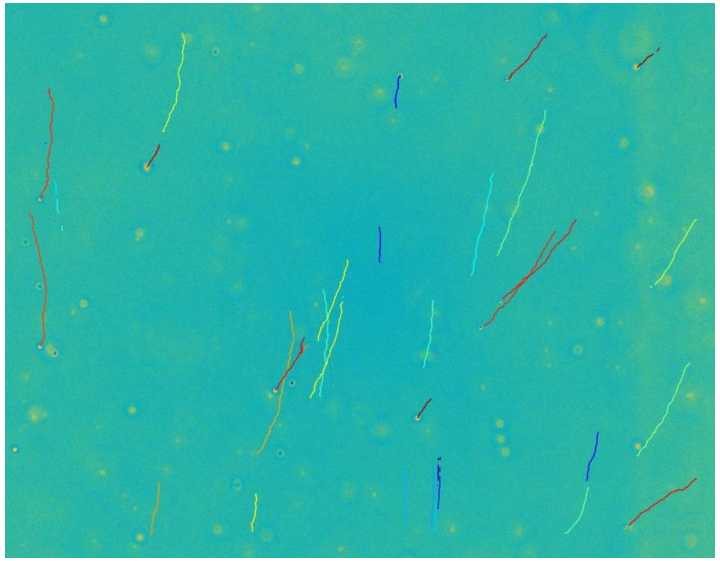
John M. Kolinski

PATT - EMSI

11.9.2024



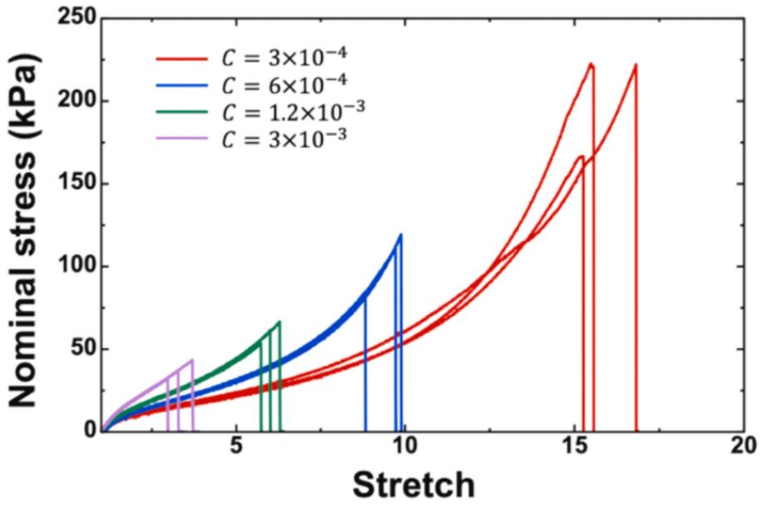




[Discussion of the wiki and the syllabus]

Module 1 in brief – analog electronics

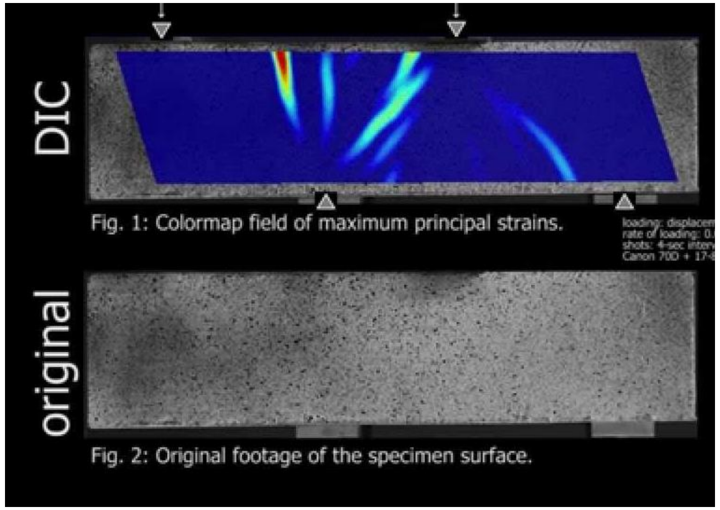
- We need an analog front-end to make sense of any quantity we wish to measure. The analog / digitized signal can then be used to measure or as part of a control loop. This is ***ubiquitous*** in mechanical sciences; thus the motivation to introduce the basics of analog circuitry.
- We will measure the **work of fracture** in a polyacrylamide hydrogel:

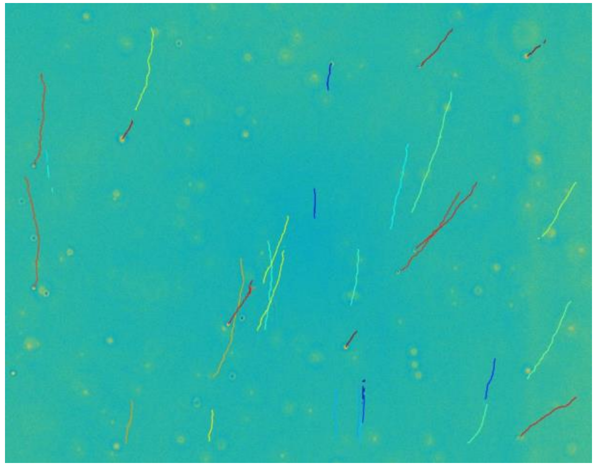


More on this later…

Modules 2 & 3

- Cracks in gels: Use **digital image correlation** to **measure stress around a crack tip**. Testing apparatus in our lab will be used. NCORR software for DIC
- Brownian motion & **micro-rheology**. Use Stokes-Einstein relation to measure viscosity from images of particles in solution. Become familiar with open-source **particle tracking** software package for Python / MATLAB





How will your grade be determined?

- **Lab reports:** For each of the 3 modules, your group will write a lab report within a group of 3 students; this will count for 100% of your grade for Mod. 1, and 80% of your grade for Mod. 2 & 3. The peer assessment in modules 2 and 3 will be considered as part of this 80% for each module.
- **Peer assessment:** Lab reports will be evaluated by other student groups as a team. The **evaluation you provide** will count for 20% for modules 2 and 3.

- Lab groups will rotate – you'll work with different people in each module

Deliverables – lab reports for each module

- The lab reports will consist of a **4-5 page two-column document**, with a focus on the scientific outcome of the experiment. **Physical Review Letters Format** – as described in the following
- **Figures and captions** are an extremely important component to the report, and should be clear, legible, and concise.
- **Writing** should be clear and concise
- Exercises unrelated to the scientific results can be included in the appendix if they are relevant to the reported results (e.g. calibration etc.)
- **Wiki contributions** (more later) should be included in the appendix and can receive additional credit toward the report grade

A grading rubrick for grading the final reports:
PRL format

1. Follows the PRL format (ca. 5 % of total).
   - Title, abstract, authors list using revtex template for PRL. No more than 5 pages for the main report, including figures, etc.
   - 1-2 paragraph introduction for background, going from general to specific.
   - Conclude introductory paragraph with the open question you will address with the report.
   - Next paragraph introduces your method to address the question - `In this letter, we will measure…. We simulate … This paragraph should consist of a concise summary of your results.
   - Specifics of the measurement / simulation approach should be provided, and supported with figures that validate the chosen approach. This portion should be anywhere from 2-5 paragraphs and 1-3 figures.
   - Results should follow – what did you find, and why are you confident in your conclusions? Support with data in figures!
   - Discussion / conclusion – 1-2 paragraphs. Review what you learned, and broaden the discussion to how your measurement / simulation advances the state of the art.
   - References, formatted appropriately.

A grading rubrick for grading the final reports:

PRL format

PHYSICAL REVIEW LETTERS **123,** 128001 (2019)

Editors' Suggestion **Featured in Physics**

Using Acoustic Perturbations to Dynamically Tune Shear Thickening in Colloidal Suspensions

Prateek Sehgal,[1,*] Meera Ramaswamy,[2,†] Itai Cohen,[2,‡] and Brian J. Kirby[1,3,§]
[1]*Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, New York 14853, USA*
[2]*Department of Physics, Cornell University, Ithaca, New York 14853, USA*
[3]*Department of Medicine, Division of Hematology and Medical Oncology, Weill–Cornell Medicine, New York, New York 10021,USA*

(Received 16 May 2019; published 17 September 2019)

Colloidal suspensions in industrial processes often exhibit shear thickening that is difficult to control actively. Here, we use piezoelectric transducers to apply acoustic perturbations to dynamically tune the suspension viscosity in the shear-thickening regime. We attribute the mechanism of dethickening to the disruption of shear-induced force chains via perturbations that are large relative to the particle roughness scale. The ease with which this technique can be adapted to various flow geometries makes it a powerful tool for actively controlling suspension flow properties and investigating system dynamics.

DOI: 10.1103/PhysRevLett.123.128001

The orders-of-magnitude increase in viscosity that arises under high shear makes dense suspensions ideal for numerous industrial applications including shock absorption, damping, soft-body armor, astronaut suits, and curved-surface polishing [1–6]. The challenge in using such shear thickening fluids, however, is that this same increase in viscosity can lead to jamming and failure of pumping and mixing equipment driving the flows. The ability to manage these limitations of this important technological material remains challenging [1,7] because

Here, we determine whether externally applied acoustic perturbations can be used to actively tune the suspension viscosity in the shear thickening regime. The advantage of this approach is that acoustic perturbations can controllably manipulate particles [36–43] and can be applied via readily available piezoelectric transducers that are bonded to otherwise fixed surfaces [40,44,45]. The key principle motivating our work is that nanoscale acoustic disturbances will locally perturb particles and break the force chains

PHYSICAL REVIEW LETTERS **123,** 128001 (2019)

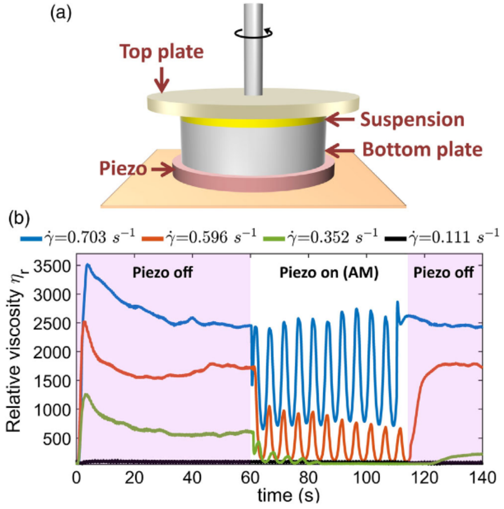


FIG. 3. Experimental setup and AM measurements. (a) The schematic of the acoustic-rheometer setup. The top plate is connected to the rheometer and the bottom plate is bonded to the piezoelectric element. The suspension is confined between the two plates. (b) The instantaneous viscosity response of $\phi = 0.53$ suspension to the gradient-direction perturbations at representative strain rates. The relative viscosity is defined as the ratio of the suspension viscosity to the solvent (dipropylene glycol, 0.11 Pa s) viscosity. Each measurement is performed at a steady $\dot{\gamma}$ for 140 s in which the AM signal is turned on at time $t \sim 60$ s for at least nine modulation cycles, followed by an off-period for the remaining time. Measurements for $\phi = 0.50$ suspension are shown in Supplemental Material, Fig. S4 [50].

For strain rates corresponding to the transition regime between the Newtonian and fully thickened state ($\dot{\gamma} = 0.596,\ 0.352\ s^{-1}$), the acoustic perturbations are sufficient to dethicken the suspension viscosity to the value in the Newtonian regime. Interestingly, the maximum viscosity during the time when AM perturbations are applied does not recover fully to the steady state value. We interpret this response to indicate that the AM frequency is too rapid for the force chains to fully form between successive oscillations at these strain rates. This picture is supported by the fact that the viscosity recovery time when the perturbations are turned off is much longer than the AM oscillation period.

We extract the magnitude of acoustic dethickening as a function of strain rates using a phase-sensitive analysis of the instantaneous viscosity response curves (Fig. 4, see Supplemental Material, Sec. IV for details [50]). We observe that the application of the acoustic perturbations decreases the viscosity substantially in the regime where the suspension thickens. This response is sensitive to the strain rate, with the largest decrease occurring in the transition regime [Figs. 4(a) and 4(b)]. We quantify this response by plotting the %Reduction in viscosity vs $\tilde{\dot{\gamma}}$, the strain rate normalized by the strain rate at the onset of thickening [Fig. 4(c)]. We find a negligible decrease in the viscosity in the Newtonian regime ($\tilde{\dot{\gamma}} < 1$), in which the force chains are mostly absent. We find the highest reduction in the transition region ($1 < \tilde{\dot{\gamma}} < 2$), in which the applied acoustic perturbations are sufficient to break up the majority of the force chains. This decrease in viscosity to nearly the Newtonian value effectively shifts the onset strain rate for thickening. Finally, we find that the %Reduction decreases and plateaus in the fully thickened

The format is simple: an **abstract summarizes the results**. The **first paragraph** is dedicated to **relevant background** information, and **culminates in the unanswered question** that the report will address. The **next paragraph** will then proceed to **explain how the work contained in the report makes progress toward addressing the question**. The **remainder** of the report is dedicated to **presenting results**. A brief **discussion and conclusion** establishes **context** for the work.

A grading rubrick for grading the final reports:

Presentation of figures

2. Figures should be clear, legible and appropriately annotated / labeled and captioned (ca. 30 % of the grade)
    - I should be able to understand what you did by reviewing only the figures and captions. They should be clear, legibly labelled, and completely captioned in concise text.
    - Any error analysis (see quantifying error) is provided with the presentation of the data.

Good vs. Bad captions:

Figure 1. Radish plants subjected to a freezing treatment.

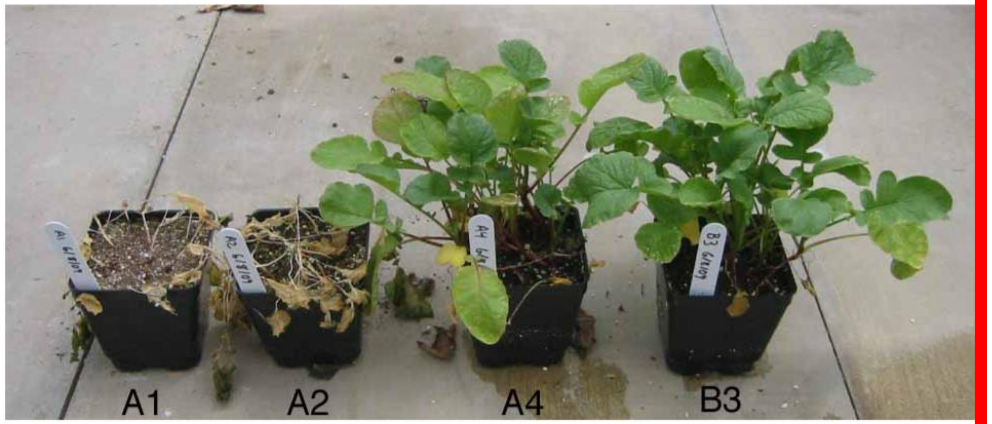
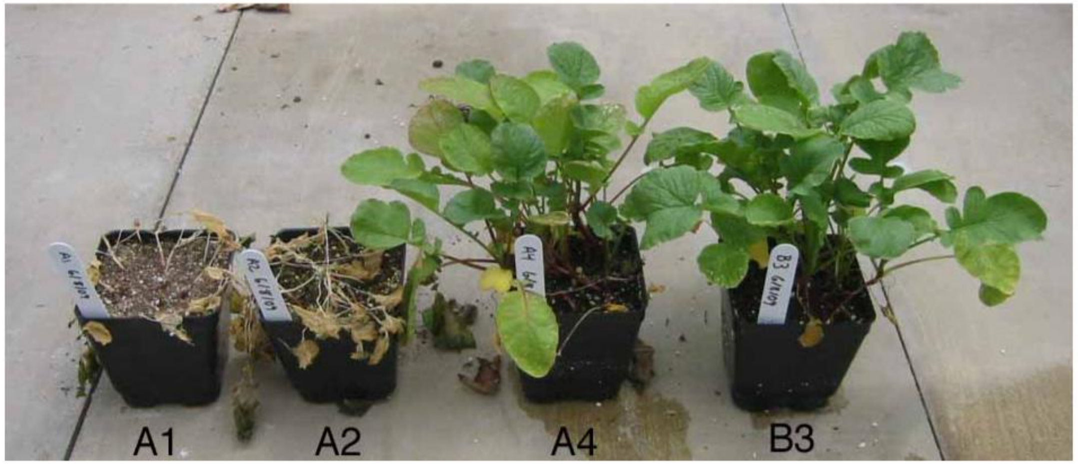


Figure 1. Radish plants showing the effects of freezing at -15C for 2h (A1 and A2) compared with control plants (A4 and B3) kept at room temperature. The plants in pots A1 and A4 were cold acclimated for 2 days at 2.5 C prior to freezing or room temperature treatments. The plants in pots A2 and B3 were not cold acclimated and were kept at room temperature (~ 25C) for 2 days prior to freezing or room temperature treatments. Following the freezing or room temperature treatments, plants were kept in a greenhouse for one week.



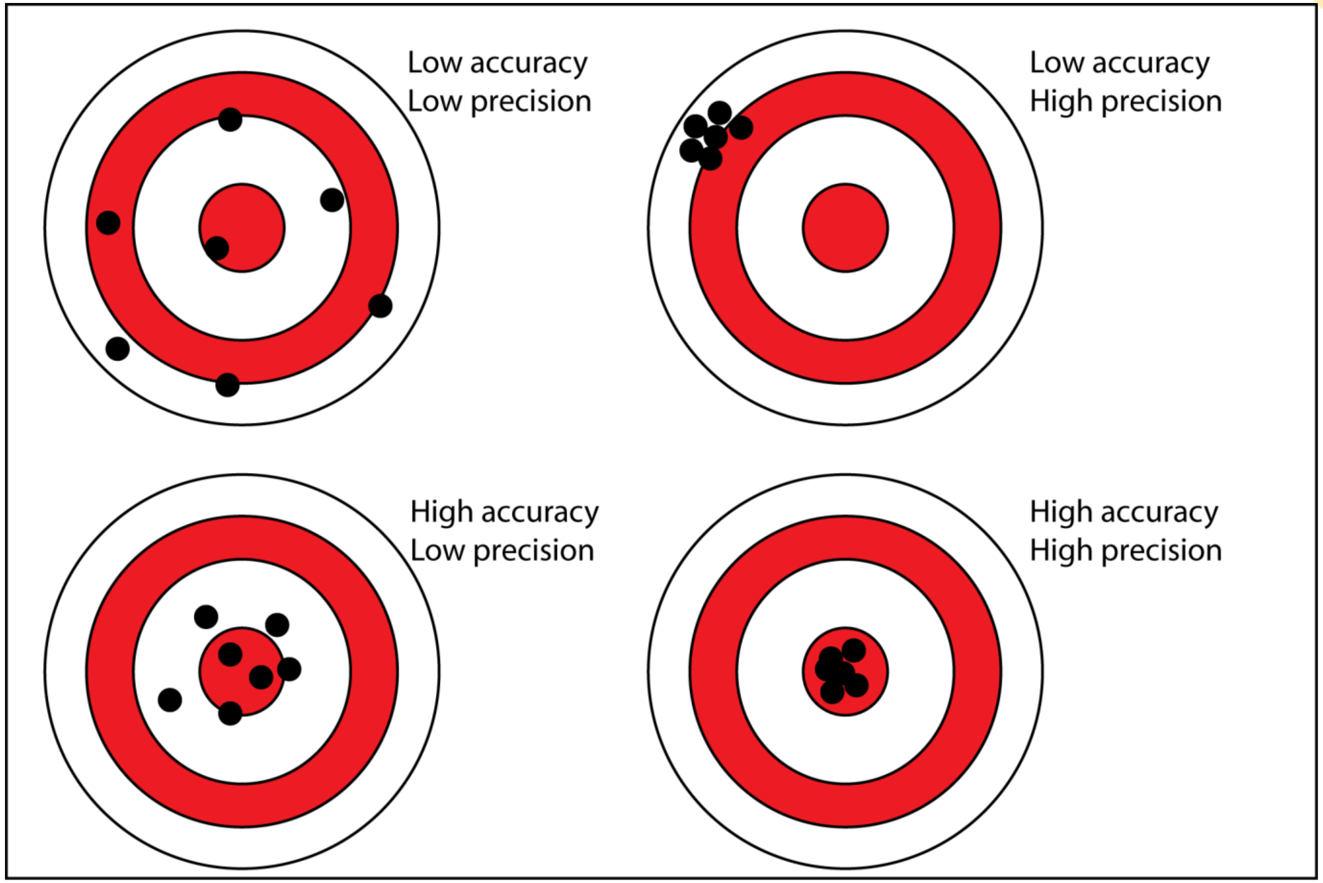
A grading rubrick for grading the final reports:

Writing and clarity

3. Writing is clear and concise (ca. 20% of total grade)
   - 5 pages with figures is SHORT!
   - Good writing: the process of eliminating *unnecessary words!*
   - Style and clarity count

A grading rubrick for grading the final reports:
Quantifying error

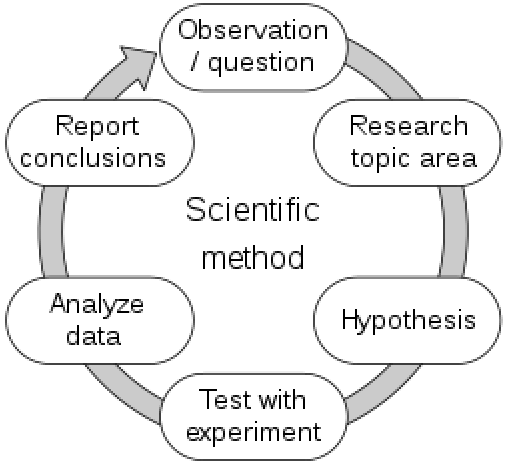
4. Appropriate use of **<u>error quantification</u>** (5%)
   - Use of error analysis where possible to generate error bars
   - Any discussion of error should **<u>identify and quantify</u>** sources of error
   - Clear and consistent use of accuracy vs. precision
   - Clear and consistent recognition of **<u>experimental error</u>** as opposed to **<u>measured fluctuations</u>** of the physical phenomenon



A grading rubrick for grading the final reports:

Applying the scientific method

5. Use of the **scientific method** (25%)
   - Science is inherently hypothesis driven – the rationale for a given hypothesis is presented in the introduction, leading to the open question the experiment intends to address
   - Confidence is built by hypothesis formulation and testing, which can rule out spurious / errant conclusions. This process should be described in the report or supplementary information.
   - Conclusions are well-supported by data, within measurement error.
   - Future prospects, prospective hypotheses suggested by the work and broad scientific outlook are presented in `discussion' or `conclusion' paragraph(s) in the report.



En.Wikipedia: Scientific_method

Contribution statement
all students submit **individually,** to me, directly

6. “CRediT offers authors the opportunity to share an accurate and detailed description of their diverse contributions to the published work.” (15% of the grade)

CRediT author statement | Elsevier

Sample CRediT author statement

Zhang San: Conceptualization, Methodology, Software **Priya Singh.**: Data curation, Writing- Original draft preparation. **Wang Wu**: Visualization, Investigation. ***Jan Jansen****:* Supervision.**:** **Ajay Kumar**: Software, Validation.: **Sun Qi:** Writing- Reviewing and Editing,

A grading rubrick for grading the final reports:
Going above and beyond

7. `Je ne sais quoi' – the effort of the measurement & attempts to reduce noise, and provide excellent, reliable measurements. (up to 5% of total grade)
   - You've learned a lot of methods for reducing noise, and enhancing measurement precision.
   - Here is your opportunity to show off – how precise are your measurements? How do they stand in comparison with the state of the art?
   - How well have you understood and quantified the various sources of error in the measurement?
   - Have you provided sufficient detail in the supplementary document for your report to support your error quantification & technical approach, including details of your simulation?
   - You can also score points for identifying paths toward an improved measurement in the discussion – perhaps changing the actuator in some way, or enhancing the load cell's precision, or changing the current range … however you might go about it, if you were given another chance, more time and unlimited equipment, what would you do to improve your measurement?

A summary of the rubrick for grading the final reports:

Grade rubrik for ME 412		Module 1, Group			
Category	Percentage	**Total percentage possible**		Remarks	
PRL Format		5			
Figures		30			
Writing		20			
Statistics and error		5			
Scientific Method		25			
Contribution statement		15		Individuals submit this. Use the CRediT form from Elsevier for the desired taxonomy:	[CRediT author statement | Elsevier]()
JNSQ		5			
Total score	0	105			

Contact hours & teaching staff

- The course is a 4 hour class. This means that you should budget 4 hours for lab activities on Wednesday morning, and an ***additional 4 hours*** off campus to complete the readings, and complete module reports.
- Chenzhuo Li, Lebo Molefe and Xinyue Wei will help help to teach the class & attend ad-hoc office hours on Tuesday afternoons.

Schedule for the class

Week	Module	Plan for contact hours	Outside of class – readings for lecture
1	I	Intro to course – grading, expectations; intro to module; intro to exercises & background – passives and filters	Reading on Oscilloscopes, review syllabus
2	I	Transistors and amplifiers	Assigned readings
3	I	Wheatstone Bridge & driving circuitry	Assigned readings
4	I	Calibrate samples with 4-wire measurement, debug / calibrate bridge circuit	Assigned readings
5	I	Measurements, report preparation and writing	Work on report
6	II	Intro. To instrumentation & experimental prep – DIC lab	Assigned readings
7	II	Prepare samples for measurement; first measurements	
8	II	Conclude experiments & process data	Assigned readings & software preparation for DIC
9	II	Measurements, report writing	Measurement & Work on report
10	II	Conclude processing & write lab report	Work on report
11	III	Lecture, discussion & exercises – Brownian motion	Prepare reading; evaluate report of alt. group from Mod II
12	III	Experimental set-up & preparation	Assigned readings & software preparation for particle tracking
13	III	Conclude experiments & process data	
14	III	Complete data processing & write reports	

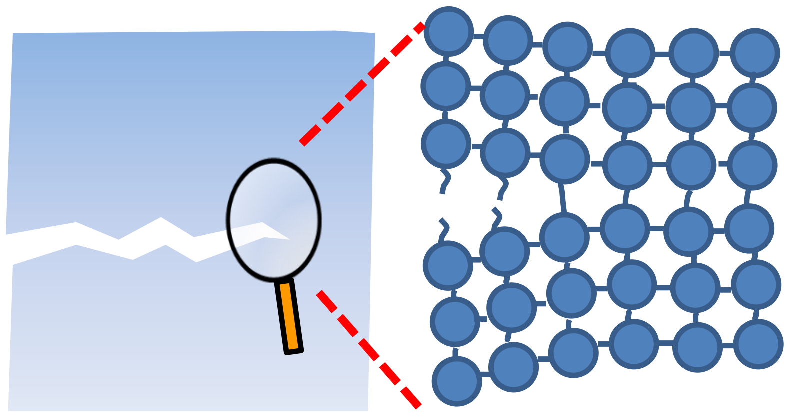
Measuring work of fracture with analog electronics

Reference: Polyacrylamide hydrogels. VI. Synthesis-property relation.
Wang, N., Nian, Guodon, Junsoo Kim, Zhigang Suo

ME 412

How do things Break?

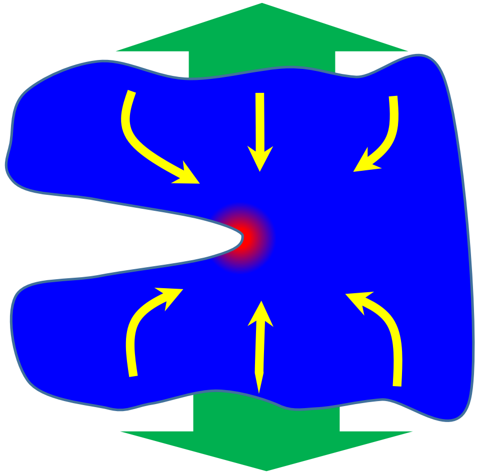
The Griffith approach (1922) –



Materials break when there is enough *elastic energy* (J/m^2) to drive a crack

Number of bonds per unit area (~10^{20}m^{-2}) × Bond energy (~10^{-19}J) = Fracture energy (1-10 J/m^2)

The energy supply comes from the elastic energy stored in the solid – due to the applied stress



➔ Fracture is a matter of energy balance

Elastic energy stored in the sample = dissipation / surface energy

$$G = \Gamma$$

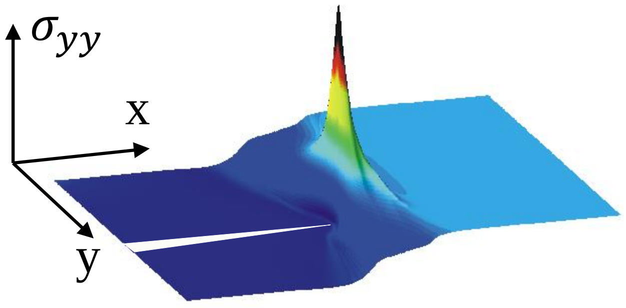
How do things Break?

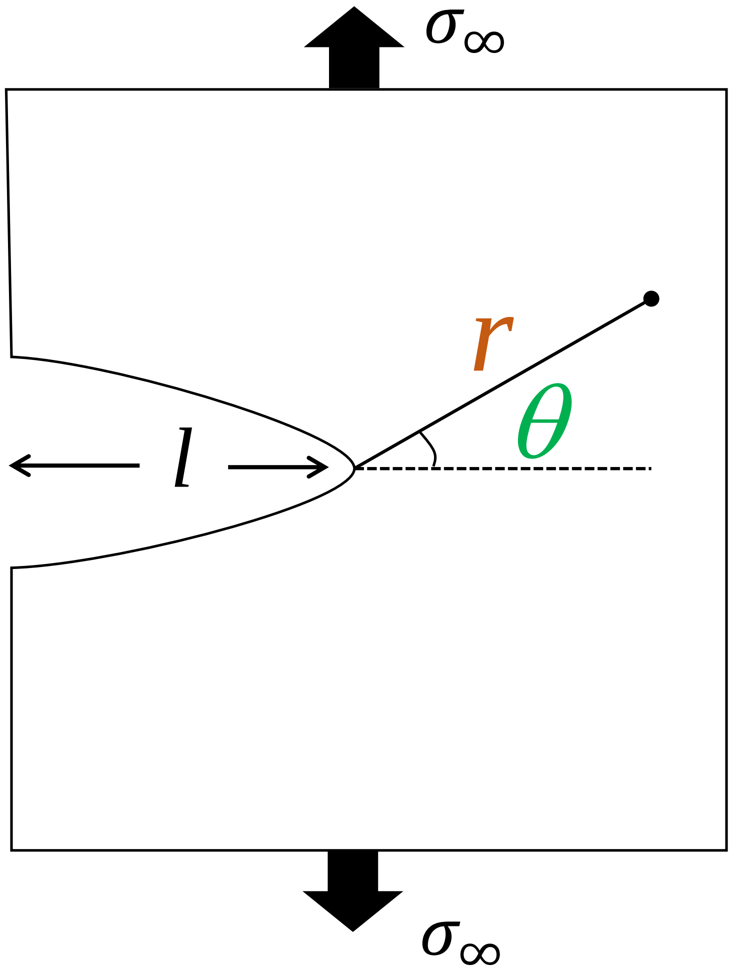
Linear Elastic Fracture Mechanics (LEFM) 1950's -

Cracks create ***singular stresses*** at their tip

As $r \to 0$, the stress ***diverges*** (Freund, 1990)

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta, \mathrm{v}) + \mathcal{O}(1)$$





K = stress intensity factor ($\sim \sigma_\infty \sqrt{l}$)

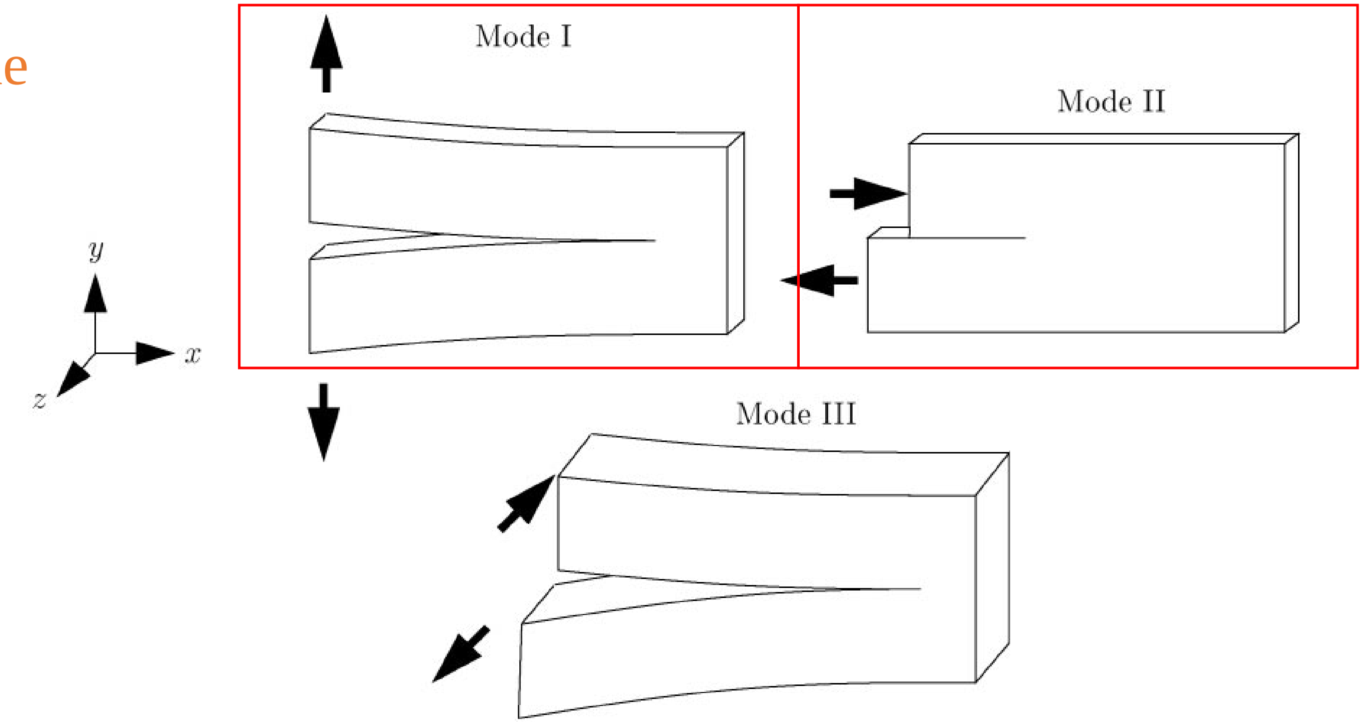
$f_{ij}(\theta, \mathrm{v})$ - dimensionless functions of ***angle*** and ***crack speed***

How do things Break?

LEFM – path selection and crack plane loading

There are ***3 fracture modes***: characterized by the **symmetry of the loading** on the crack plane.

$$\sigma_{ij} = \sum_{\alpha=\mathrm{I}}^{\mathrm{III}} \frac{\boldsymbol{K}_{\alpha}}{\sqrt{2\pi r}} f_{ij}^{\alpha}(\mathrm{v}, \theta)$$

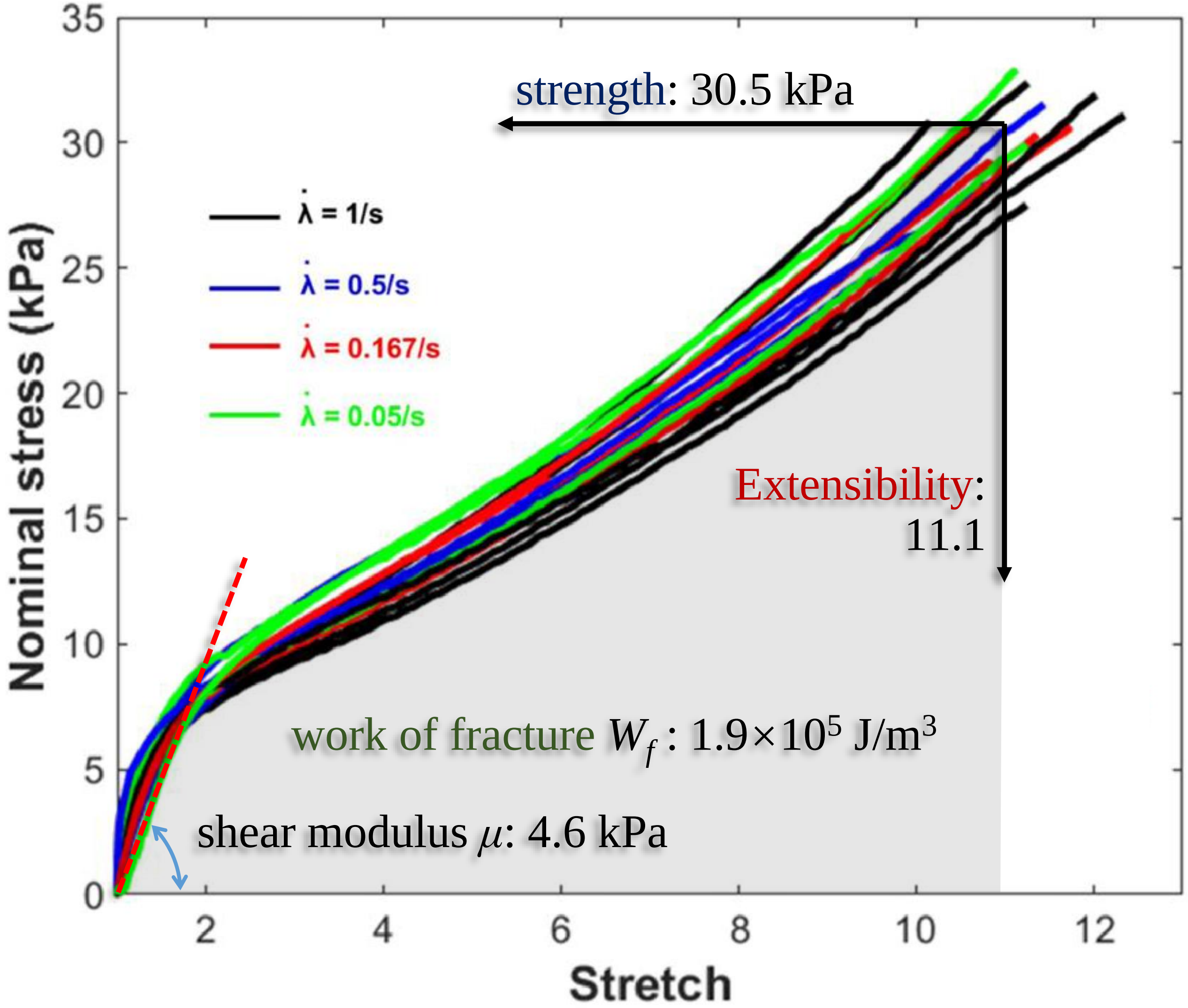


Because the ***stress is singular,*** there is ***no a-priori path selected*** for a crack

Our ***applied loading*** is ***tensile*** loading → Mode I

But remember …. Mode II

A brief introduction to Work of Fracture



Strength	stress of fracture
Extensibility	stretch of fracture
Work of fracture	the area beneath the stress–stretch curve

Calculation of shear modulus

Neo-Hookean material:

$$W = \frac{1}{2}\mu\left(\lambda_1^2 + \lambda_2^2 + \lambda_3^2 - 3\right)$$

Clamp constrain: $\lambda_1 = 1$,

Incompressible: $\lambda_3 = 1/\lambda_2$,

$$W = \frac{1}{2}\mu\left(\lambda_2^2 + \lambda_2^{-2} - 2\right)$$

Nominal stress $s_2 = dW/d\lambda_2$,

$$s_2 = \mu\left(\lambda_2 - \lambda_2^{-3}\right)$$

Small deformation $\lambda_2 \to 1$,

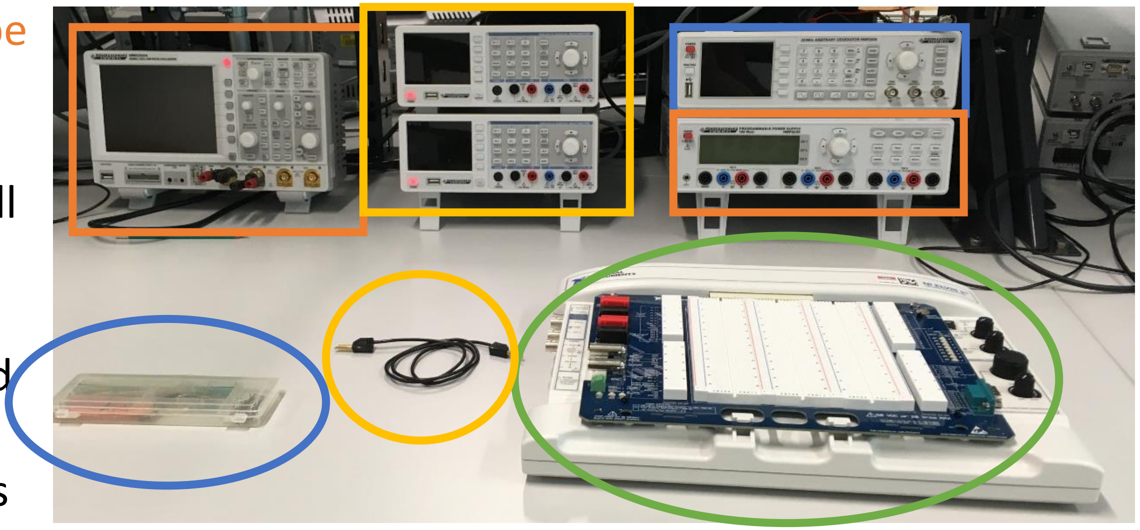
$$s_2 = \underline{4\mu}(\lambda_2 - 1)$$

Linear

Upload your data in an excel work book – calibrated loading curves, and the associated work of fracture – on the wiki

Equipment for analog electronics module

You'll be using the lab benches in MED 2 2419. The lab bench layout is standard at each desk, and includes a 3-channel power supply, a function generator, two multi-meters and an oscilloscope (see the layout at right).



Also available are a standard breadboard and jumper wire, as well as BNC connectors (not shown) and banana plug connectors.

Circuit components are found near the entrance to the room, and include all components from passives (resistors, capacitors, inductors and diodes) to actives, up to and including the op-amps and precision voltage references.

On day one of your on campus presence, you'll form your group for the semester by our algorithm. Groups are formed at random.

Analog electronics I: Fundamental laws, Passives, V-dividers.

Two Fundamental laws: Ohm and Kirchoff

Ohm's law & the hydrodynamic analogy

$$V = I R \quad \text{(Ohm)}$$

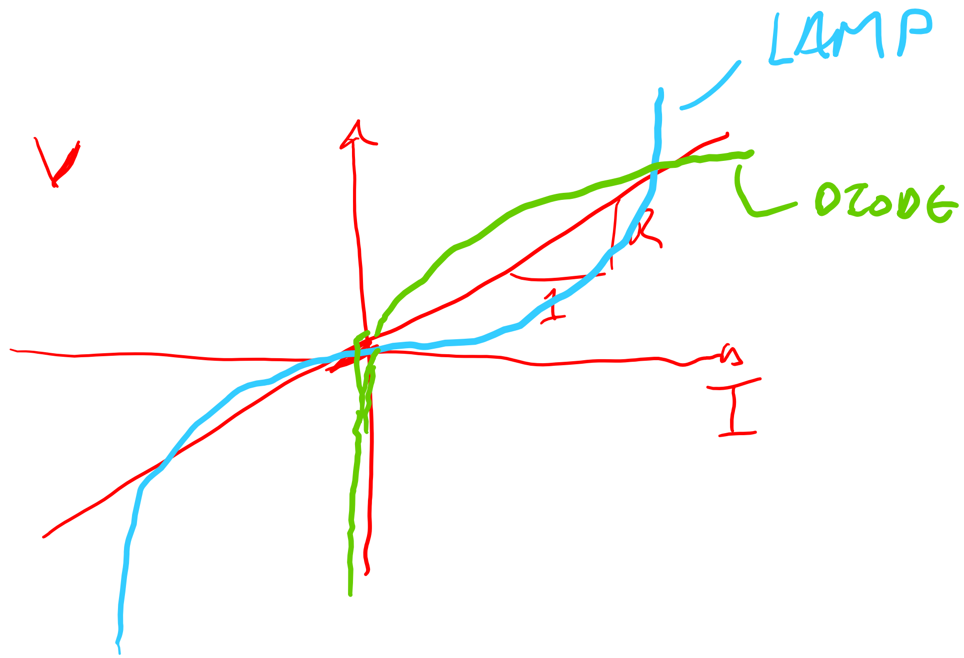
VOLTAGE — CURRENT — RESISTANCE

HYDRODYNAMIC ANALOGY:

$$H = J \cdot R$$

H: HYDROSTATIC PRESSURE; J: FLUID FLUX; R: HYDRODYNAMIC RESISTANCE

Ex:



POWER DISSIPATION:

$$P = IV = I^2 R$$

DIMENSIONS: [I] = CURRENT / CHARGE/TIME

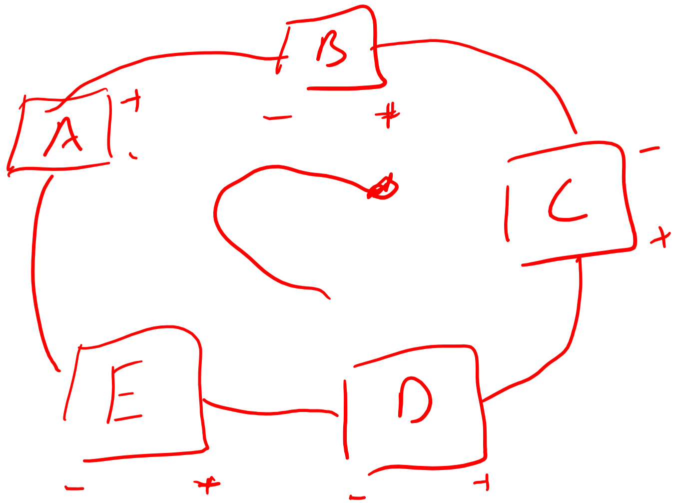
[V] = WORK (ENERGY)/CHARGE

[IV] = WORK/TIME → DIMENSIONS OF POWER.

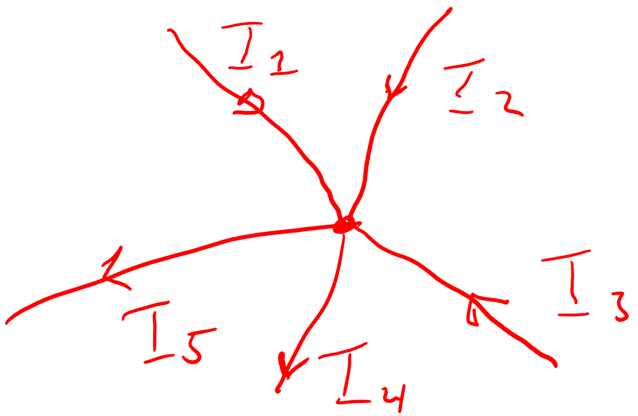
Two Fundamental laws: Ohm and Kirchoff

Kirchoff's laws I & II

I: SUM VOLTAGES AROUND A CLOSED LOOP IS ZERO:



II: SUM OF CURRENTS AT A NODE IS 0:

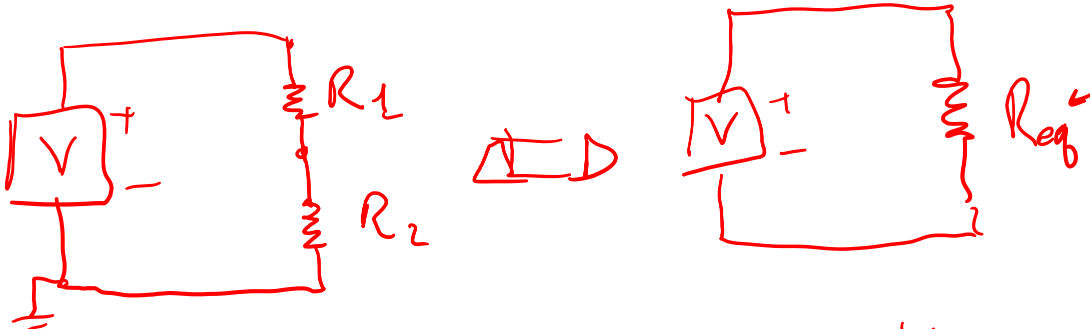


$$\sum_{i=1}^{5} I_i = 0$$

Two Fundamental laws: Ohm and Kirchoff

Kirchoff's laws examples

RESISTORS IN || & IN SERIES:



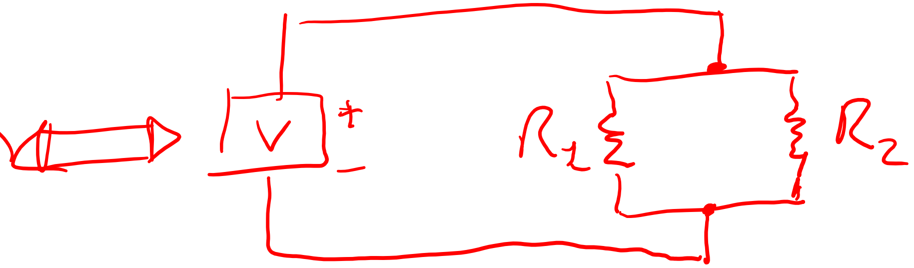
$$I_{TOT} = I_1 = I_2$$

$$V_{TOT} = V_1 + V_2$$

BY OHM: $I_1 R_1 + I_2 R_2 = I_{TOT}(R_1 + R_2)$

$$\Rightarrow R_{eq} = \frac{V_{TOT}}{I_{TOT}} = R_1 + R_2$$

IN PARALLEL:



$$V_{TOT} = V_1 = V_2$$

$$I_{TOT} = I_1 + I_2$$

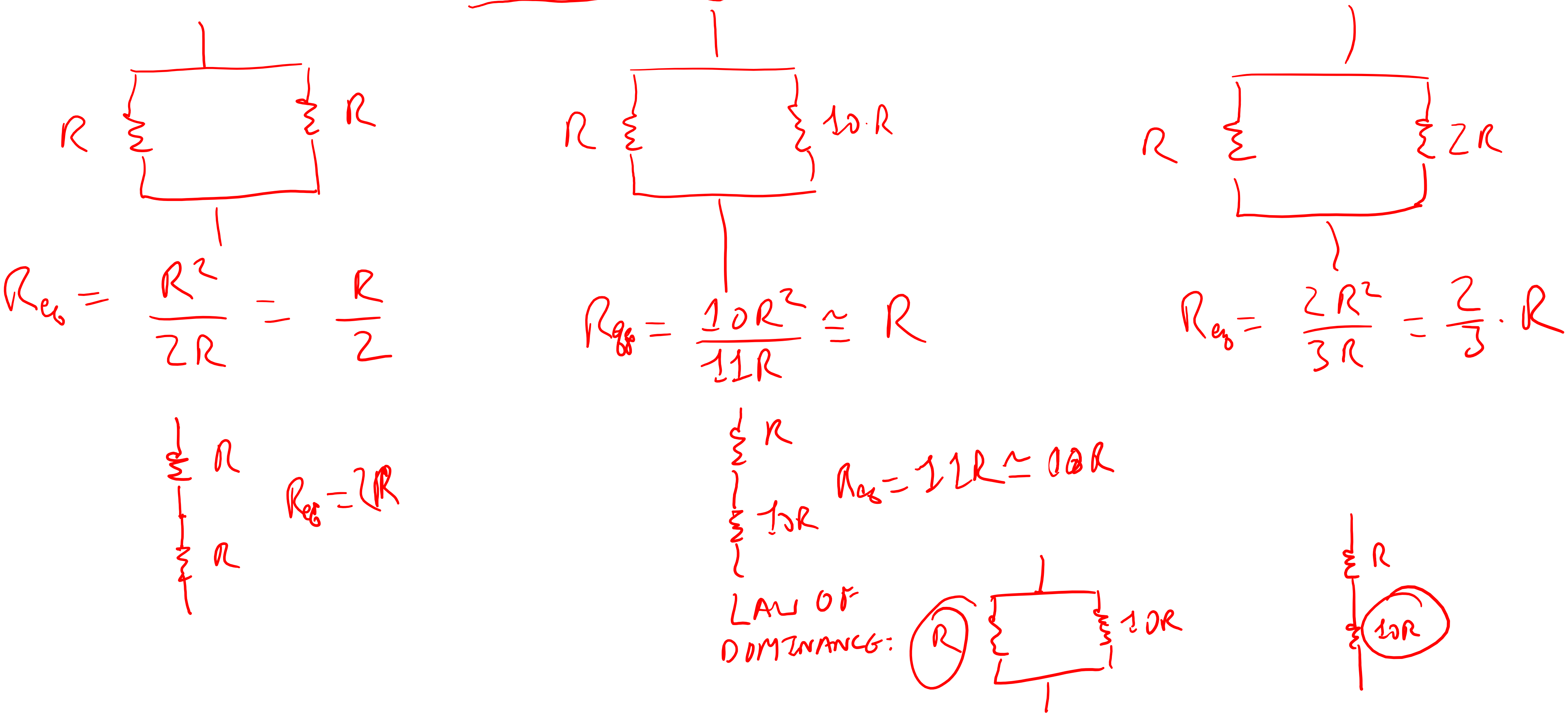
OHM: $I_1 = \frac{V_1}{R_1}$; $I_2 = \frac{V_2}{R_2}$

$$I_{TOT} = \frac{V}{R_1} + \frac{V}{R_2} = \frac{V_{TOT}}{R_{eq}}$$

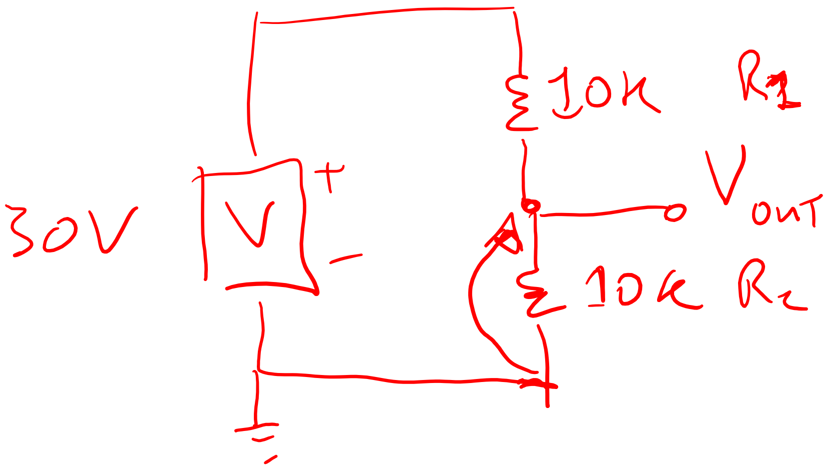
$$\Rightarrow \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_{eq}} \Rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Two Fundamental laws: Ohm and Kirchoff

Kirchoff's laws examples & the law of dominance

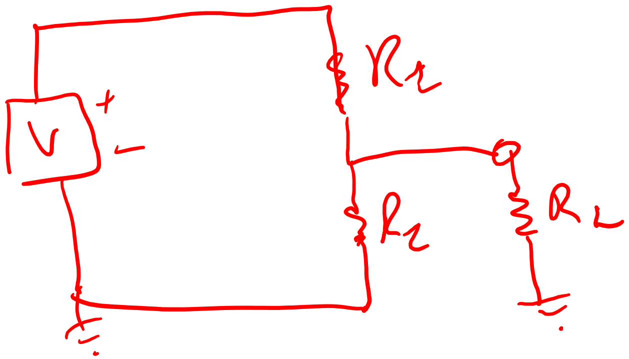


A device: V-divider



$$I = \frac{V_{in}}{(R_1 + R_2)} \;;\quad V_{out} = I R_2 = 15V.$$

APPLY THE DEVICE:



Thevenin equivalent circuits

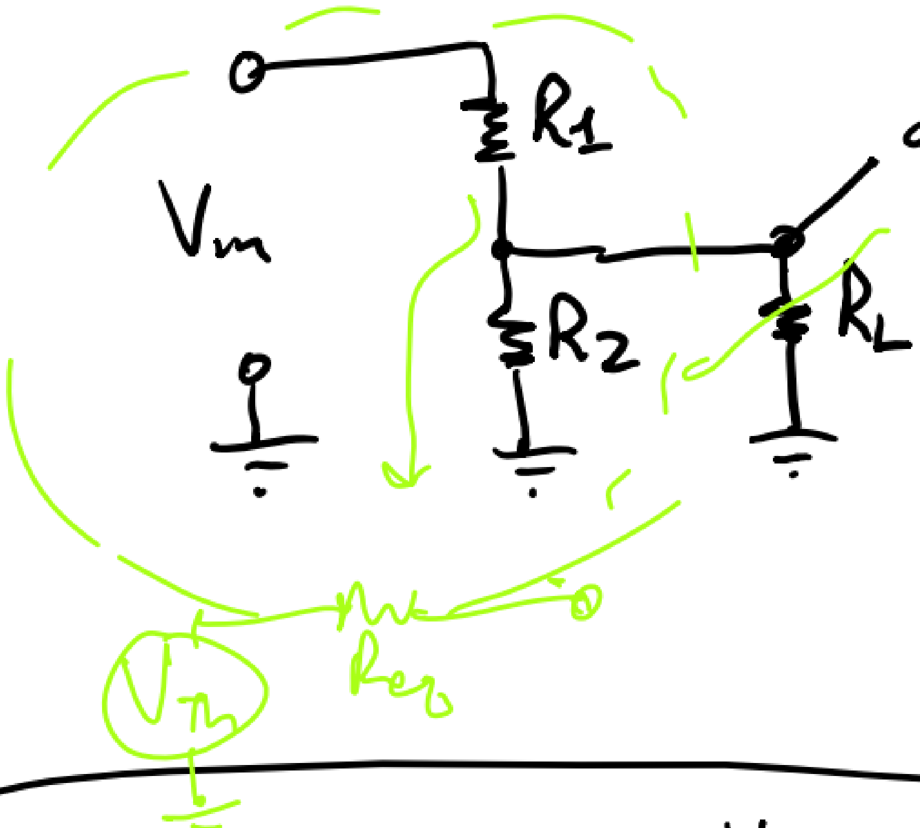
- For any two nodes in a circuit, irrespective of the circuits complexity (only linear circuit elements @ steady-state): An equivalent circuit composed of a voltage source U_{TH} and a resistance R_{TH} can be found.

* V_{TH} & R_{TH} can be found in 2 steps:
  1. $V_{TH} = V$ (open circuit) – no load!
  2. $R_{TH} = \dfrac{V_{TH}}{I \text{ (short circuit)}}$

R_{TH} is the output impedance of such a circuit

The voltage divider revisited: the Thevenin equivalent circuit and output impedance

Let's revisit the humble voltage divider circuit from last week:



1. V open circuit: $V_a = \frac{V_{in} R_2}{R_1 + R_2} = V_{Th}$.

2. $R_{Th} = \frac{V_{Th}}{I(\text{short})}$! If $R_L = 0\,\Omega$, a short circuit is achieved.

If this circuit is shorted, we find:

$$I = \frac{V_{in}}{R_1}.$$

Thus, $R_{Th} = \frac{V_{Th}}{I} = \frac{R_1 R_2}{R_1 + R_2}$. Note, this is $R_1 \| R_2$!

Reactive circuit elements & transient circuit response: capacitors

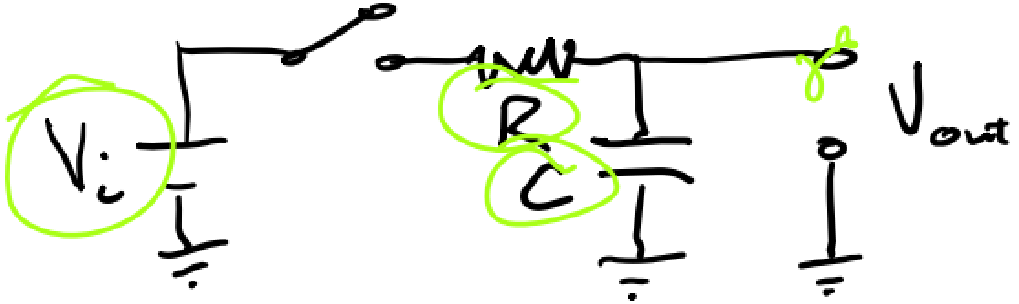
A ***capacitor with capacitance*** *C* stores charge (*Q*) in proportion to the applied voltage *V*. This can be expressed with an equation as:

$$Q = CV$$

Q is hard to quantify or measure in practice; instead, we reformulate this expression for current *I* to make it more manageable, by taking the time derivative of both sides:

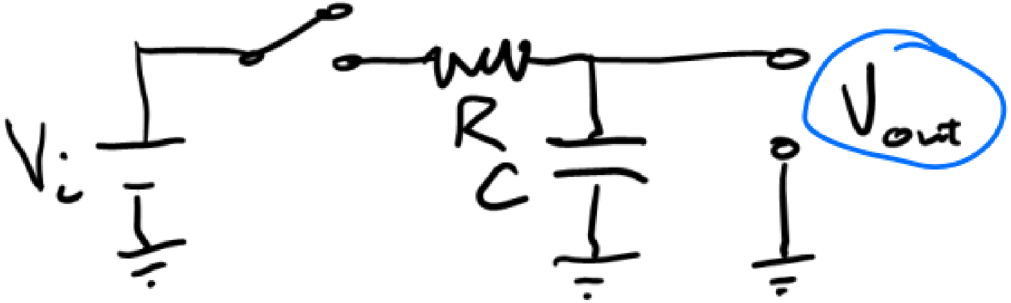
$$\frac{dQ}{dt} = C\frac{dV}{dt} \rightarrow I = C\frac{dV}{dt}$$

Since *I* now depends on the *temporal behavior* of *V*, we call capacitance a `reactive' circuit element -> current `reacts' to changes in voltage. A natural question is how this manifests itself with a simple circuit: let us look at the transient response of a simple circuit:



here, $I = \frac{V_i - V(t)}{R} = C\frac{dV}{dt}$.

solving this ODE: $V(t) = V_i + Ae^{-t/RC}$.

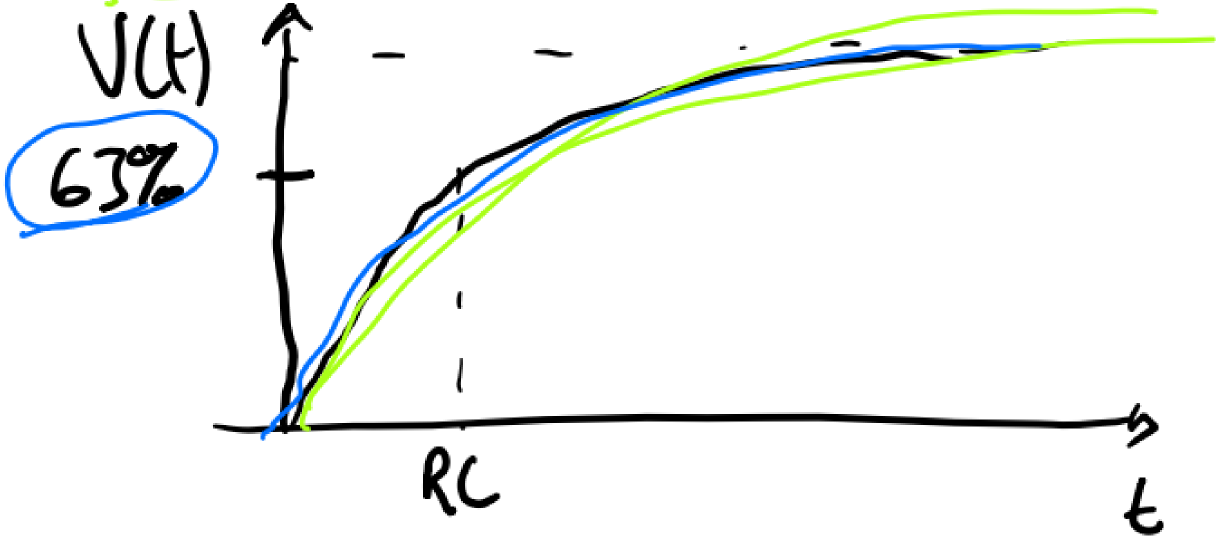


Solving this ODE: $V(t) = V_i + Ae^{-t/RC}$

We can find A by considering the initial condition:

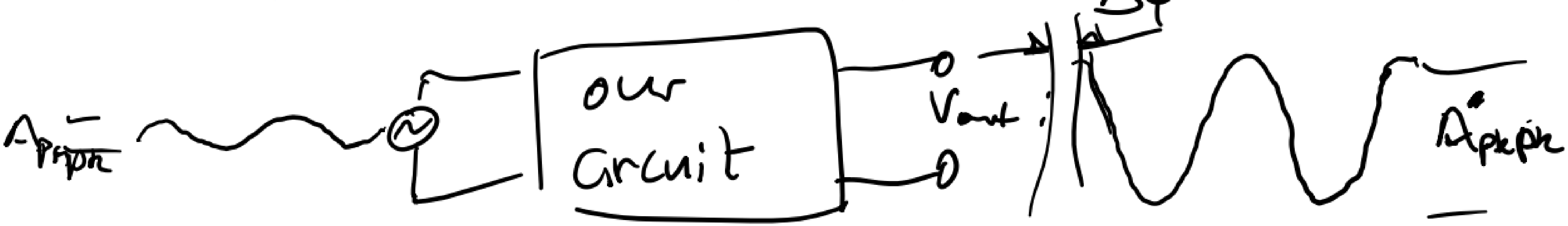
$$V(0) = 0 = V_i + Ae^{0} \Rightarrow A = -V_i.$$

Thus, $V(t) = V_i\left(1 - e^{-t/RC}\right)$:



A/C circuit analysis – looking at sinusoidal* signals

Since our circuits at this stage are *Linear*, the output of a sinusoidally driven circuit is itself a sinusoid with the *same frequency* and at most a modified ***amplitude*** and ***phase***



Sinusoids are parameterized by an ***amplitude*** and a ***phase*** – this structure lends itself well to a description with complex variables, which also have two quantities of merit – the ***real*** and ***imaginary*** part. The phasor description, with e.g. $V = V_0 e^{j\varphi} = V_0\,(\cos\varphi + j\sin\varphi)$; for a signal oscillating with frequency ω, the phase of this signal is ωt

* For the math experts, the Fourier series is a sum of sinusoids, and with appropriate weights can approximate any periodic function over an interval. Thus analyzing the response to an arbitrary sinusoid, and the linear property of RLC circuits means we can describe the circuits response to an *arbitrary* forcing.

Generalizing resistance: Impedance and the transfer function for `forcing' (V) to `response' (I)

Let's first consider a sinusoidal voltage applied to a capacitor:

Here, $I = C\frac{dV}{dt} = C\omega V_0 \cos(\omega t)$

Neglecting the phase, we can obtain the current as:

$I = \frac{V}{1/\omega C}$; thus the capacitor behaves like an ω-dependent resistor!

RECALL: $I = \frac{V}{R}$

To account for phase correctly, we'll write $V = Re\left(V_0 e^{j\omega t}\right)$.

Since $I = C\frac{dV}{dt}$, $\Rightarrow I(t) = Re\left(j\omega C V_0 e^{j\omega t}\right) = Re\left(\frac{V_0 e^{j\omega t}}{-j/\omega C}\right)$.

Thus, we define a reactance X_c for the capaci:

$$X_c = \frac{V}{I} = \frac{-j}{\omega C}$$

(n.b. this is structurally similar to R from Ohm's law! $R = \frac{V}{I}$.)

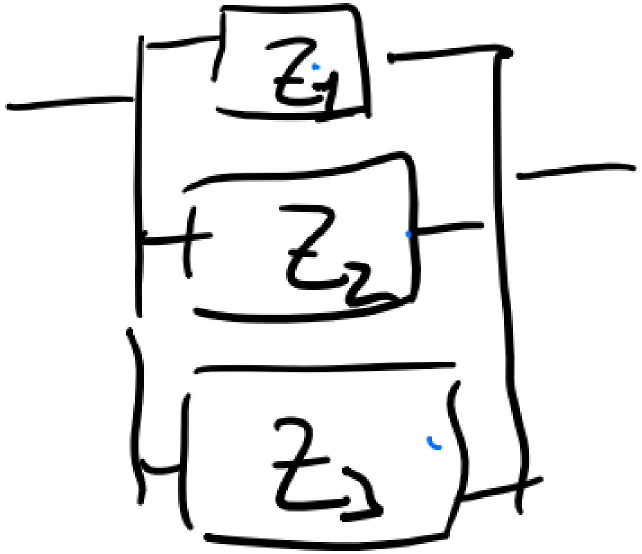
We can now define a generalized Impedance, Z, which in general is complex:

$$I = \frac{V}{Z} \iff V = IZ.$$

Z's follow R's rules for addition in || & series:

: $Z_{eq} = Z_1 + Z_2 + Z_3$.

In ||:



:

$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}}.$$

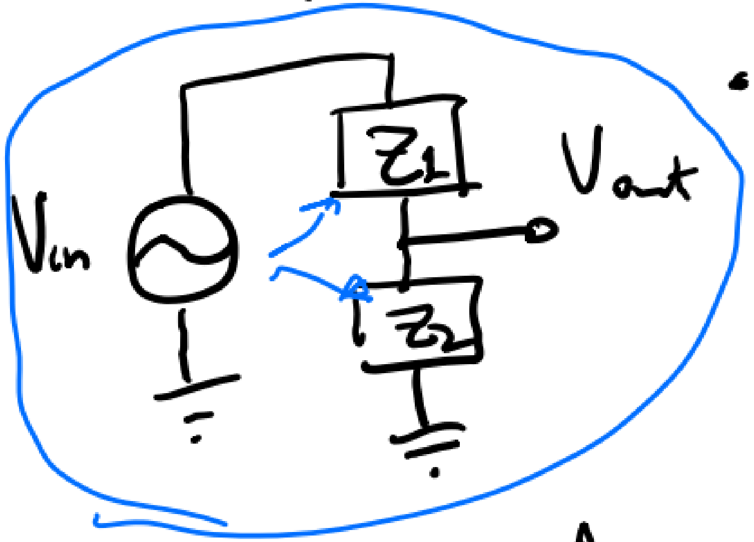
Generalized impedance for passive circuit elements:

$$Z_R = R \quad \text{(resistors)}$$

$$Z_I = j\omega L \quad \text{(inductors)}$$

$$Z_C = -j/\omega C \quad \text{(capacitors)}.$$

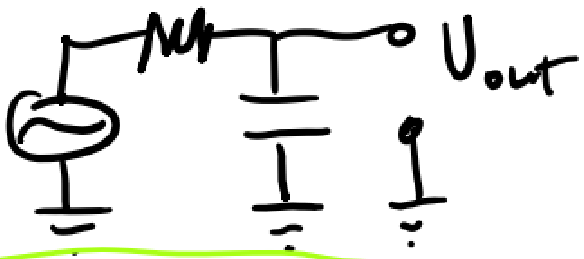
Generalizing the V-divider: frequency dependent response and filter devices



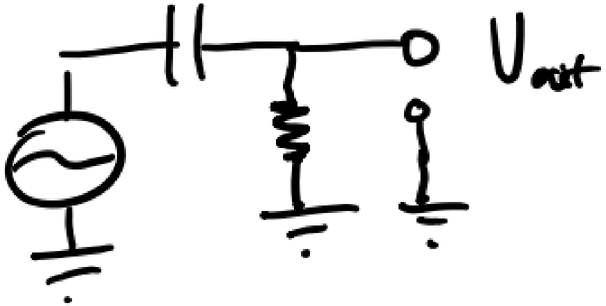
- The analysis is essentially the same as before:
  - $I = \frac{V_{in}}{Z_{tot}}$, $Z_{tot} = Z_1 + Z_2$.
  - $V_{out} = I Z_2 = \frac{V_{in} Z_2}{Z_1 + Z_2}$.

But now, Z is in general complex – and thus ω-dependent!

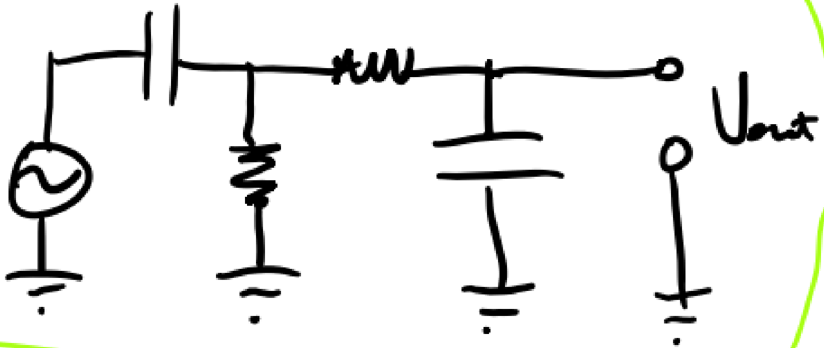
Low-pass filter:



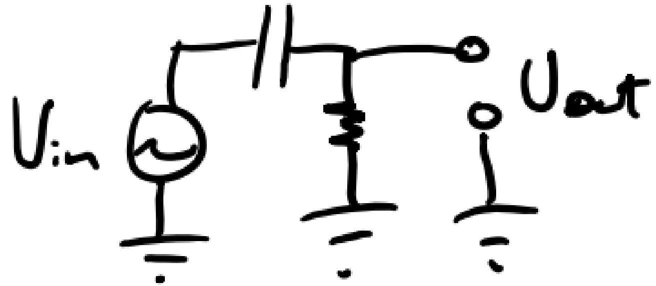
high-pass filter:



band-pass filter:



Analysis of the high-pass filter



By generalized Ohm's law: $I = \frac{V_{in}}{Z_{tot}} = \frac{V_{in}}{R - j/\omega C}$

$$= \frac{V_{in}(R + j/\omega C)}{R^2 + (1/\omega C)^2}$$

$$\Rightarrow V_{out} = I Z_R = \frac{V_{in}(R + j/\omega C)R}{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

Since we'll focus on amplitude, not phase, we'll work w/

$$|V_{out}| = (V_{out}^* V_{out})^{1/2} = \frac{R V_{in}}{\left[R^2 + \left(\frac{1}{\omega C}\right)^2\right]^{1/2}} = \frac{R\omega C}{(1 + (R\omega C)^2)^{1/2}} V_{in}$$