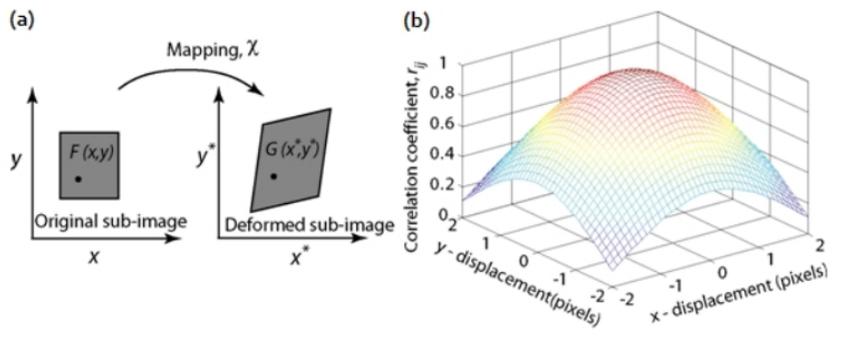
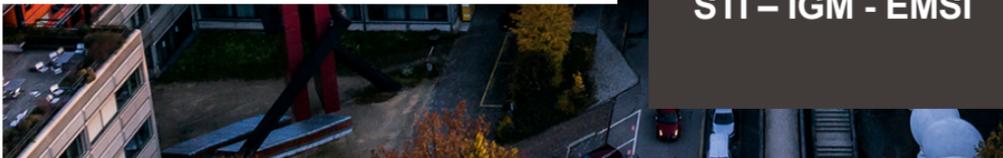




2D Digital Image Correlation with open-source NCORR

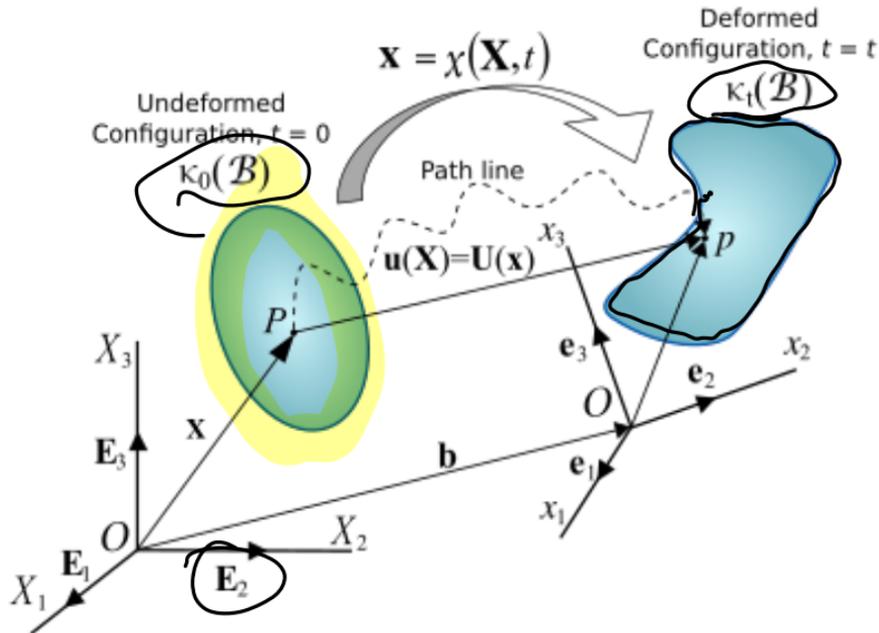


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What problem does DIC address?

DIC is an **optical method** used to track **full-field displacements**



- The main coordinate directions forming a right-handed basis are indicated by \mathbf{E}_i (i is an index from 1-3)
- \mathbf{X} is the **location of a material point P** in the **undeformed / reference configuration**
- \mathbf{x} is the **laboratory coordinate** of the material point \mathbf{X} at time t
- \mathbf{u} is the **displacement vector** from \mathbf{X} to \mathbf{x}

The basics of deformation: *getting from displacements to strains*

In continuum mechanics, the kinematics of **deformation** are entirely described by the **deformation gradient tensor F** :

$$F = I + \frac{\partial u_i}{\partial X_j}$$

Here, I is the identity tensor, and \mathbf{u} , \mathbf{X} are the displacement vector and material coordinate defined before

This tells a piece of the story; essentially, $d\mathbf{x} = \mathbf{F}d\mathbf{X}$ however, we are interested in **specific displacements** – in particular **strains** as these are the fundamental quantities used in **Solid Mechanics** to find the **stress distribution** (more on that to come later)

To get to the non-linear strains of interest, we do some simple manipulations. First we map the lengths of material elements to the lengths of deformed elements:

$$\underline{d\mathbf{x} \cdot d\mathbf{x}} = \underline{\mathbf{F}d\mathbf{X} \cdot \mathbf{F}d\mathbf{X}} = \underline{d\mathbf{X} \cdot (\mathbf{F}^T \mathbf{F})d\mathbf{X}}$$
$$\underline{ds^2} = \underline{d\mathbf{X} \cdot \mathbf{C}d\mathbf{X}}$$

Here we've introduced the right Cauchy-Green deformation tensor $\mathbf{C} = \mathbf{F}^T \mathbf{F}$

The basics of deformation: *getting from displacements to strains*

Recalling the definition of F , and substituting in to the definition for C :

$$F = I + \frac{\partial u_i}{\partial X_j} \rightarrow C = F^T F = \left(I + \frac{\partial u_i}{\partial X_j} \right)^T \left(I + \frac{\partial u_i}{\partial X_j} \right) = I + \frac{\partial u_i}{\partial X_j} + \left(\frac{\partial u_i}{\partial X_j} \right)^T + \left(\frac{\partial u_i}{\partial X_j} \right)^T \left(\frac{\partial u_i}{\partial X_j} \right)$$

Taking note that in the absence of deformation, $C = I$; this corresponds to rigid body motion or **rotation**.

Because we are interested exclusively in **deformation** insofar as we **don't** want to include rigid body motion in our analysis of strain and stress distribution, we take only the **non-identity portion** of C , which now includes deformation of the body out to second-order; this is called the **Lagrange strain tensor E** :

$$E_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial X_j} + \frac{\partial u_j}{\partial X_i} + \frac{\partial u_j}{\partial X_i} \frac{\partial u_i}{\partial X_j} \right)$$

E 's components in 2D for coordinates \mathbf{x} , \mathbf{y} and displacements \mathbf{u} , \mathbf{v} :

$$\begin{aligned} E_{xx} &= \frac{1}{2} \left(2 \frac{\partial u}{\partial X} + \left(\frac{\partial u}{\partial X} \right)^2 + \left(\frac{\partial v}{\partial X} \right)^2 \right) \\ E_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X} + \left(\frac{\partial u}{\partial X} \frac{\partial u}{\partial Y} \right) + \left(\frac{\partial v}{\partial X} \frac{\partial v}{\partial Y} \right) \right) \\ E_{yy} &= \frac{1}{2} \left(2 \frac{\partial v}{\partial Y} + \left(\frac{\partial u}{\partial Y} \right)^2 + \left(\frac{\partial v}{\partial Y} \right)^2 \right) \end{aligned}$$

These are used by NCORR to calculate strains from the measure displacement fields, and account for material stretching as well as shear.

The basics of linear elasticity

In most applications of DIC, you might be interested to know not just the strain field, but also the **stress field**. This can be used to find where stresses are **highly concentrated and**, and allow you to do in-situ testing on real samples, thus providing insight into sample stability among other important aspects related to structural engineering.

Here we'll briefly introduce the foundations of linear elasticity:

The goal of elasticity theory is to map our **deformation field** onto a **stress field**. In the most general sense, for fully-3D linear elasticity, there are **81 elastic coefficients (!)** in the elasticity tensor **C** that map from the 9 independent coefficients of **strain** to the 9 independent coefficients of **stress**:

$$\tau_{ij} = \hat{C}_{ijkl} \epsilon_{ij}$$

But, both the strain and stress tensors are symmetric as long as there are not body torques; thus we reduce the number of **C's** coefficients from 81 -> 36 (keep in mind, we're now mapping 6 strain components onto 6 stress components and $6 \times 6 = 36$).

The basics of linear elasticity

The next step is more subtle. We have to realize that the concept of 'elasticity' implies a **stored energy function**, $U(\mathbf{E}_{ij})$, called the **strain energy function**; This positive definite function U is defined such that

$$\tau_{ij} = \frac{\partial U(\mathbf{E}_{ij})}{\partial E_{ij}}$$

This allows us to further reduce the number of independent coefficients of our elasticity tensor $\hat{\mathbf{C}}$ from 36 to 21. At this point, we've reached the most general form of the **anisotropic elasticity tensor**; this is relevant for composite materials. While this is clearly an important limit, I'll proceed to develop the elastic coefficients for **isotropic materials**:

Suppose our material is **homogeneous** and **isotropic**:

- The elastic properties do not change as a function of position in the material
- The elastic properties are independent of the direction of deformation.

Then we can write our elasticity tensor as the sum of the 3 isotropic fourth order tensors using constants λ , α , and β as:

$$\hat{\mathbf{C}}_{ijkl} = \lambda \delta_{ij} \delta_{kl} + \beta \delta_{ik} \delta_{jl} + \alpha \delta_{il} \delta_{jk}$$

Isotropic

2 coeffs

Then we can rewrite stress in terms of strain as:

$$\tau_{ij} = \hat{\mathbf{C}}_{ijkl} \mathbf{E}_{kl} = \lambda \delta_{ij} \delta_{kl} \mathbf{E}_{kl} + \beta \delta_{ik} \delta_{jl} \mathbf{E}_{kl} + \alpha \delta_{il} \delta_{jk} \mathbf{E}_{kl} = \lambda \mathbf{E}_{kk} \delta_{ij} + (\beta + \alpha) \mathbf{E}_{ij} = \lambda \mathbf{E}_{kk} \delta_{ij} + 2\mu \mathbf{E}_{ij}$$

The basics of linear elasticity

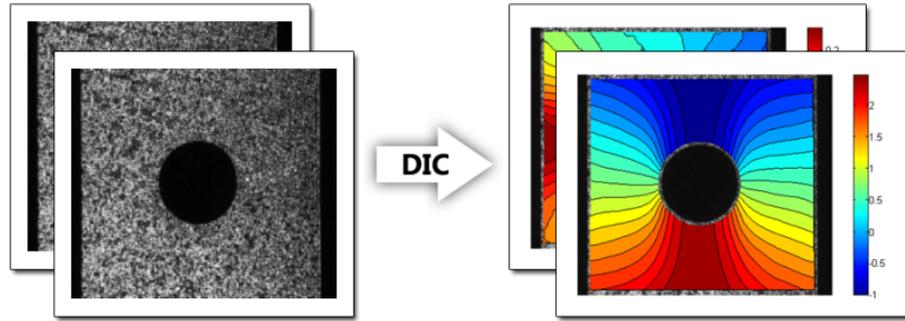
Thus we've shown that from 81 independent coefficients, for homogeneous isotropic linear elasticity, we have **2 independent** coefficients, called the 'Lamé coefficients'

$$\tau_{ij} = \lambda E_{kk} \delta_{ij} + 2\mu E_{ij}$$


TABLE 2.2 Relationships between Elastic Moduli

μ	k	λ	E	ν
$\frac{3(k-\lambda)}{2}$	$\lambda + \frac{2\mu}{3}$	$k - \frac{2\mu}{3}$	$\frac{9k\mu}{3k+\mu}$	$\frac{\lambda}{2(\lambda+\mu)}$
$\lambda \left(\frac{1-2\nu}{2\nu} \right)$	$\mu \left[\frac{2(1+\nu)}{3(1-2\nu)} \right]$	$\frac{2\mu\nu}{(1-2\nu)}$	$2\mu(1+\nu)$	$\frac{\lambda}{(3k-\lambda)}$
$3k \left(\frac{1-2\nu}{2+2\nu} \right)$	$\lambda \left(\frac{1+\nu}{3\nu} \right)$	$3k \left(\frac{\nu}{1+\nu} \right)$	$\mu \left(\frac{3\lambda+2\mu}{\lambda+\mu} \right)$	$\frac{3k-2\mu}{2(3k+\mu)}$
$\frac{E}{2(1+\nu)}$	$\frac{E}{3(1-2\nu)}$	$\frac{E\nu}{(1+\nu)(1-2\nu)}$	$3k(1-2\nu)$	$\frac{3k-E}{6k}$

Now, on to DIC...



As we already covered, the general problem that DIC solves is it will map from an *image* onto a *field of displacements*, from which we can *calculate strains*.

The strain field can then be used to find *stresses* provided we have a known *constitutive law*.

Be aware that not all materials will enable us to measure their stress state from the measured strains.

*The following presentation follows the algorithm used in NCORR, which is state-of-the-art as of 2015.

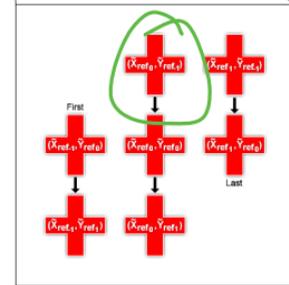
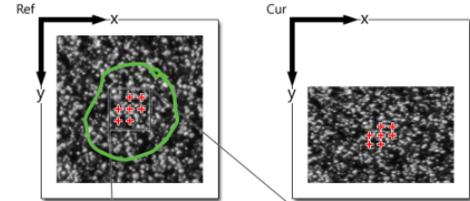
Assuming we have a speckled sample and some images, *how do we measure displacements?*

A key concept: the subset, S .

We will select a few material points as indicated by the red crosses to form a subset.

These material points will be used to identify the local displacement vector from the *reference configuration (measured in X)* to the *current configuration (measured in x)*

Typically, the mapping from the reference to the current configuration is assumed to be a *linear first order transformation*.



$S = \{(-1,0), (-1,1), (0,-1), (0,0), (0,1), (1,-1), (1,0)\}$
 (first i, first j) = (-1,0)
 (last i, last j) = (1,0)

$$\tilde{x}_{cur_i} = x_{ref_i} + u_{rc} + \frac{\partial u}{\partial x_{rc}} (x_{ref_i} - x_{ref_c}) + \frac{\partial u}{\partial y_{rc}} (y_{ref_j} - y_{ref_c}) \quad (i, j) \in S$$

$$\tilde{y}_{cur_j} = y_{ref_j} + v_{rc} + \frac{\partial v}{\partial x_{rc}} (x_{ref_i} - x_{ref_c}) + \frac{\partial v}{\partial y_{rc}} (y_{ref_j} - y_{ref_c})$$

This defines a transformation vector \underline{p} :

$$\underline{p} = \left\{ \underline{u} \quad \underline{v} \quad \frac{\partial u}{\partial x} \quad \frac{\partial u}{\partial y} \quad \frac{\partial v}{\partial x} \quad \frac{\partial v}{\partial y} \right\}^T$$

Measuring displacements, continued:

For computational convenience, we also allow the reference subset to translate in the reference frame; thus we introduce tilde variables that represent the deformation of the original subset once we've compared it to the **current subset**:

$$\tilde{x}_{ref_i} = x_{ref_i} + u_{rr} + \frac{\partial u}{\partial x_{rr}} (x_{ref_i} - x_{ref_c}) + \frac{\partial u}{\partial y_{rr}} (y_{ref_j} - y_{ref_c})$$
$$\tilde{y}_{ref_j} = y_{ref_j} + v_{rr} + \frac{\partial v}{\partial x_{rr}} (x_{ref_i} - x_{ref_c}) + \frac{\partial v}{\partial y_{rr}} (y_{ref_j} - y_{ref_c})$$

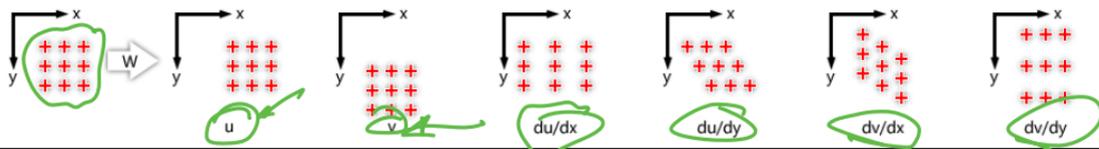
And here comes the key statement where we optimize the correlated mapping (the 'C' part in DIC):

We want to find the **optimal** p_{rc} mapping from the **reference** to the **current subset**, such that

- $p_{rr} = 0$,
- The $X(x_{ref}, y_{ref})$ best match $x(x_{cur}, y_{cur})$ in the reference and current images, respectively, for each value of X in the subset S

This optimal value of p_{rc} we'll call p_{rc}^*

Linear warpings of the subset include these operations:



A small note on the 'C' in DIC:

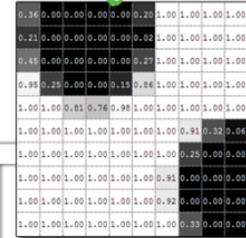
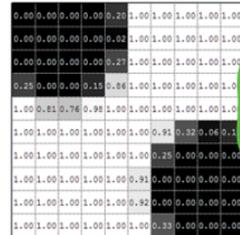
We need a means of comparing our mapping from the intensity values in the **reference image** to the intensity values in the **current image**:

$$C_{CC} = \frac{\sum_{(i,j) \in S} (f(\tilde{x}_{ref_i}, \tilde{y}_{ref_j}) - f_m)(g(\tilde{x}_{cur_i}, \tilde{y}_{cur_j}) - g_m)}{\sqrt{\sum_{(i,j) \in S} [f(\tilde{x}_{ref_i}, \tilde{y}_{ref_j}) - f_m]^2} \sqrt{\sum_{(i,j) \in S} [g(\tilde{x}_{cur_i}, \tilde{y}_{cur_j}) - g_m]^2}}$$

$$C_{LS} = \sum_{(i,j) \in S} \left[\frac{f(\tilde{x}_{ref_i}, \tilde{y}_{ref_j}) - f_m}{\sqrt{\sum_{(i,j) \in S} [f(\tilde{x}_{ref_i}, \tilde{y}_{ref_j}) - f_m]^2}} - \frac{g(\tilde{x}_{cur_i}, \tilde{y}_{cur_j}) - g_m}{\sqrt{\sum_{(i,j) \in S} [g(\tilde{x}_{cur_i}, \tilde{y}_{cur_j}) - g_m]^2}} \right]^2$$

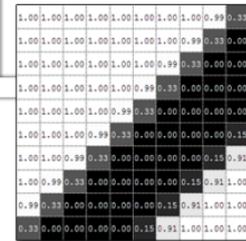
Here, f and g are the **reference** and **current** image intensities, respectively; f_m and g_m are the mean intensity values for the **subset**.

We'll use C_{LS} as this can be minimized in the non-linear optimization we'll perform to find the \mathbf{p}_{rc}^*
 -> note the behavior of these functions as they do or don't match a region:



$$C_{CC} = 0.8281$$

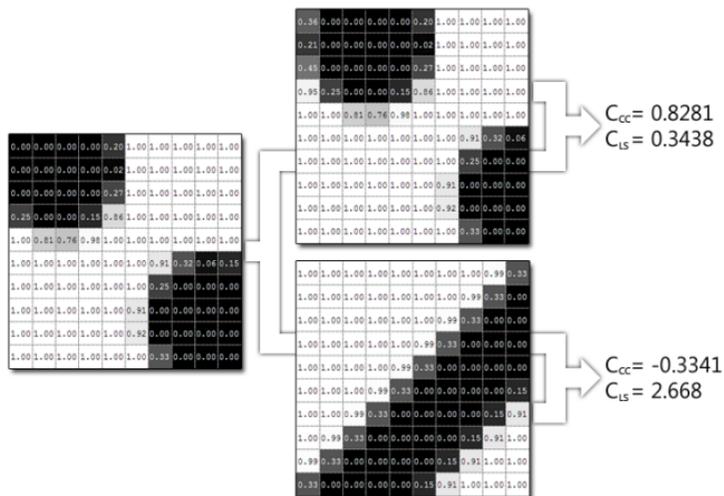
$$C_{LS} = 0.3438$$



$$C_{CC} = -0.3341$$

$$C_{LS} = 2.668$$

A small note on the 'C' in DIC:

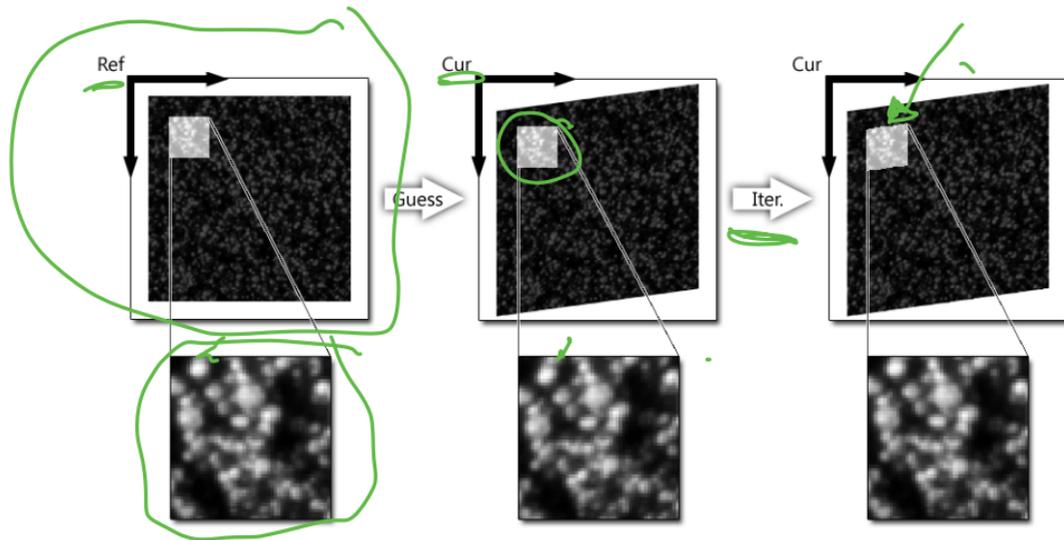


Now, C will vary with the selected mapping of p_{rc} :

This has important implications for the experiment: it essentially sets how far your material can displace from one frame to another

-> essentially, displacements should be sufficiently small for the correlation to have a **well-defined minimum** as p_{rc} is varied; otherwise, you will not be measuring the real displacement (!)

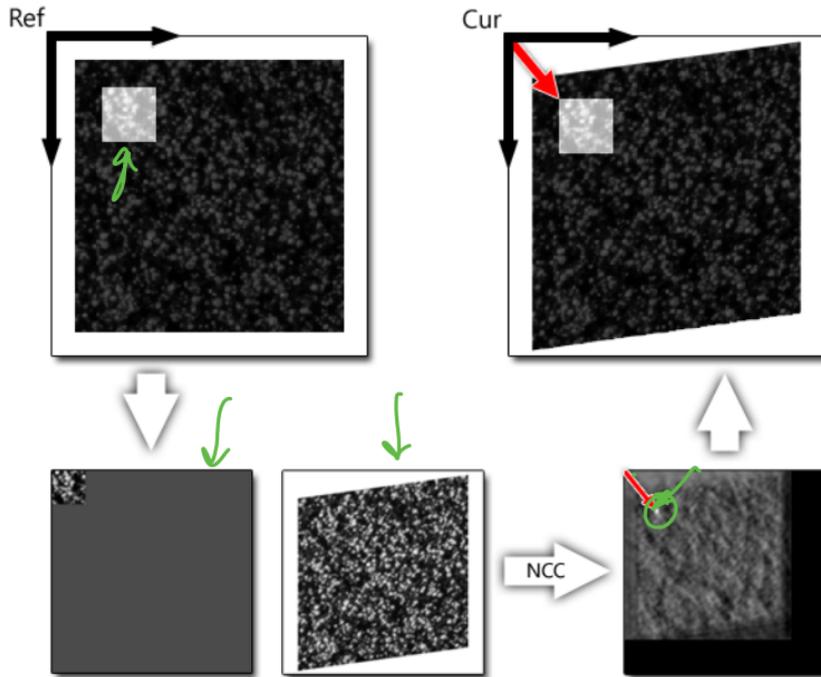
Closing the loop: non-linear optimization to find \mathbf{p}_{rc}^* :



Here we show a schematic depiction of the optimization scheme that we'll use to find \mathbf{p}_{rc}^*

1. Select a reference subset
2. Guess a \mathbf{p}_{rc}
3. Iterate until C_{LS} is minimized, identifying \mathbf{p}_{rc}^*

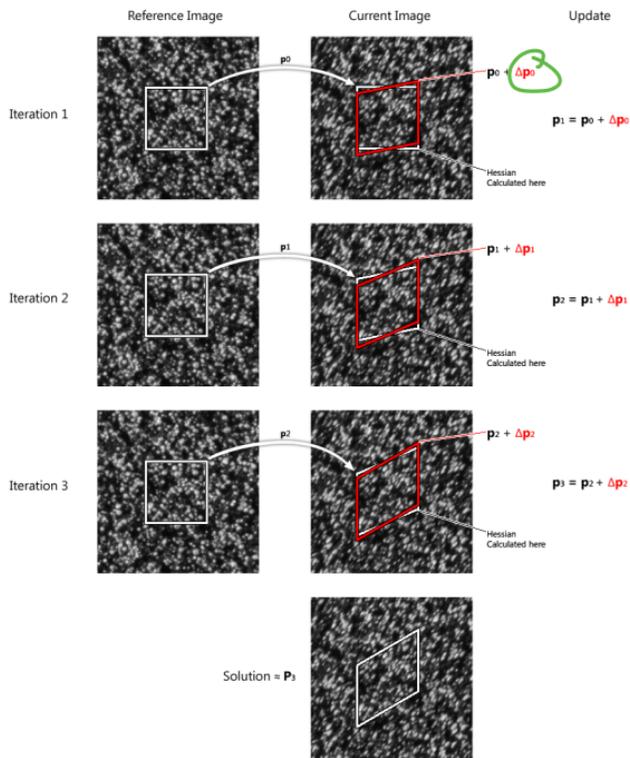
Carrying out the correlation: *fast normalized cross-correlation (NCC)*



1. We take our reference subset, and pad it so it matches the size of our current image.
2. We carry out fast NCC to obtain the correlation maximum
3. This allows us to identify the position in the current image of the best-matched value.

At this stage, we have a ***rough idea of the position*** of the displaced material coordinates, but ***we haven't yet accounted for their deformation.***

Optimizing from our first guess: Method 1: FAGN (forward additive Gauss-Newton iteration)



At left, you see an example reference image, and a current image that has undergone some shear-deformation. We already have p_0 , as this was our initial guess obtained with NCC. Now, we proceed by:

1. Calculate the second-order derivative of C_{LS} in the *current image* with respect to each component of p
2. Determine an increment Δp_0 to add to our initial guess on the basis of inverting the second-order derivative of

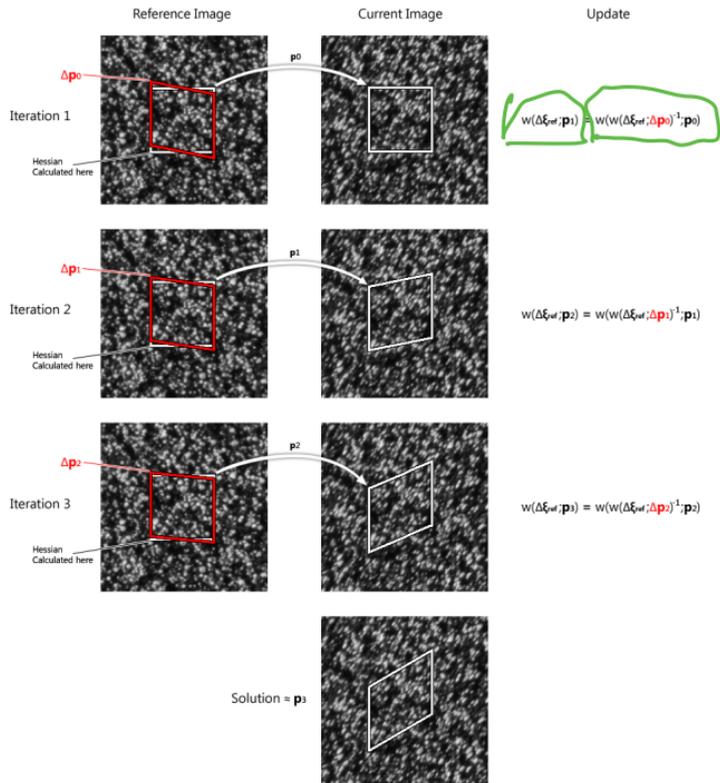
C_{LS}

3. Add this to form a new mapping p_1
Repeat until the C_{LS} is sufficiently small (a defined threshold for agreement of our mapping).

Note: this method is computationally expensive, because the derivative of the correlation function has to be calculated to second order in each component of p at each step of the iteration (!)

Optimizing from our first guess:

Method 2: Inverse Compositional method



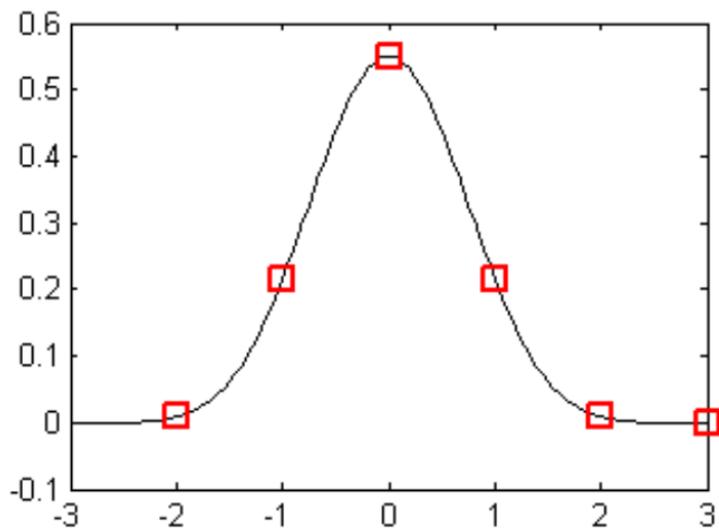
At left, you see an example reference image, and a current image that has undergone the same shear-deformation as before. We already have p_0 . Now, we proceed by:

1. **Calculate the second-order derivative of C_{LS} in the *reference image* with respect to each component of p
2. Determine an increment Δp_0 *in the reference image* to alter our reference subset
3. Add this to form a new mapping p_1

Repeat until the C_{LS} is sufficiently small (a defined threshold for agreement of our mapping).

Note: this method is significantly cheaper than the forward additive method, because the derivative of the correlation function only has to be calculated to second order in each component of p once (!)

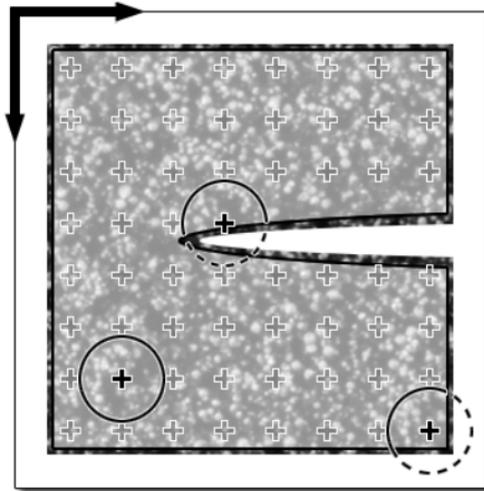
A quick remark: interpolation scheme



Bi-quintic spline interpolation (kernel shown at left) is used to ensure that high-frequency noise is suppressed. This locally smooths the image slightly, and can enable sub-pixel resolution of the gradients we need to calculate to optimize our mapping.

This step is necessary to ensure our solution converges (!)

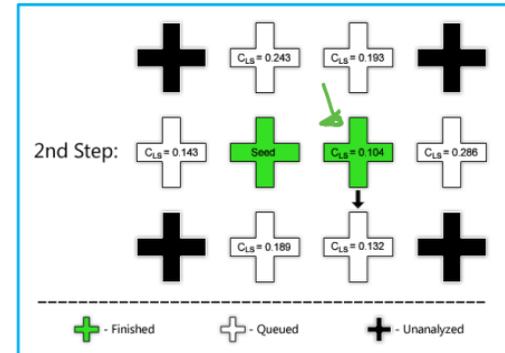
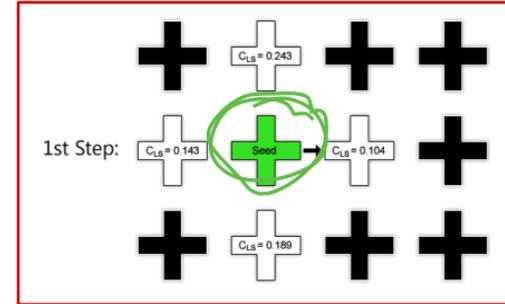
Now we have found the ideal mapping for an individual subset... what about the rest of the image?



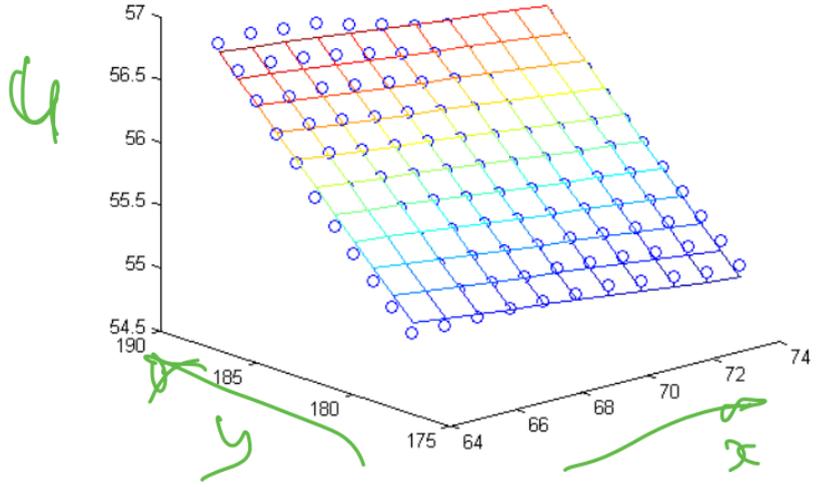
Typically DIC will be carried out for an *entire image*, and this will include *many subsets*.

NCORR uses a reliability-guided DIC method to essentially 'stitch' the neighboring subset together, and thus obtain the displacement field over the entire region of interest. This is carried out in several steps:

1. Select a seed subset:
2. Using the p_{rc} obtained for the seed, Calculate the neighbor's C_{LS}



Now that we have the full displacement field, how do we obtain the strains?



We have an over-constrained system in the sense that we have a displacement measurement corresponding to **each subset**.

Thus, we can simply fit a plane to the displacement data using e.g. least-squares, and calculate its gradients to find the strain components of the Lagrange strain tensor:

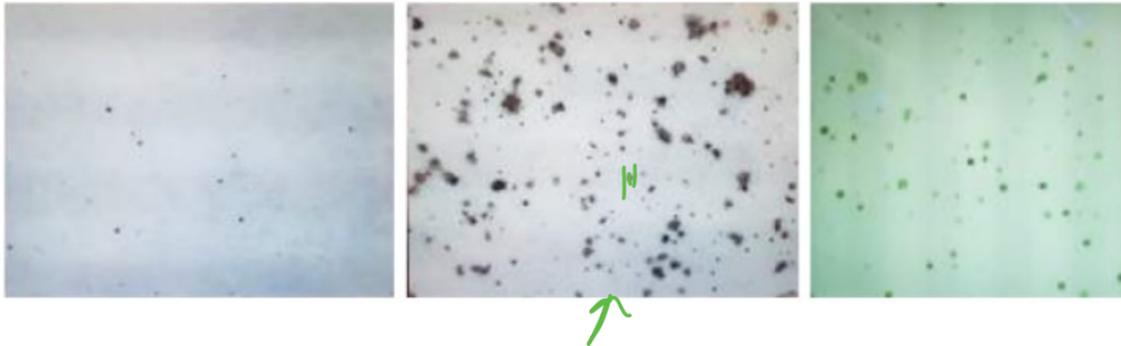
$$\begin{aligned} E_{xx} &= \frac{1}{2} \left(2 \frac{\partial u}{\partial X} + \left(\frac{\partial u}{\partial X} \right)^2 + \left(\frac{\partial v}{\partial X} \right)^2 \right) \\ E_{xy} &= \frac{1}{2} \left(\frac{\partial u}{\partial Y} + \frac{\partial v}{\partial X} + \left(\frac{\partial u}{\partial X} \frac{\partial u}{\partial Y} \right) + \left(\frac{\partial v}{\partial X} \frac{\partial v}{\partial Y} \right) \right) \\ E_{yy} &= \frac{1}{2} \left(2 \frac{\partial v}{\partial Y} + \left(\frac{\partial u}{\partial Y} \right)^2 + \left(\frac{\partial v}{\partial Y} \right)^2 \right) \end{aligned}$$

Here it is important to note that this will **smooth the strain data** and could lead to **misinterpretation of the data**

We've covered the fundamentals ... *how do we apply DIC in the lab?*

A central component to effective DIC measurement is the **pattern application**. The pattern must:

- Be **sufficiently small** compared to the **resolution of the displacements & strains** that you require
- Be **fixed to the sample** – moves **with the sample, not on its own**.
- Provide **sufficient contrast** for the correlation to be calculated effectively

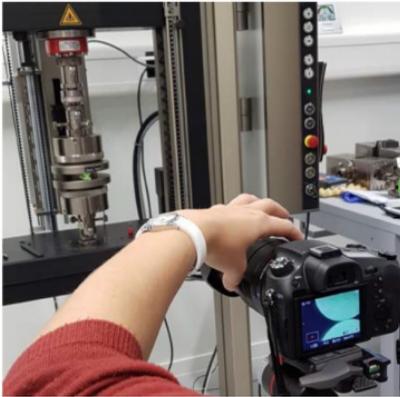


Aim for **homogeneity of pattern size**, and the **highest density** that you can resolve optically
Your strain unit resolution will be the equivalent of approximately 3 pattern elements (e.g. paint droplets)

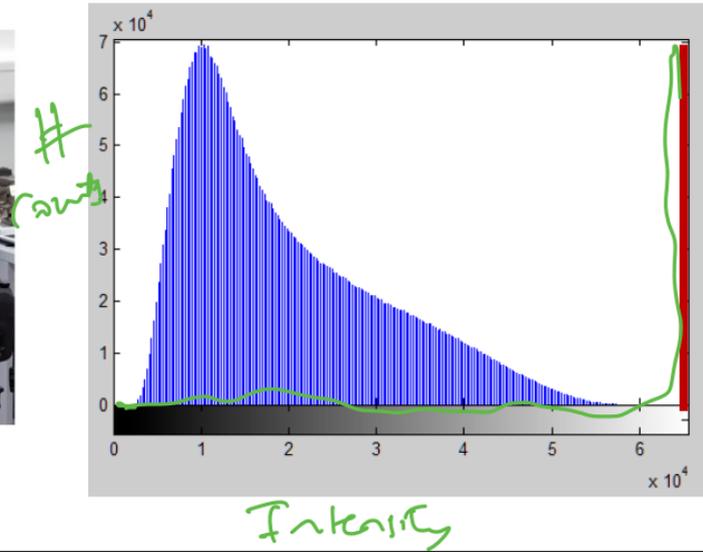
We've covered the fundamentals ... *how do we apply DIC in the lab?*

The other key laboratory component to your set up is your optical system. Here you have to ensure that:

- Your **resolution** is sufficient for your **pattern**
- Your pattern is **in focus**
- You resolve **the entire area of the sample** that you are **interested in analyzing**

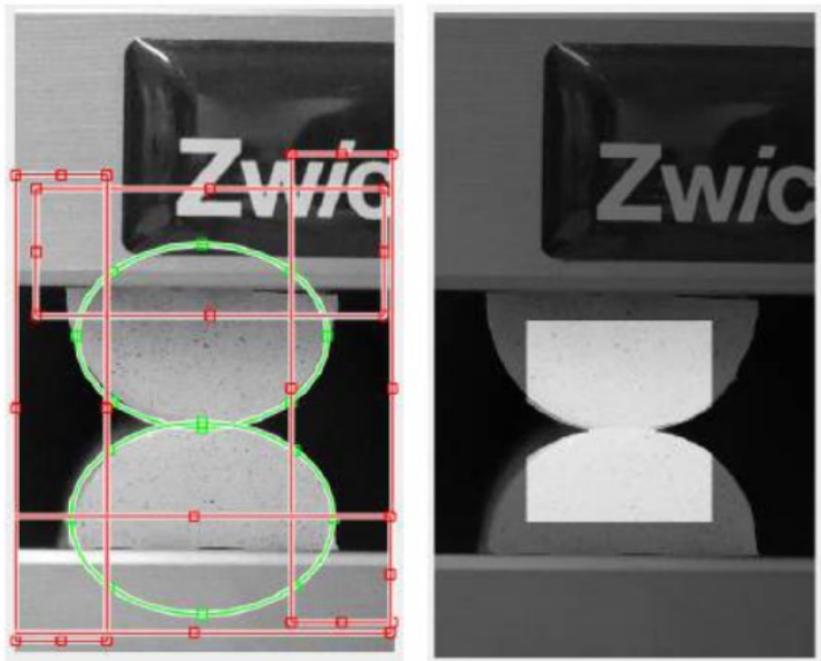


You might need a **high-power lens**



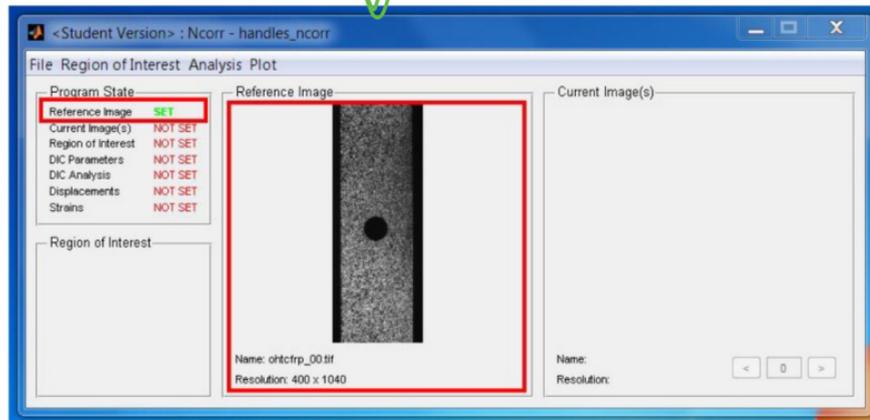
Your histogram (number of counts vs. intensity value) should not indicate **saturation (in red at left)**, and you should attempt to use the full bit-depth of the sensor

Nuts and bolts: complete analysis with NCORR



Begin with a **reference image** . This should be cropped and converted to 8-bit to ensure that the software runs rapidly and efficiently.

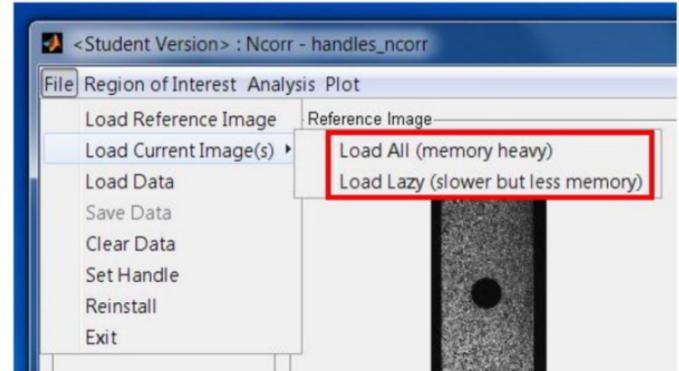
Here a screen shot of the GUI:



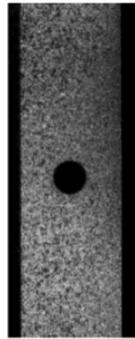
Nuts and bolts: complete analysis with NCORR

Load your **current images** where the material has undergone **displacement** into the software.

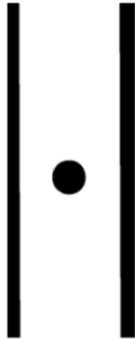
You have the option to **load all the images into memory**, provided you have enough memory, **to speed up** the processing



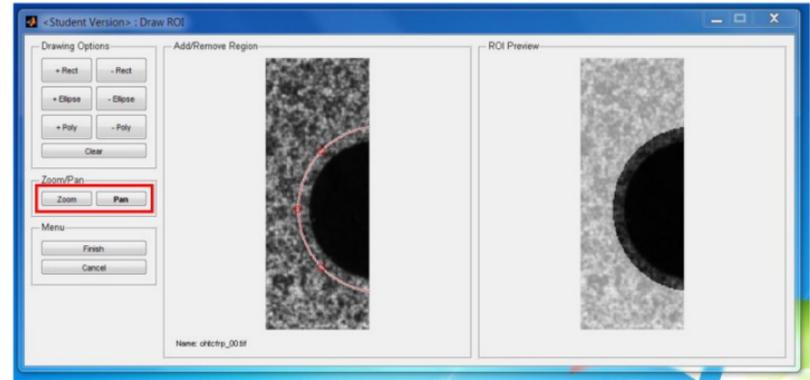
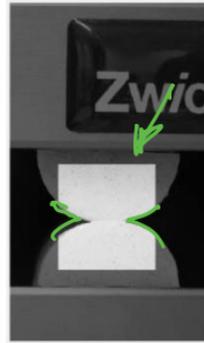
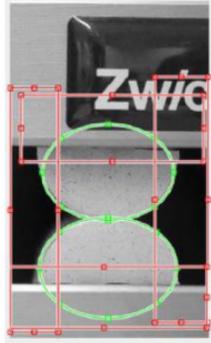
Nuts and bolts: complete analysis with NCORR



Sample



ROI



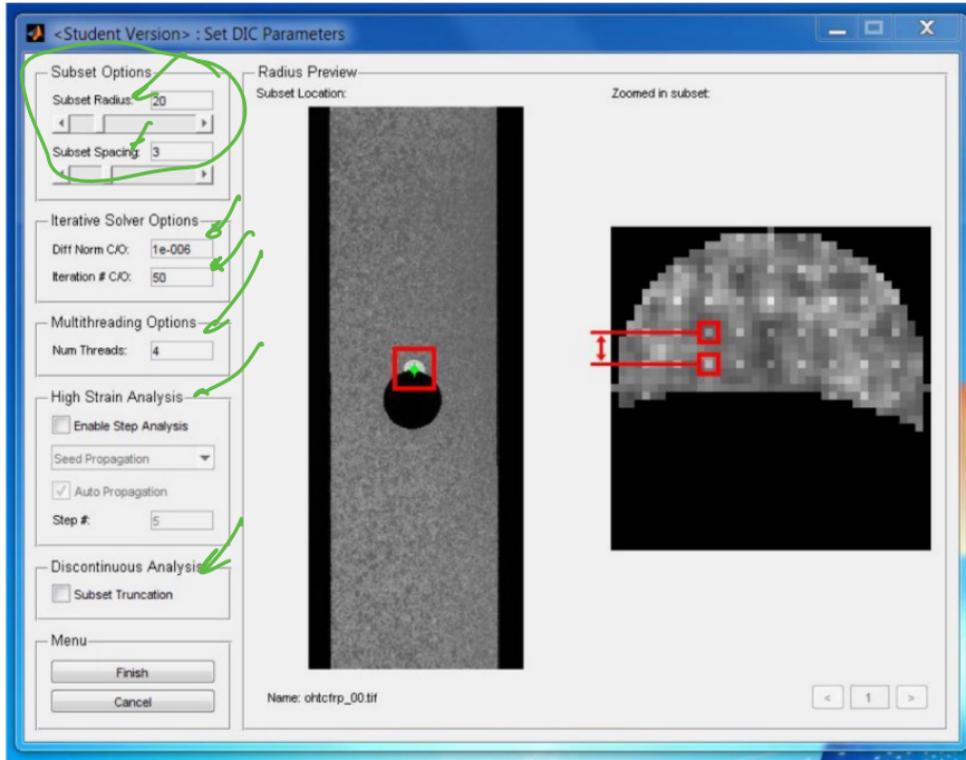
After carefully cropping all of the images and loading them into the software, you will select a region of interest (ROI) using the reference image.



NCORR can account for even complex geometry, as shown above, center

Edges must be treated carefully to ensure convergence of the correlation. It is very challenging to measure displacements along the edge of the sample

Nuts and bolts: complete analysis with NCORR



Once you selected the ROI, you can proceed to set the parameters for the correlation calculations.

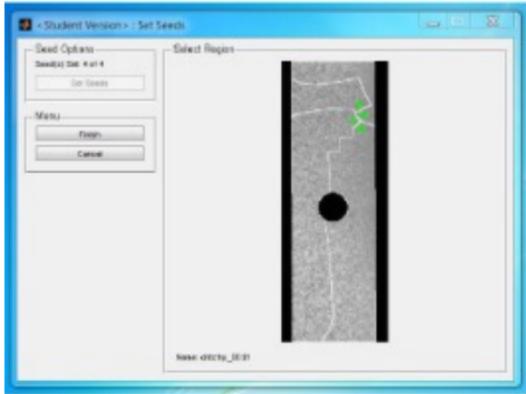
You'll designate the subset including its size and spacing. This should be done to ensure you have enough high- and low- intensity points to get a good correlation measurement.

The solver options set the threshold when the iteration will stop, and set a limit to your accuracy.

Step analysis can be used to handle large strains if you have large deformation. It will provide an intermediate step every 5 frames or so to update the reference.

Subset truncation can be used when you have subsets that might overlap with an ROI boundary

Nuts and bolts: complete analysis with NCORR

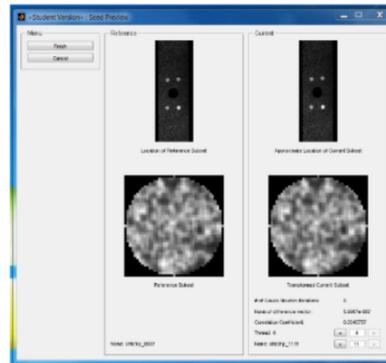
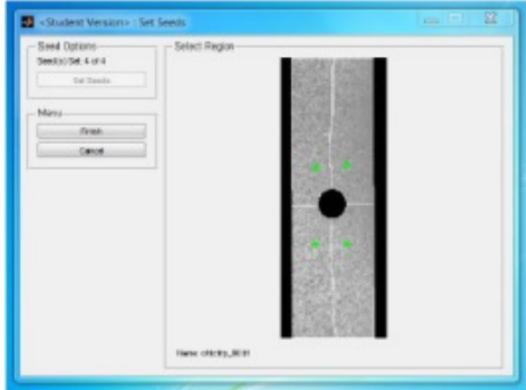


If you're using multi-threading (recommended), you'll have the option to place seeds. This should be done so as to ensure that the regions are of equal size with relatively smooth borders.

Bad example (at top) shows unequal regions and rough borders.

Good example (at bottom) shows approximately equal regions and smooth borders.

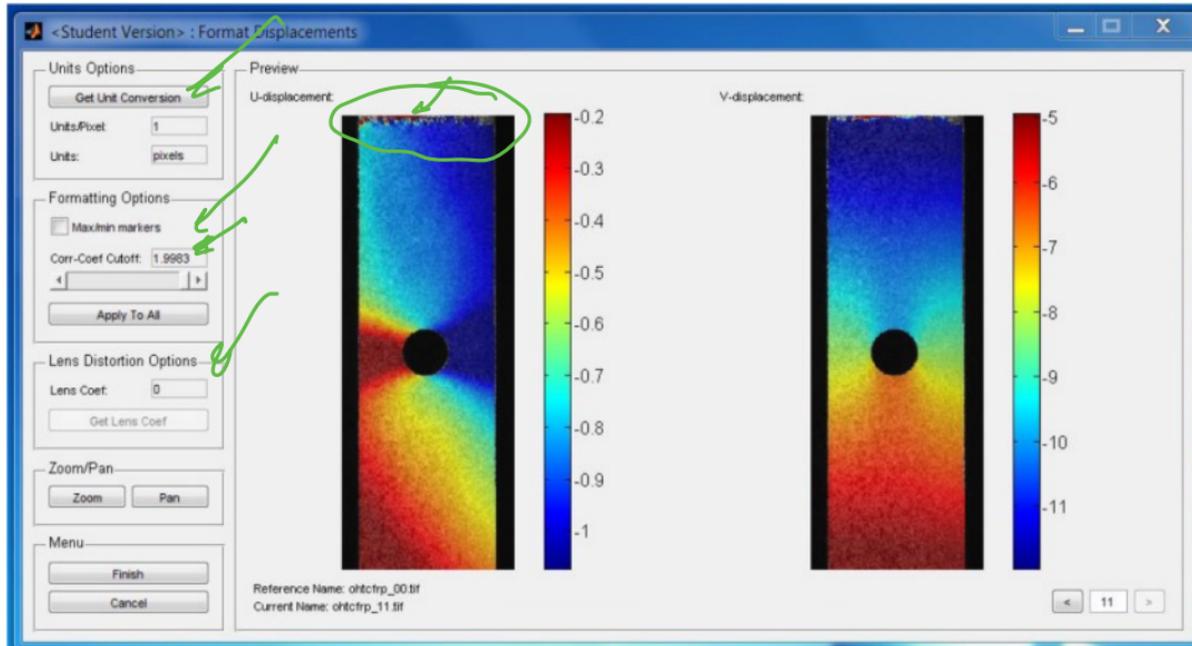
Each seed will define a region to be calculated by a single core of the PC. This parallelizes the process and it will go faster.



The seed preview tool will show you what the DIC calculation determines for a given seed and a given reference frame. If your seed placement is 'successful,' you'll be able to evaluate how well it mapped the reference to the current configuration for each seed, and every picture to be analyzed. It usually fails badly if it fails, letting you know you shouldn't trust the data.

Nuts and bolts: complete analysis with NCORR

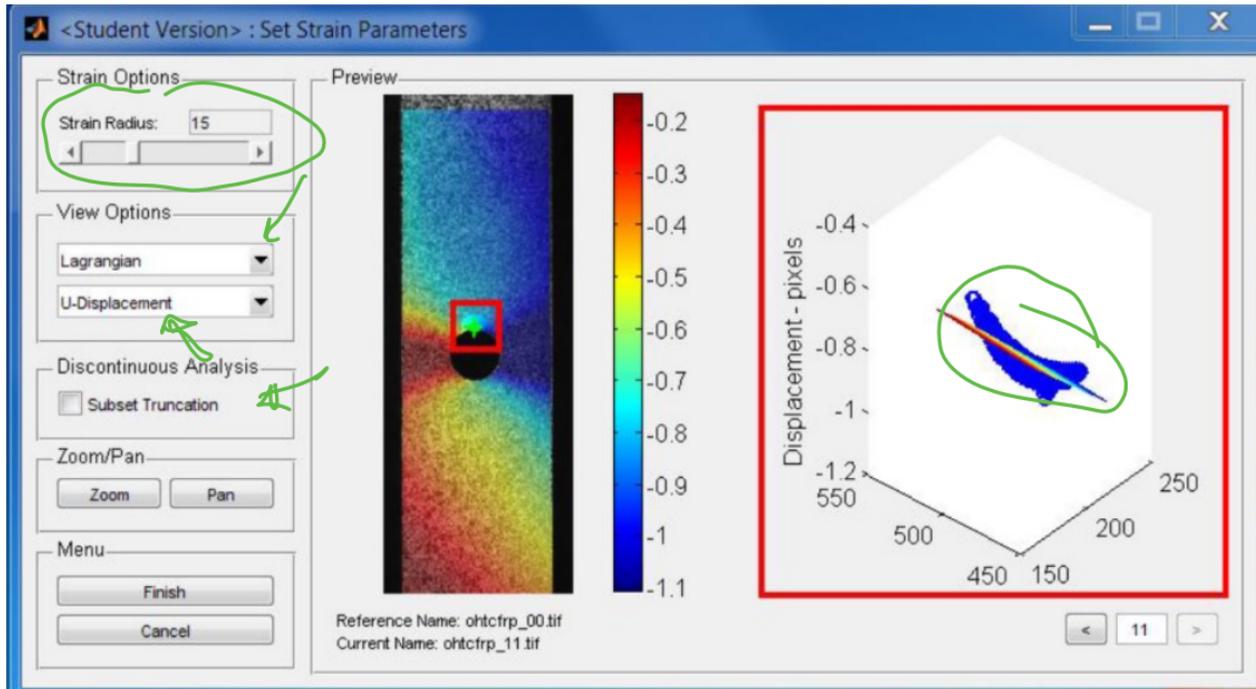
Now NCORR has calculated displacement fields using its algorithm. At this stage, we are ready to view the displacement fields:



Here you'll define your length-scale if you know the reference scale of the image. You can also drop points that didn't have a low correlation coefficient cut-off during DIC. Optical issues, such as barreling and other lens corrections, can be applied here.

Nuts and bolts: complete analysis with NCORR

Now that we've formatted displacements, we're ready to calculate strains:



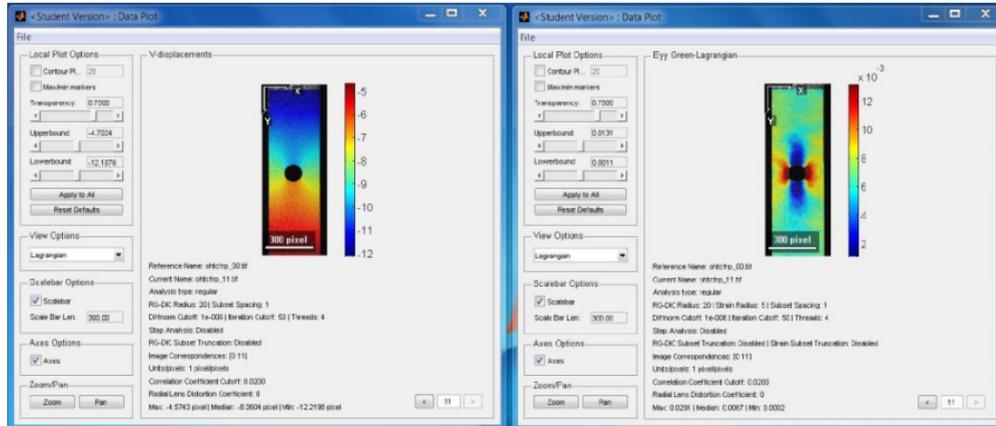
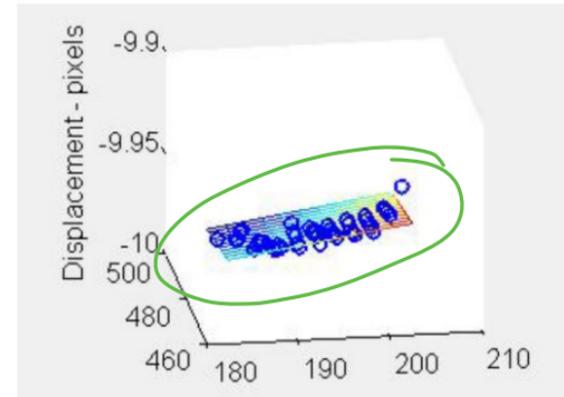
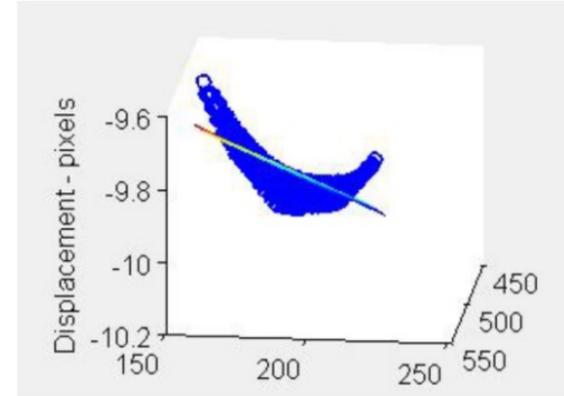
Here you'll define the region size for the strain calculation. This is uniform over the entire image. Using the 3D displacement plot at right, you can see how much distortion you have as you vary the strain radius. The preview shows the strain fields. You can view this in both the reference (Lagrangian) and current (Eulerian) configurations; this is the advantage of DIC over PIV.

Nuts and bolts: complete analysis with NCORR

The strain radius is important: you want to select it so that you have an accurate measurement of strain, but that the displacements are nearly planar (see images at right)

This is something that you'll have to determine based on your required strain resolution and error tolerance for your measurement.

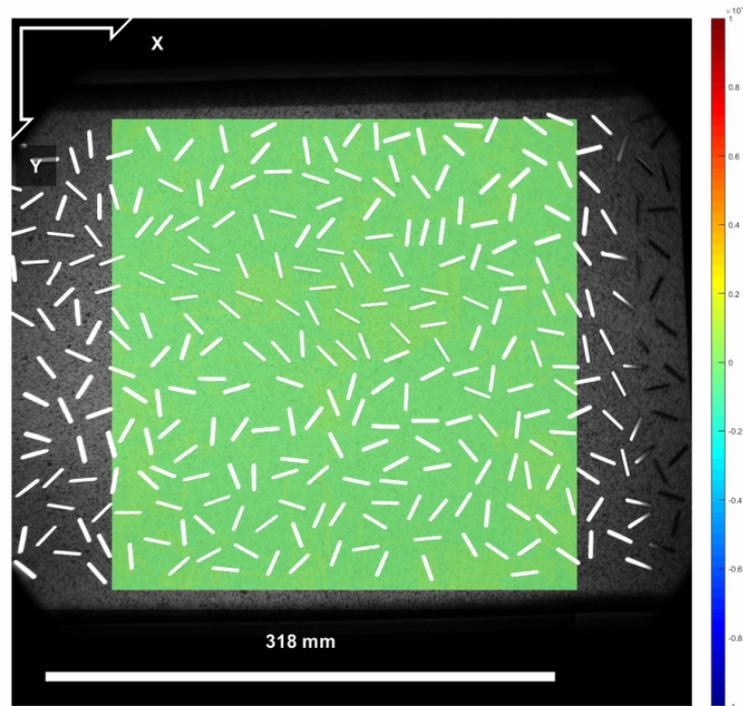
After this, you can plot your data...



Nuts and bolts: complete analysis with NCORR

... and even output the strain fields (here ε_{xy}) as a gif!

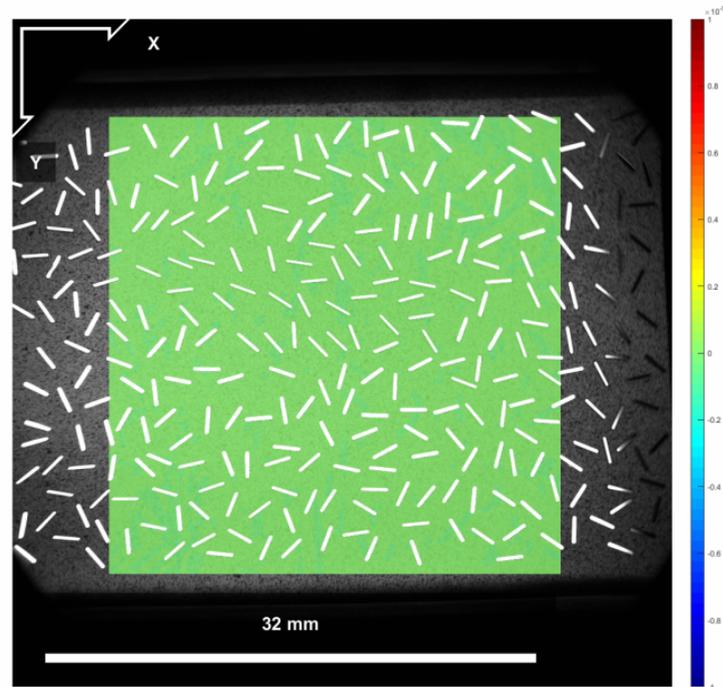
Eulerian:



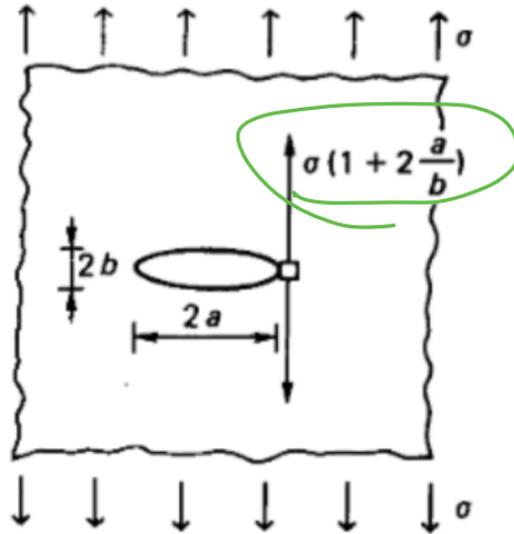
Nuts and bolts: complete analysis with NCORR

... and even output the strain fields (here ε_{xy}) as a gif!

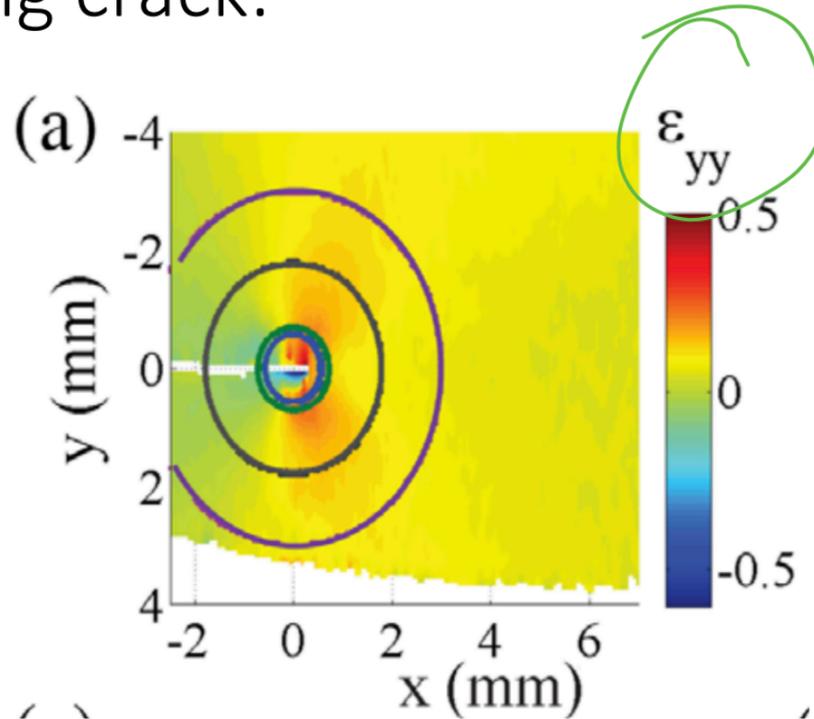
Lagrangian:



We will be measuring the stress around an elliptical void:

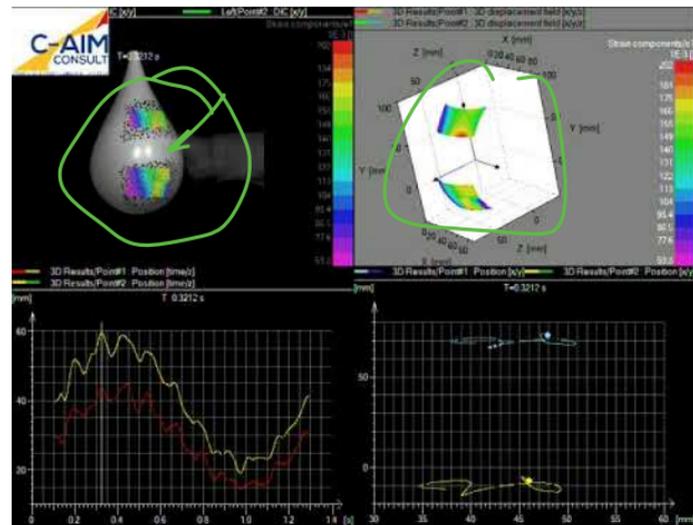
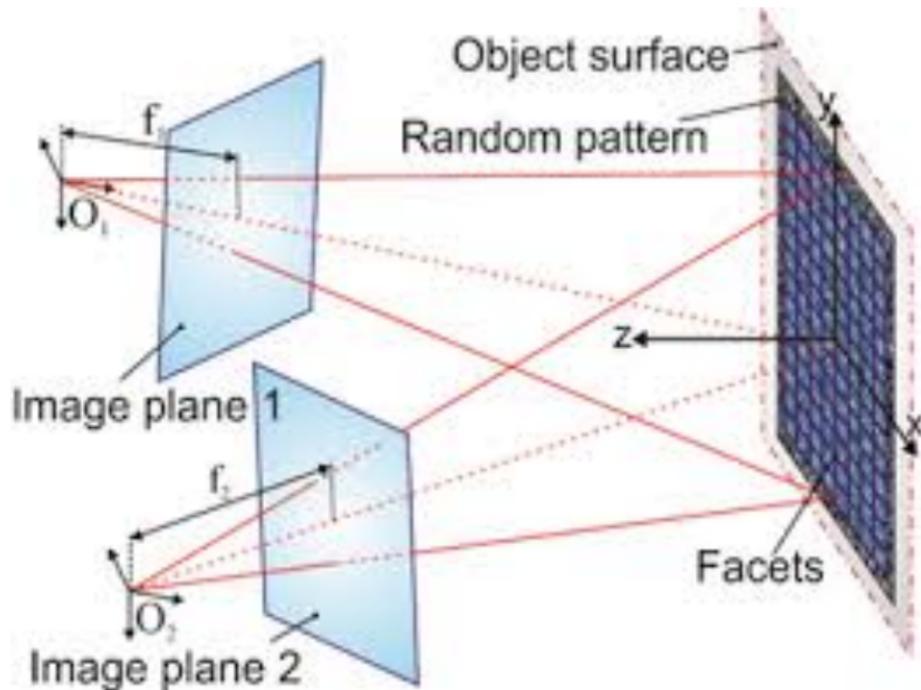


Or evaluating the stress field of a dynamically propagating crack:



Or measuring a problem of your choice!

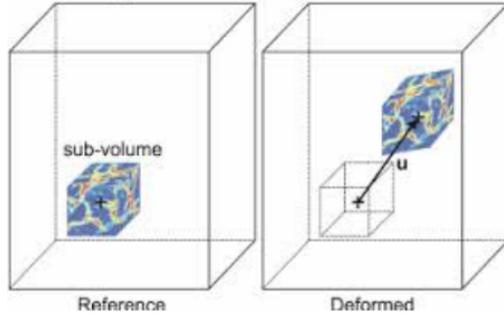
Extensions and generality of DIC:



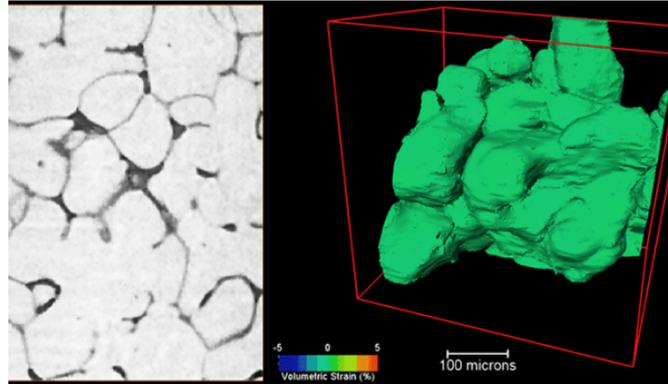
Stereo DIC: planar strain fields on curved surfaces

Extensions and generality of DIC:

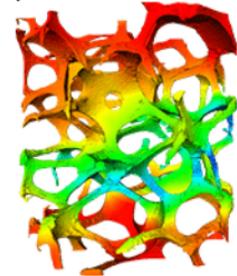
3D image data



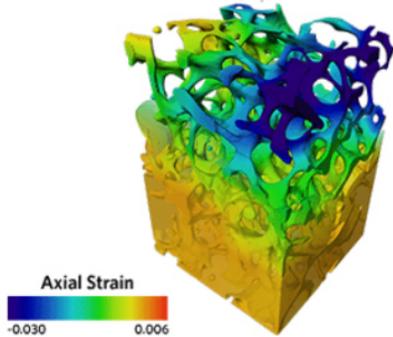
Zirconia plasticity:



Alu foam under compression:

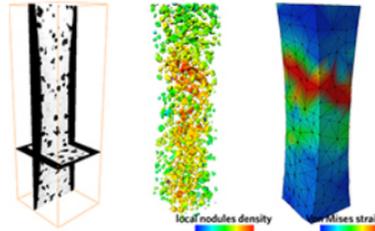


Bone-cement interface:



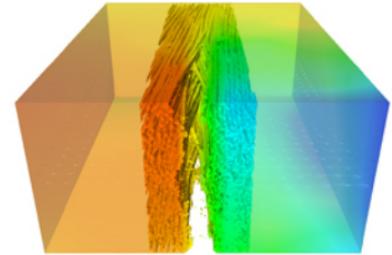
Data courtesy of Dr. Gianluca Tuzi at University of Portsmouth

Necking around inclusions



Data courtesy of F. Hill at LMT Gachon

Composite fracture:



Data courtesy of L. Lazaranthos & T. Morgeneyer (Centre des Matériaux Mines ParisTech), and L. Helfen (European Synchrotron Radiation Facility, Karlsruhe Institute of Technology)

DVC: **fully 3D** volumetric correlation (requires 3D data)



Merci

John Kolinski