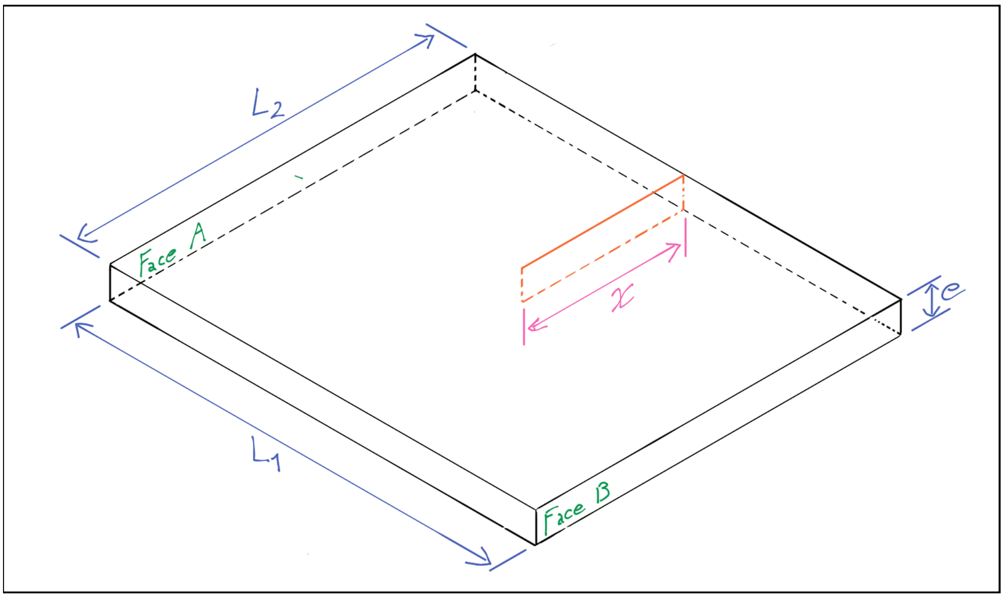
GROUP 3: Relationships between the sample resistance and the crack length

The goal of this text is to provide and verify analytical relations between the crack length and the measured electrical resistance of Aluminium coating on top of PMMA samples.

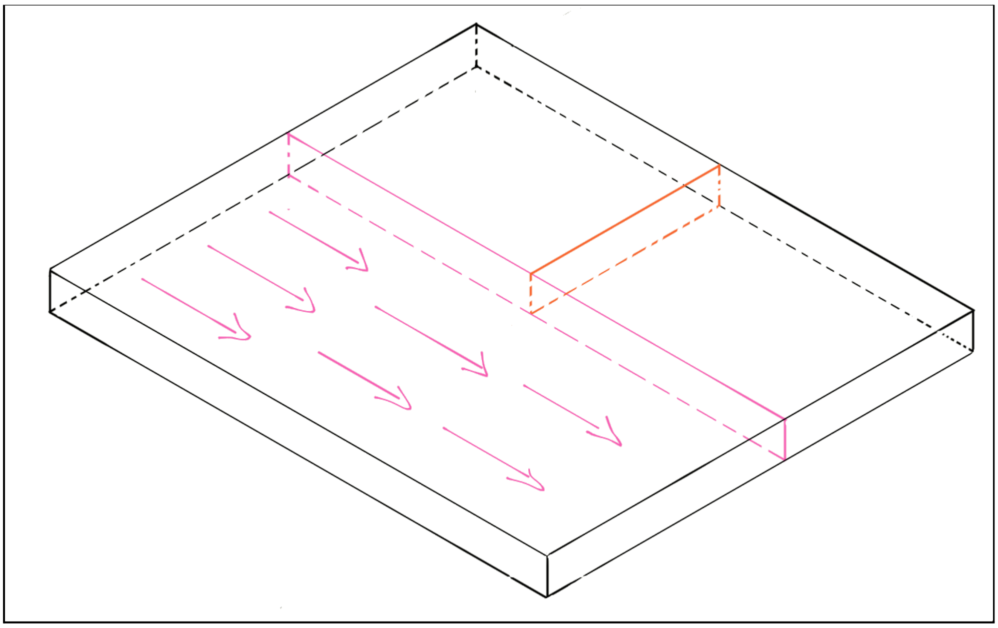
To do that, we will use the drawing below:



The figure shows a rectangular part (representing the Aluminium coating). The electrical resistance is measured between Face A and Face B. The crack (shown in orange) has a length of x, while the coating has dimensions of L1, L2 and e (thickness). We will assume that the speed of electrical signal (speed of light order) is way faster than the speed of propagation of the crack (a few km/s). We hence can say that only x impacts the resistance, and not dx/dt.

RECTANGULAR MODEL

As a first approximation, we can assume that electricity flows using the Rectangular path:



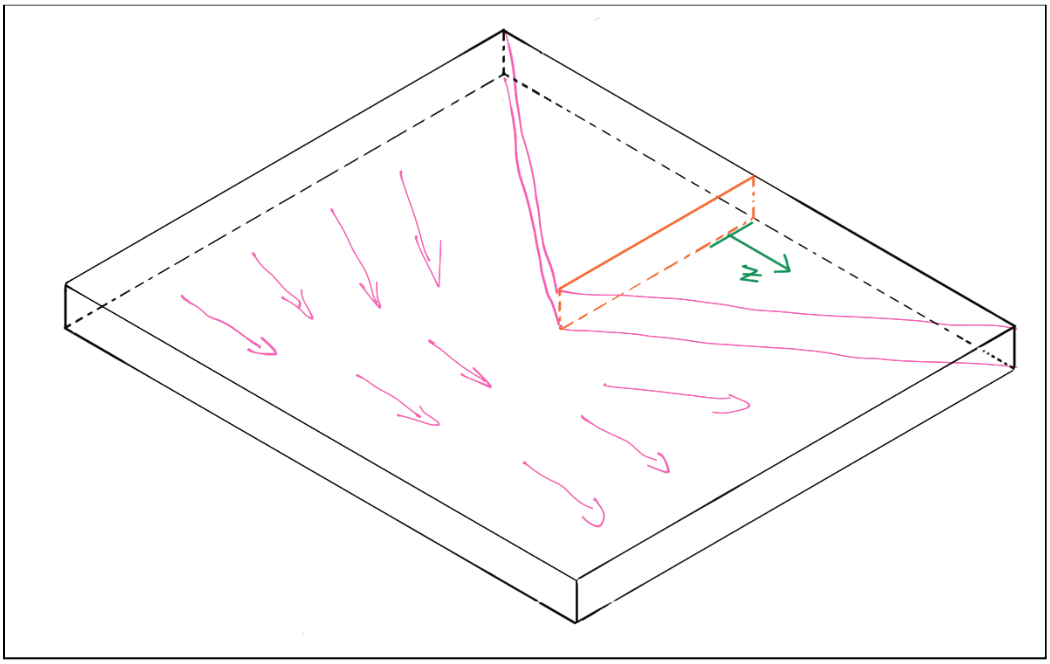
This implies that the resistance would have a value of

$$R_{rectangular} = \frac{L_1\sigma}{e(L_2 - x)};$$

With σ being the electrical resistivity [Ohm.m]

TRAPEZOIDAL MODEL

Rectangular model supposes that the electricity flows parallel to one side of the sample. However, we may also think than electricity could be flowing from the whole Face A, to the uncut part of the coating, to Face B (as is shown below).



If this is the case, and if we suppose that electricity follows a trapezoidal shape, we have the following relationship (we suppose the crack to be in the middle of the sample):

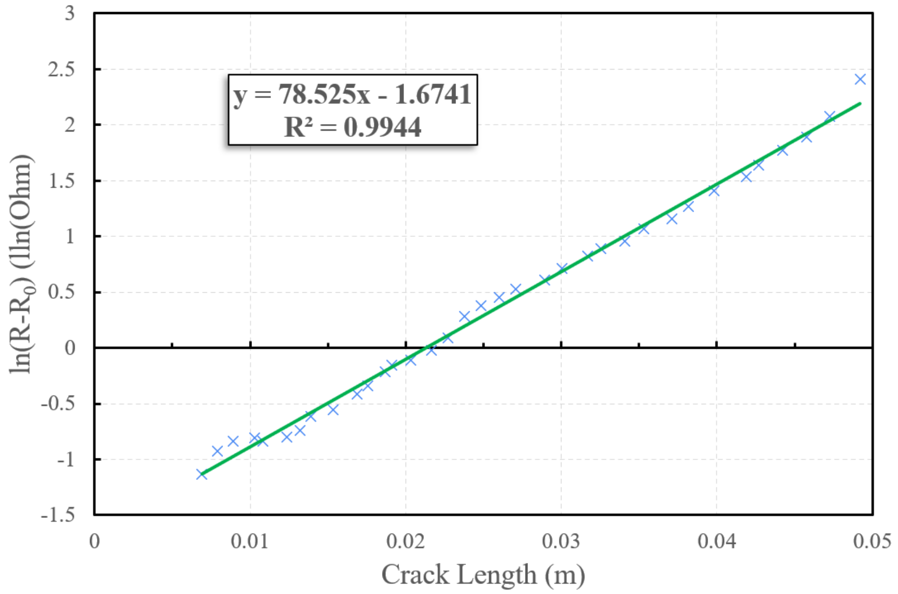
$$dR_{trapezoidal} = \frac{\sigma dz}{e[L_2 - x(1 - \frac{2z}{L_1})]}$$

Integrating from z=0 to z=L1/2 and multiplying the result by 2 (because of symmetry) leads to:

$$R_{trapezoidal} = \frac{-L_1\sigma}{xe} Log(1 - \frac{x}{L_2})$$

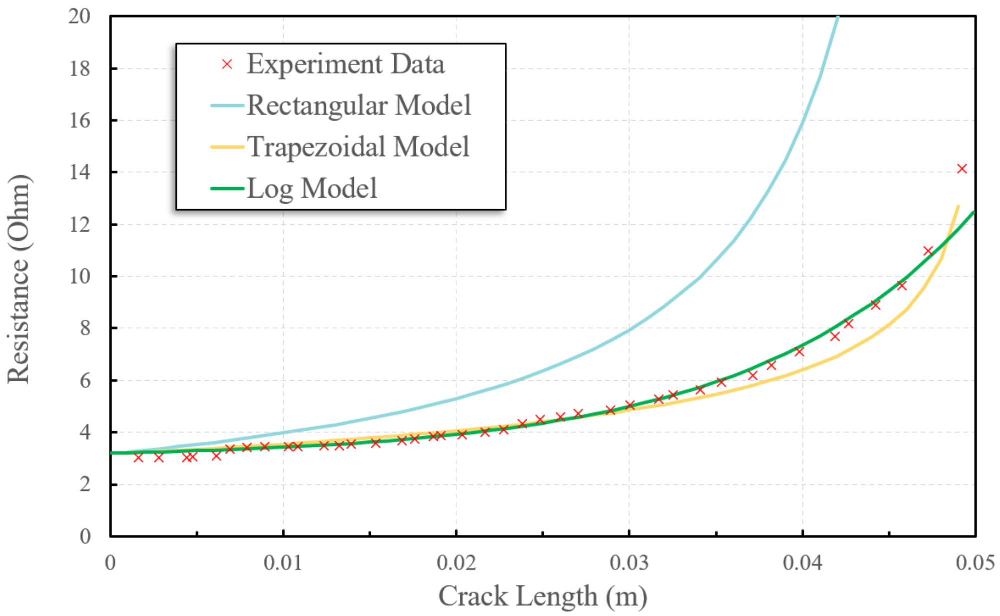
EXPERIMENTAL MODEL

We decided to also test an experimental procedure, aiming at fitting a certain model to experimental data. To do so, we measured (using a 4-wire configuration) the resistance of the coated sample for different values of crack length. The measured data is being processed by applying a Log function: we realize that we can express Log(R-R(x=0)) =78.525*x-1.6741



COMPARING THE MODELS

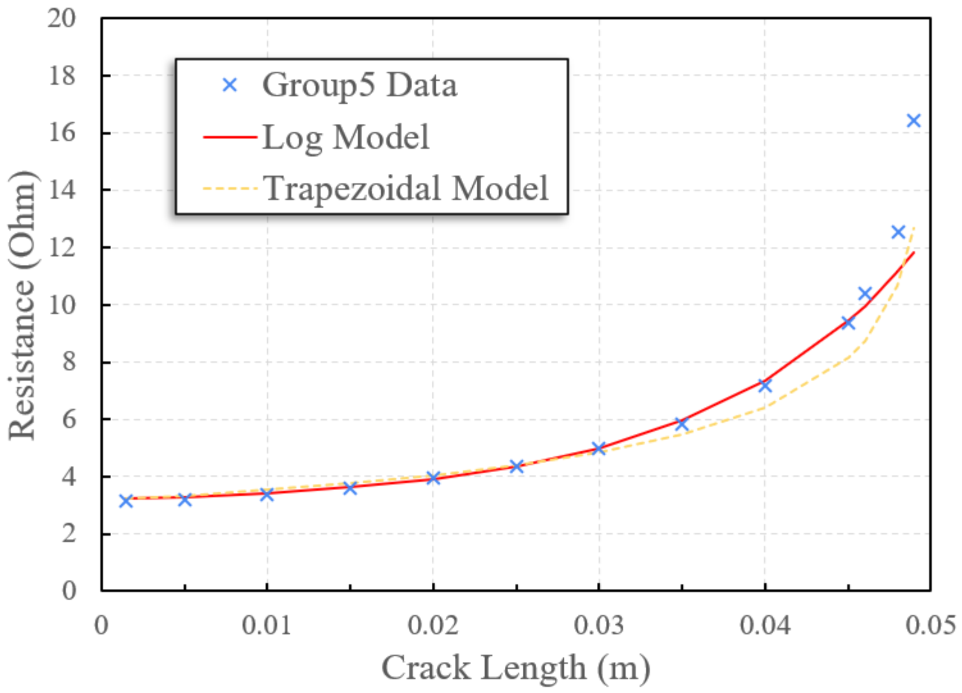
Now, in addition of knowing two analytical models for R(x), we know an exponential experiment-based model. We decide to plot the two analytical models, the experiment-based model (Log model) and the real experimental data. Results are shown below.



We notice that the trapezoidal model is relatively close to the experimental data. The experiment-based log model gets even closer. The rectangular model on the other hand is not reliable.

GROUP 5 DATA COMPARAISON

We noticed that Group 5 produced data for resistance with respect to the crack length x. We call that data seriesGroup5 Data. We decided to plot their data, our trapezoidal and our Log model (experiment-based) in order to check once again the validity of both the trapezoidal and the Log model. Results are shown below.



The Log and Trapezoidal model keep up. However, the Log model is closer, hence more viable in our case.