$$\frac{L^2}{T} \doteq D = \frac{kT}{\eta\, a\, 6\pi}$$

?

goal

Random Walkers:

3 Rules:

1. Each walker steps R or L at each time interval, τ.
   - Step size is fixed by the particle's velocity, v:

$$\delta = v\tau$$

2. Probability that a walker goes R is 1/2
   " L is 1/2.
3. Particles don't interact or collide.

Now, we'll look at an ensemble of N walkers:

Let $x_i(n)$ be the i'th particle's position after n steps.

By our rules, we can write down i's position as a function of position at $n-1$ step:

$$x_i(n) = \underline{x_i(n-1) \pm \delta}$$

Considering the ensemble, we can calculate the mean displacement after n steps:

$$\langle x(n)\rangle = \frac{1}{N}\sum_{i=1}^{N} x_i(n) = \frac{1}{N}\sum_{i=1}^{N}\left[x_i(n-1) \pm \delta\right]$$

$$= \frac{1}{N}\sum_{i=1}^{N} x_i(n-1) = \langle x_i(n-1)\rangle$$

The mean position doesn't change.

Intuitively, though, we expect the particles to disperse.

Here we'll average the square displacements. (Always >0).

$$\underline{x_i(n)^2} = x_i(n-1)^2 \pm 2\delta x_i(n-1) + \delta^2.$$

$$\underline{\langle x^2(n)\rangle} = \frac{1}{N}\sum_{i=1}^{N} x_i^2(n) = \frac{1}{N}\sum_{i=1}^{N}\left[\underbrace{x_i^2(n-1)}\,\underbrace{\pm 2\delta x_i(n-1)}_{0} + \delta^2\right]$$

$$= \langle x^2(n-1)\rangle + \delta^2$$

$$\langle x^2(0)\rangle = 0,\quad \langle x^2(1)\rangle = \delta^2,\quad \langle x^2(2)\rangle = 2\delta^2$$

$$\ldots\ \langle x^2(n)\rangle = n\delta^2$$

n steps corresponds to time $t = n\tau$

$$\Rightarrow n = \frac{t}{\tau} \qquad \doteq \frac{L^2}{T}.$$

$$\langle x^2(t)\rangle = \frac{t}{\tau}\delta^2 = \underbrace{\left(\frac{\delta^2}{\tau}\right)}_{D} t.$$

In fact, $D = \frac{\delta^2}{2\tau} \Rightarrow \langle x^2(t)\rangle = 2Dt.$

In 2-D: Same applys to "y-axis".

$$\Rightarrow \langle x^2(t)\rangle = 2Dt \quad \& \quad \langle y^2(t)\rangle = 2Dt$$

$$\langle r^2(t)\rangle = \langle x^2(t) + y^2(t)\rangle = \langle x^2(t)\rangle + \langle y^2(t)\rangle = 4Dt.$$