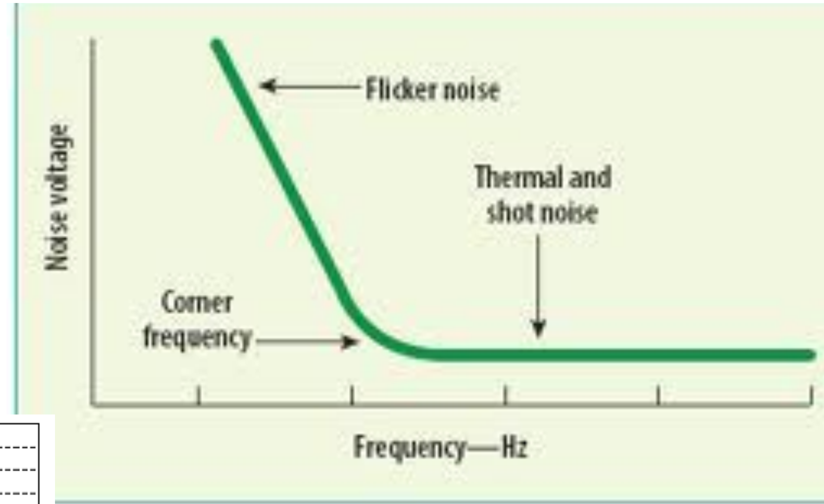
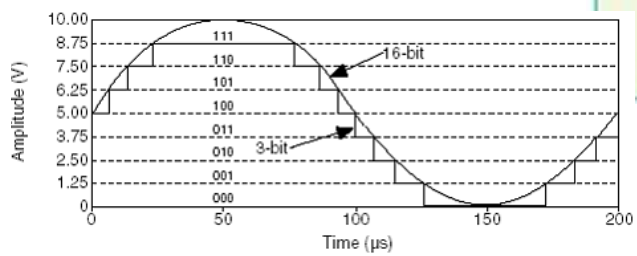
Methods of noise reduction

- White noise reduction: Impedance / Temperature / Bandwidth reduction
- Shot noise reduction: Current reduction
- 1/f noise reduction: Band shifting
- Quantization noise reduction: Dithering
- High-frequency techniques

Noise reduction

- For each type of noise:
  - Thermal noise (Johnson) $V_n^2 = 4k_B T\,\mathrm{R}\,B$
  - Current noise (shot) $I_n^2 = 2eIB$
  - Amplifier (flicker) noise (1/f)
  - Quantization noise





- We can "attack" each part of these equations:
  - For thermal noise: Reduce T, R or B
  - For current noise: reduce I or B
  - For 1/f noise: increase f
  - For quantization noise: dithering

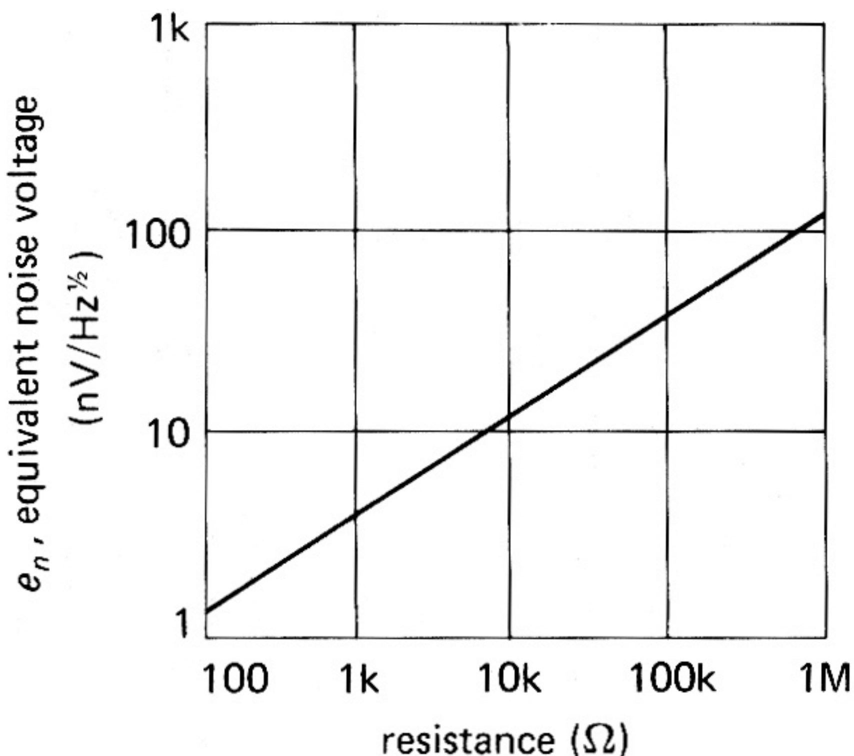
White noise reduction: Impedance reduction (if possible)

Reducing the source impedance (if possible) gives two benefits:

- Reduction in thermal noise voltage
- Reduction in capacitive interference coupling (see later)
- Example: Using 20 kHz bandwidth, a 1 MΩ sample at room temperature generates thermal noise of 20 μV. Reducing the resistance to 10 kΩ would reduce it to 2 μV.
- The best case: superconductors (R_s = 0),

metallic samples (low R_s)

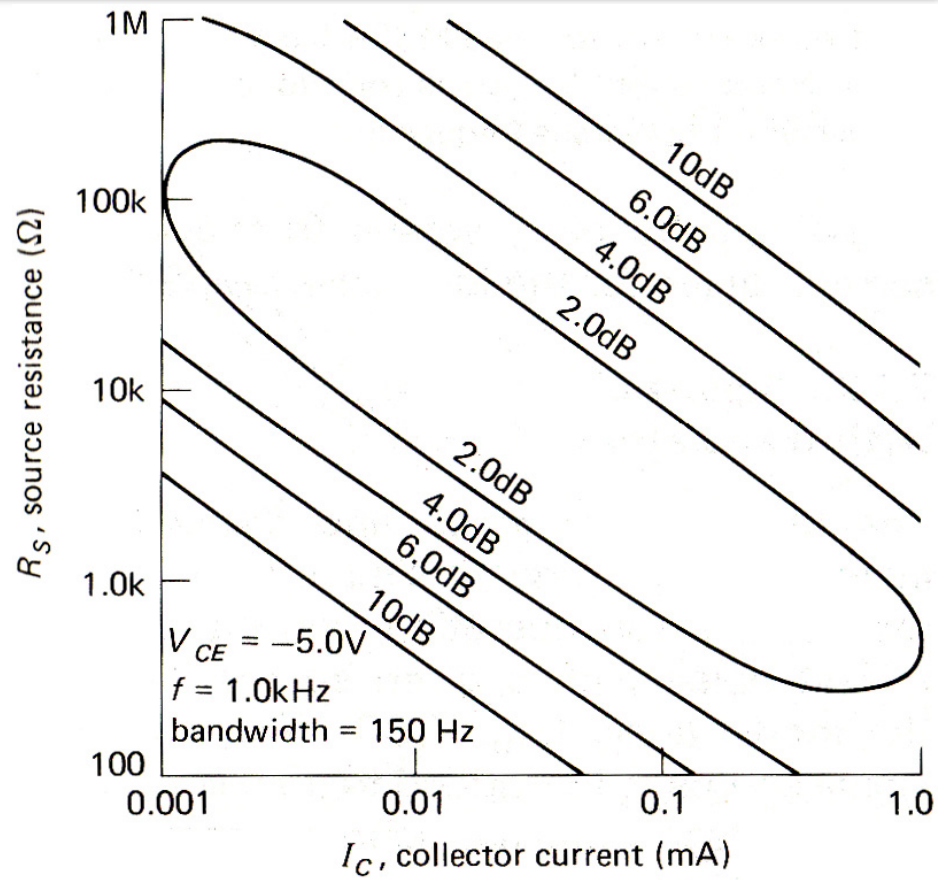
- Sometimes this can not be done:
  - Chemical electrodes (high Z)
  - Biological probes (High Z)
  - Semiconductors
  - Photodetectors (High R)



White noise reduction: Impedance matching (if possible)

In case of very low source impedance, and only for AC, it's worth using an Impedance-matching Transformer. It increases the signal by the turns ratio N_2/N_1, and increases the equivalent source resistance by the square of the turns ratio N_2^2/N_1^2.

- The transformer doesn't change the thermal SNR at the input: Thermal noise voltage increases by the same ratio as the signal.
- However, the amplifier works at a higher input resistance, so its NF is lower.



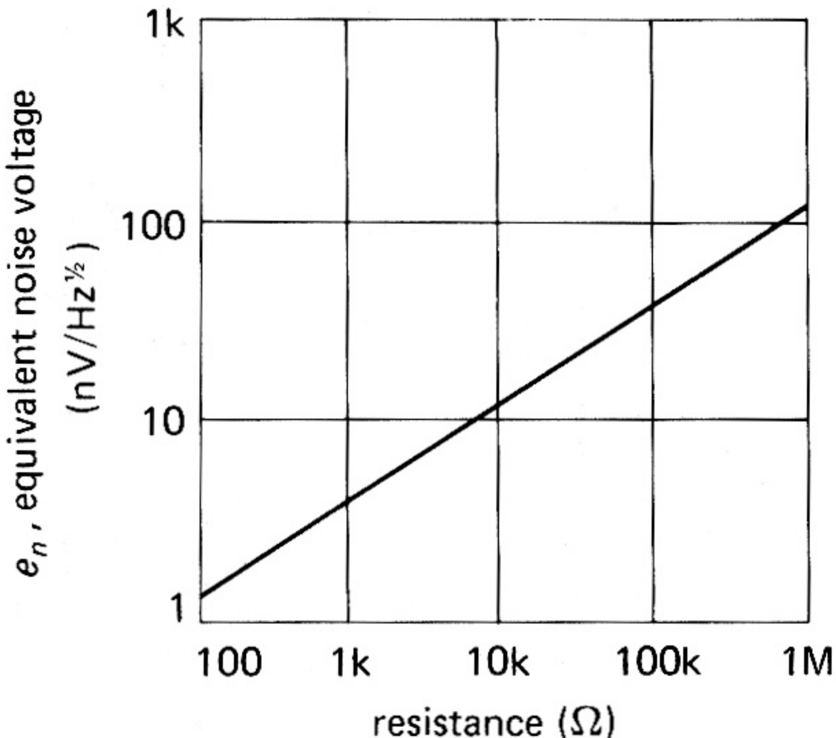
- For high frequencies, impedance matching is essential, to avoid reflections!
- 50Ω is the standard RF impedance (75Ω for video)

White noise reduction: Temperature reduction (if possible)

Reducing the source temperature (if possible) also reduces the thermal noise voltage

- Example: Using 20 kHz bandwidth, a 1 MΩ sample at room temperature generates thermal noise of 20 μV. Reducing the temperature to 4.2 would reduce it to 2.5 μV.
- Can be used in laboratories: when studying samples at low temperatures
- Industrial uses (thermoelectric coolers): cooled photodetectors and image sensors, cooled RF preamps, space electronics
- In most cases, temperature reduction

  is not possible!
- Even worse: high-temperature electronics



White noise reduction: Bandwidth reduction

As reducing the source resistance and temperature is not always possible, the main tool for noise reduction is bandwidth reduction.

- This usually means: reduce noise by measuring more slowly
- This is equivalent to taking more measurements and averaging the results: for random noise, we can reduce the fluctuations by a factor N if we average over N^2 samples (measurement time will increase by N^2). The same will happen if we reduce the bandwidth by N^2, as noise is proportional to N.
- If the sampling (or another factor) is limiting the signal bandwidth, there is no point in having higher bandwidth at the amplifier. Therefore filtering should be used
- In some cases this is not practical, as in image taking:
  - Video frame rate: 20 Hz. Picture size: 640x480 pixels. BW=20*640*480=6.1 MHz!
  - CL image: 1s per spectrum, 100x100 pixels. Total time: 2.8 hr! Sample must be very stable…

Bandwidth reduction: Passive, active and digital filters

The best way of bandwidth reduction is by low-pass filtering. This can be done by different methods:

- Passive filter: made of RC or LC network, it's simple to install and can be added almost anywhere in the circuit. However, usually it's limited to first-order (single pole), so high-frequency reduction is limited (slope of 20 dB/decade)

- Active filter: usually part of the amplifier (sometimes the preamp) circuit, it can provide better performance and more flexibility (even programmability) than the passive one, at the cost of extra complexity.

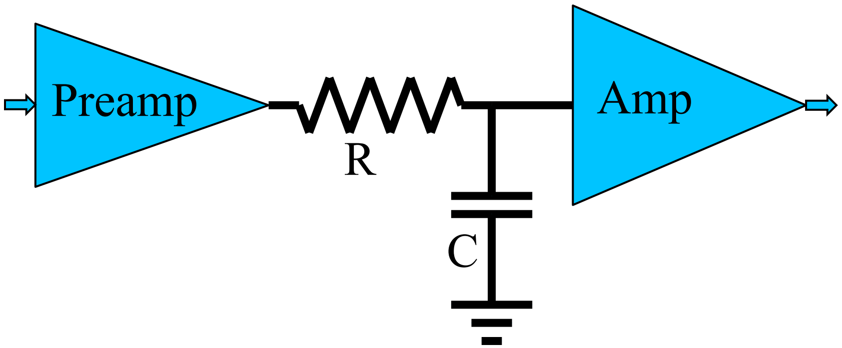
- Digital filter: can be implemented in software, the signal from the A/D converter is processed by algorithms which provide signal filtering (averaging) of any type. It's easy to implement, but depends on good pre- and amplifier, ADC (should have enough dynamic range). Analog filtering might be required for correct operation of the ADC (Nyquist limit).

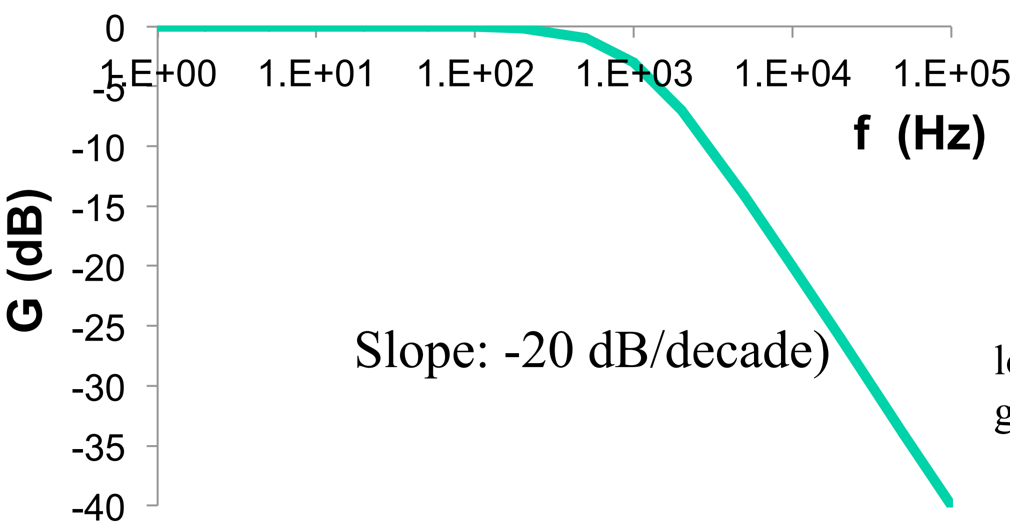
Low-pass filter examples 1

Simple RC filter:

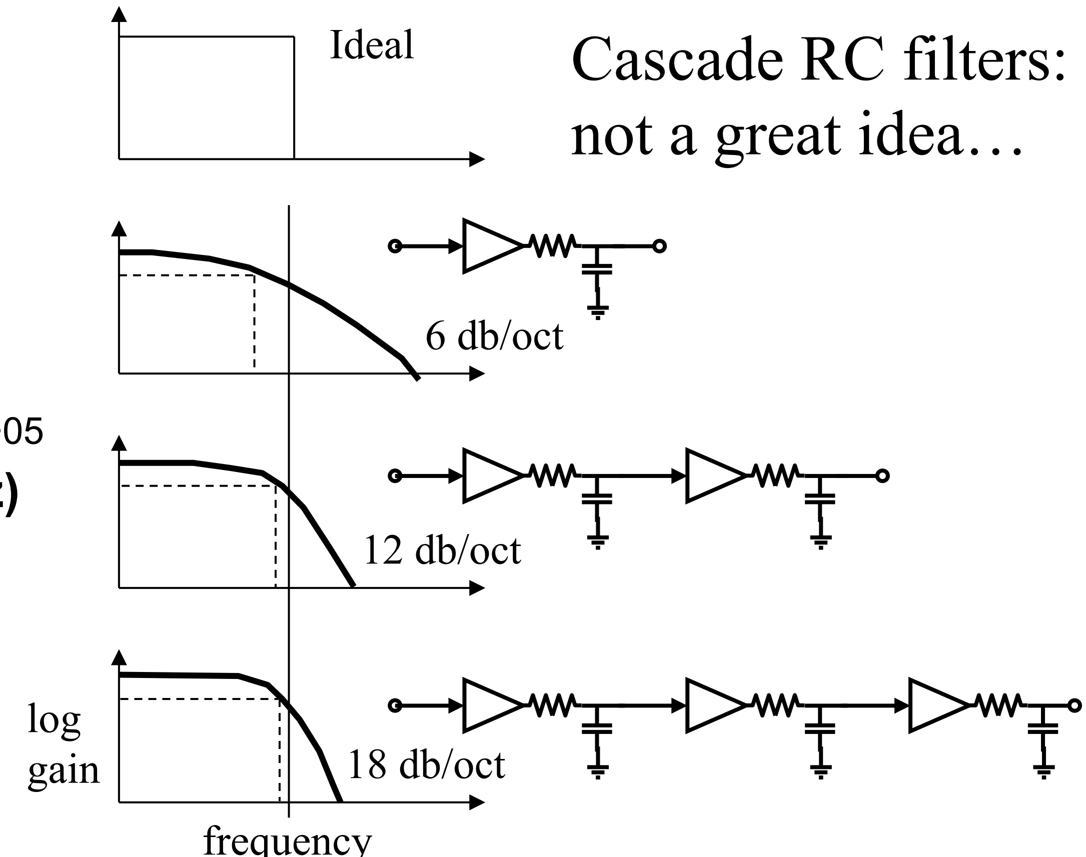
- To limit the bandwidth to 1 kHz, we can use a 100kΩ resistor with 1.6 nF capacitor.

The filter should be placed between a pre-amp and amp, to buffer the input and output





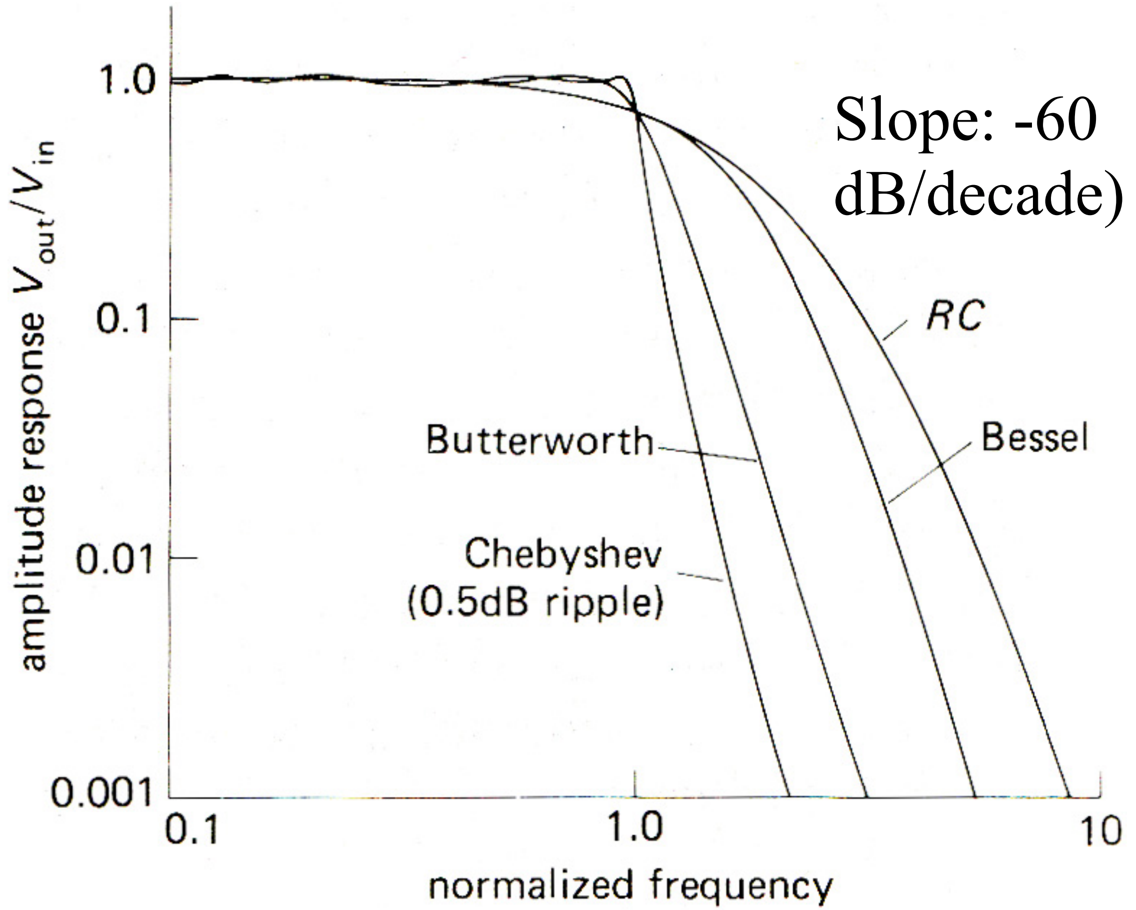
Cascade RC filters: not a great idea…

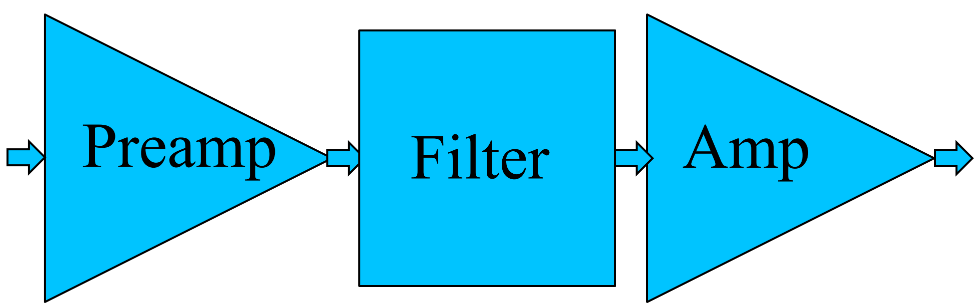


Low-pass filter examples 2

Active filter: Same bandwidth - 1 kHz - but:

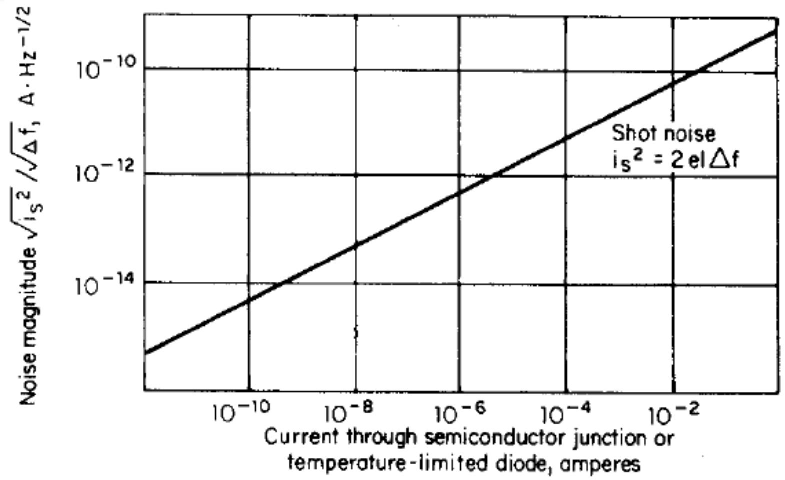
- Has built-in input/output buffering
- Has higher slopes
- Can have tailored frequency response (slope, phase/delay response, etc.)
- Same can be done in software (digital filter)





Shot noise reduction

Shot noise is proportional to the square root of DC (bias) current.



- In DC measurements, the *relative* S/N is proportional to the DC current.
- For low frequency AC measurements, low bias current reduces noise
- Bandwidth reduction is useful (same as in white noise reduction)
- When individual electrons or photons need to be counted, gated integration or multi-channel scaling can be used (see below)

1/f noise reduction: Band shifting

In many systems, the signal is DC. Noise reduction by bandwidth limiting is thus an excellent solution: just apply a low-pass filter! However, there are two problems:

- Amplifier noise is usually highest at DC (1/f noise)
- DC interference (thermal emf, etc.) and drift can be important

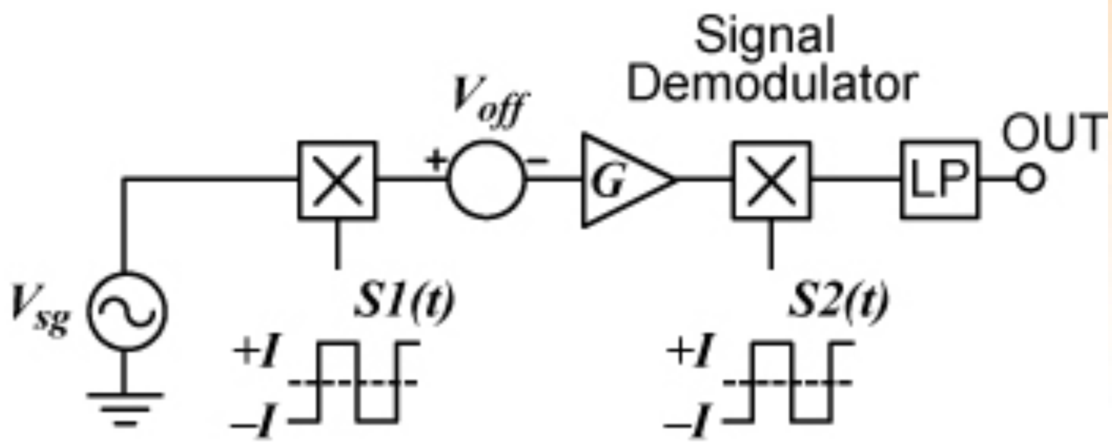
The solution is then band shifting. The most widely used instrument to accomplish this is the lock-in amplifier. The method consists of these steps:

- Transform the input signal to AC at a reference frequency (far from line freq.)
- Use a good AC preamplifier at this frequency (with extra line filtering)
- Filter the amplified signal at the reference frequency with the wanted bandwidth
- Transform the filtered amplitude again to DC

There are even commercial chopper-stabilized DC amplifiers, where an incoming DC signal is internally switched on/off electronically, amplified and converted back to DC.

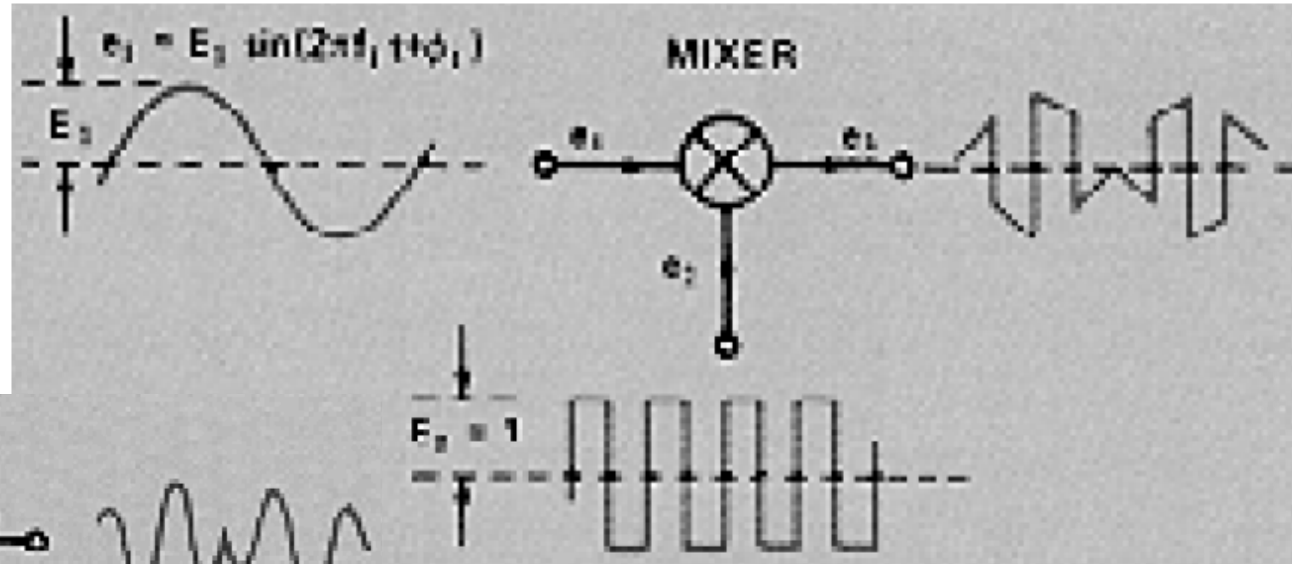
Band shifting: How to get AC signals

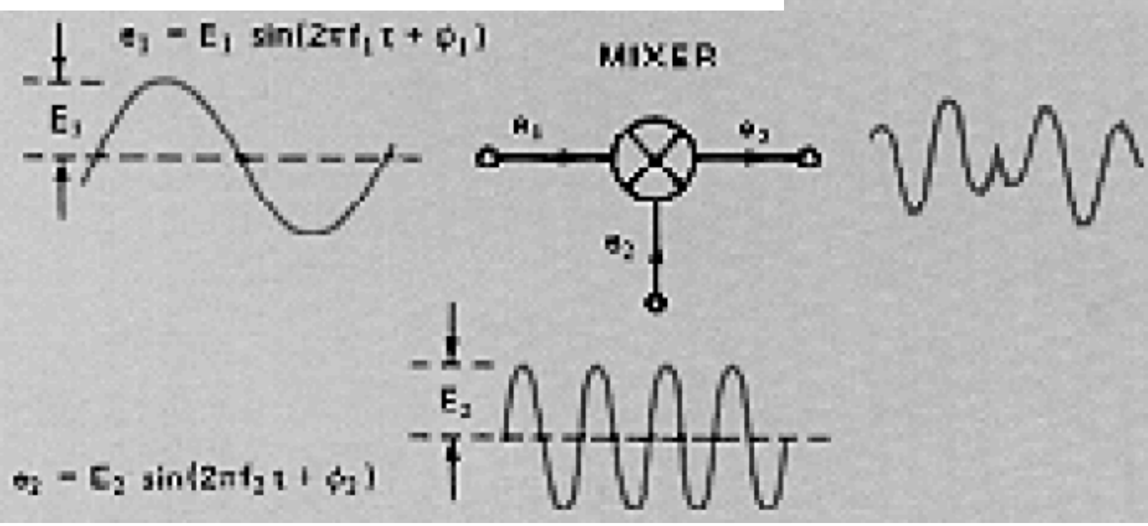
- In many systems, passive (resistive) transducers provide the signal: strain gauges, potentiometers, resistance thermometers, etc. In this case, it's enough to replace the DC drive by an AC drive at the reference frequency.
- In other cases, the system is stimulated by an external excitation source: voltage or current source in electrical measurements, light source in optical measurements, etc. The source can be transformed to AC, or light can be chopped.
- There are even commercial chopper-stabilized DC amplifiers, where an incoming DC signal is internally switched on/off electronically and then amplified and converted back to DC.



The Phase-sensitive detector (PSD)

- The PSD multiplies the signal (+noise) with a reference signal, either sinusoidal (e.g. analog multiplier) or square (e.g switch). Here are the waveforms:



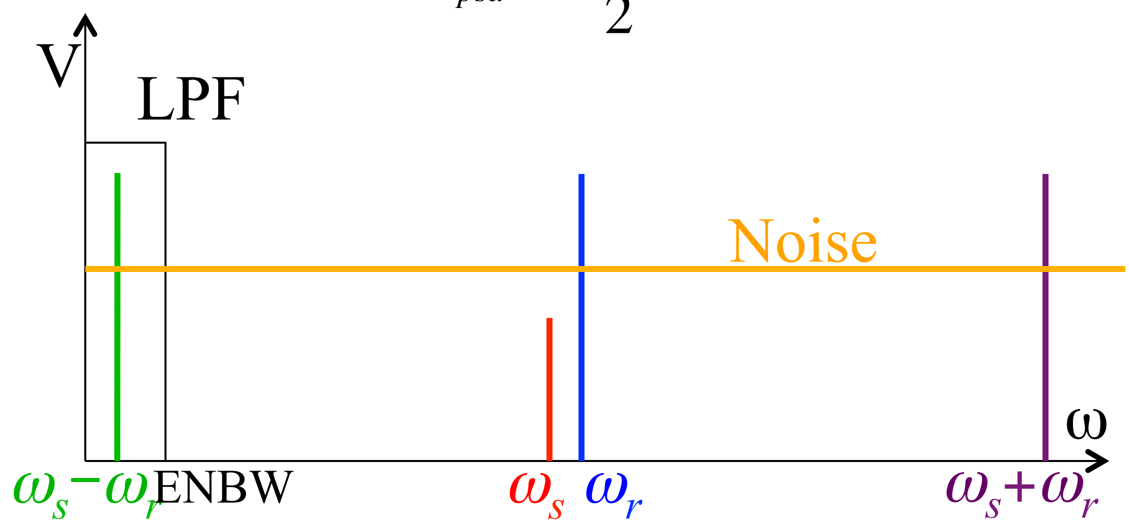


How the PSD works

- To analyze the operation of the PSD we use the Fourier transform = spectrum
- Suppose we have a signal: $V_s sin(\omega_s t)$ and a reference: $V_r sin(\omega_r t+\phi)$. Their multiplication in the PSD creates sum and difference frequencies:

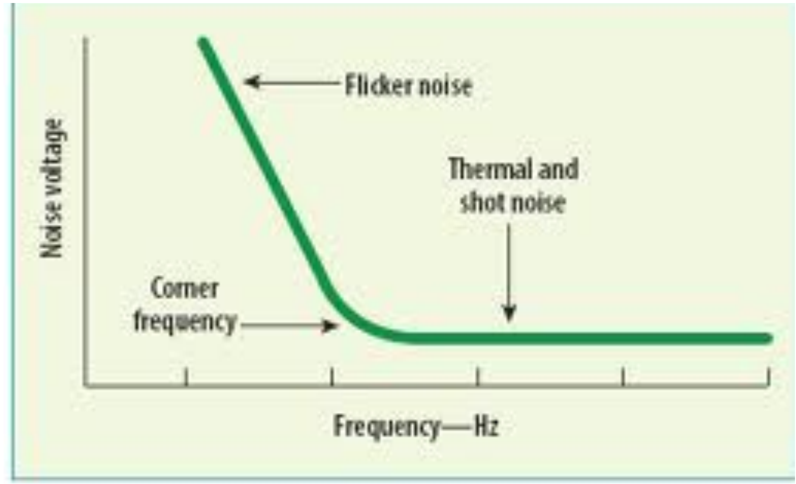
$$V_{psd} = V_s \sin(\omega_s t)\cdot V_r \sin(\omega_r t+\phi) = \frac{V_s V_r}{2}\left[\cos\left((\omega_s-\omega_r)t-\phi\right)-\cos\left((\omega_s+\omega_r)t+\phi\right)\right]$$

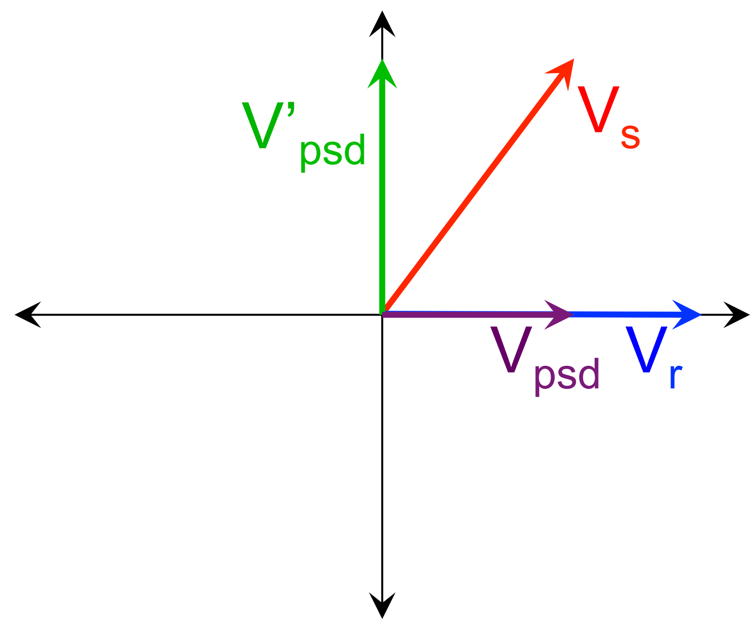
- A low-pass filter eliminates the sum frequency, as well as most of the noise.
- In the special case where $\omega_s = \omega_r$, the output is DC: $V_{psd} = \frac{V_s V_r}{2}\cos(\phi)$
- In this case the output depends on the *relative phase* between the signal and reference (can be tuned to zero)
- The noise bandwidth is reduced from ω_s to the filter's bandwidth: NEBW=1/4T.



The benefits of using PSD

- Noise reduction: The low-pass filter reduces the noise bandwidth from ω_s (typically kHz) to ENBW (usually <1Hz).
- More noise reduction: preamp works above *1/f* noise.
- No drift: pure AC measurement
- Phase measurements: $V_{psd} = \frac{V_s V_r}{2}\cos(\phi)$
- If we change the reference phase by $\pi/2$, we get: $V'_{psd} = \frac{V_s V_r}{2}\sin(\phi)$
- Combining these, we can know the phase relationship between signal and reference: Vector measurement

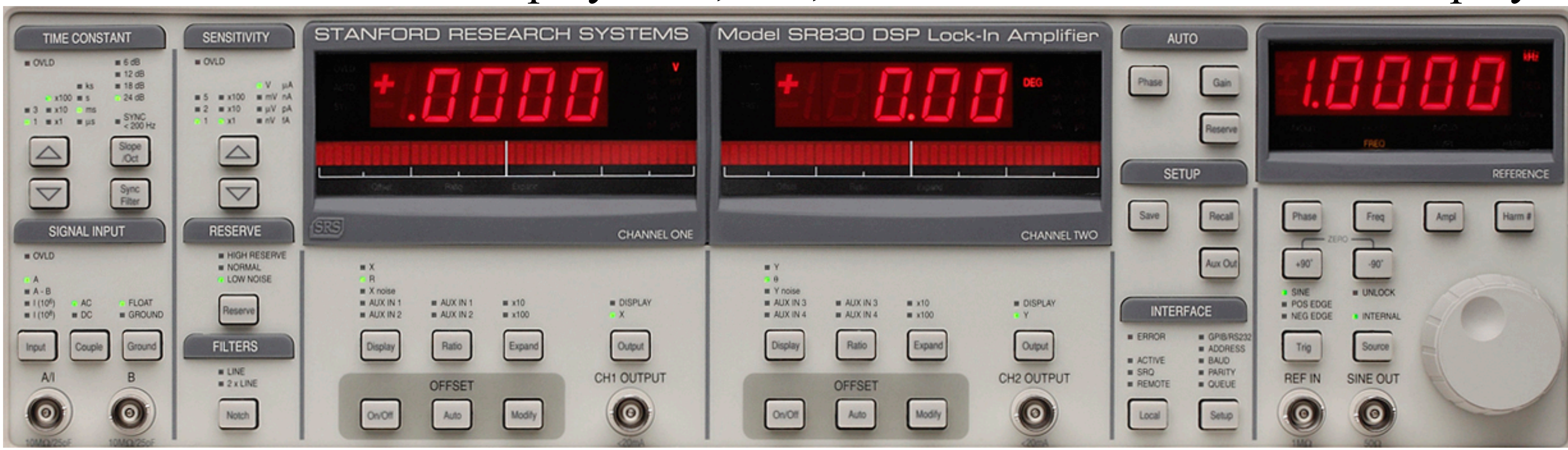




The Lock-in Amplifier

- Combines pre-amplifier, filters, PSD, reference generator … in one programmable instrument.

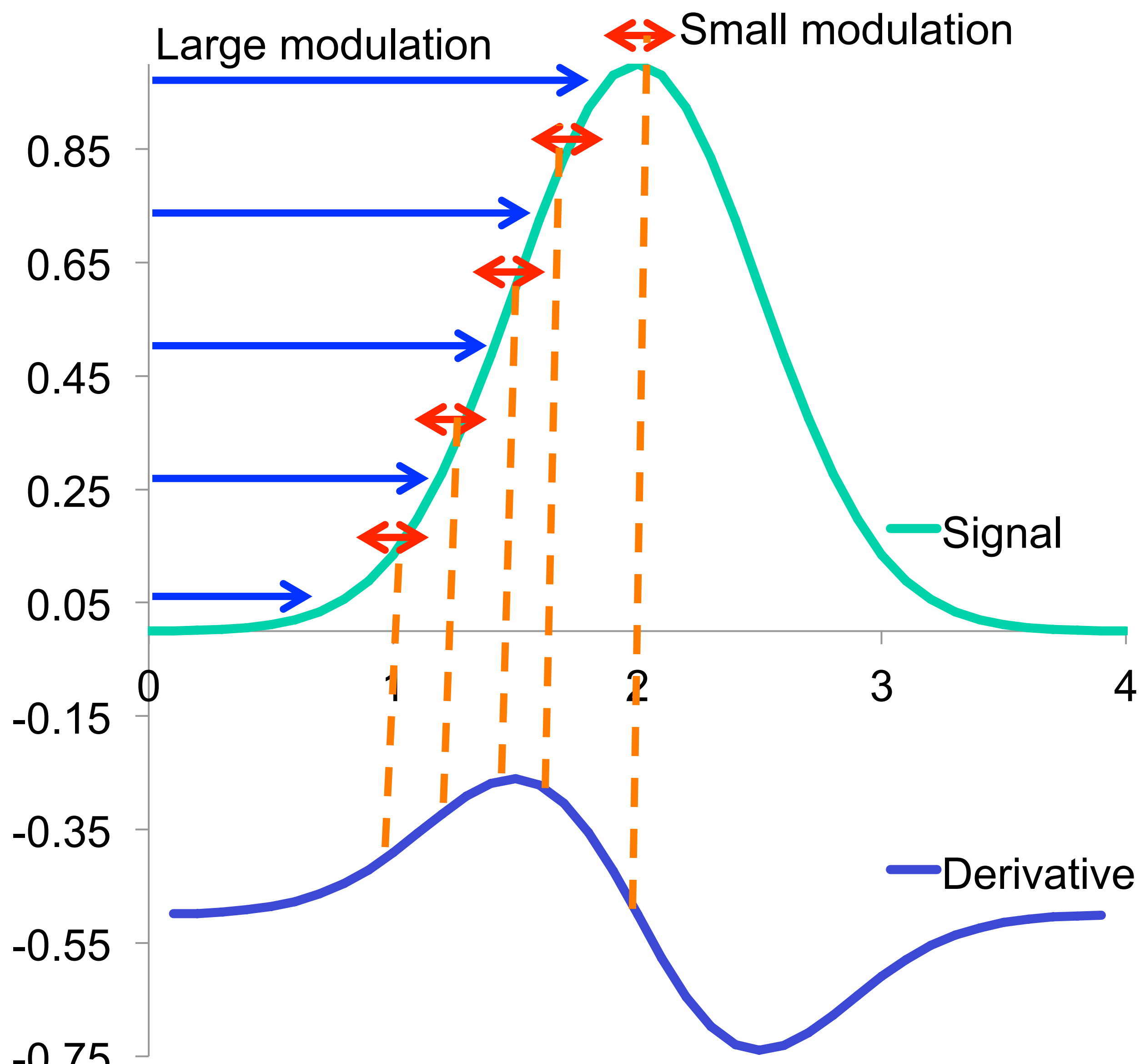
Gain TC Display: X/Y, R/θ, etc. Ref. display



Inputs (differential, current) Filters Outputs: X/Y, R/θ, etc. Reference In/Out

Two ways to measure…

- Large modulation: excitation goes from 0 to full value at each cycle. Example: Light chopper
- Small modulation: small excitation amplitude riding on slowly varying "DC" signal sweep. Example: AC I/V curve measurements, showing dI/dV vs. V curves

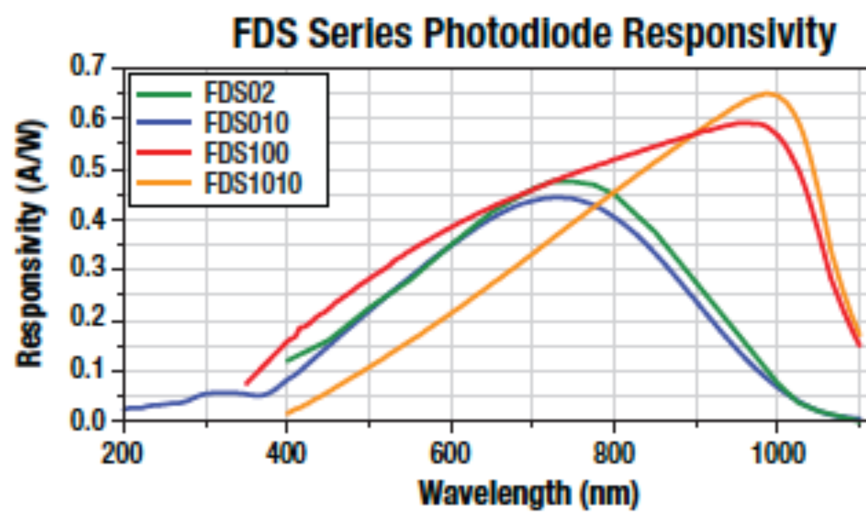


Example of LIA use: photodiode

- We measure light with a Si photodiode. Main specifications: NEP=$5{\cdot}10^{-14}$ W·Hz$^{-1/2}$, rise time= 1 ns (BW=350 MHz), S = 0.5 A/W. Noise current: I_n=$2.5{\cdot}10^{-14}$ A·Hz$^{-1/2}$.

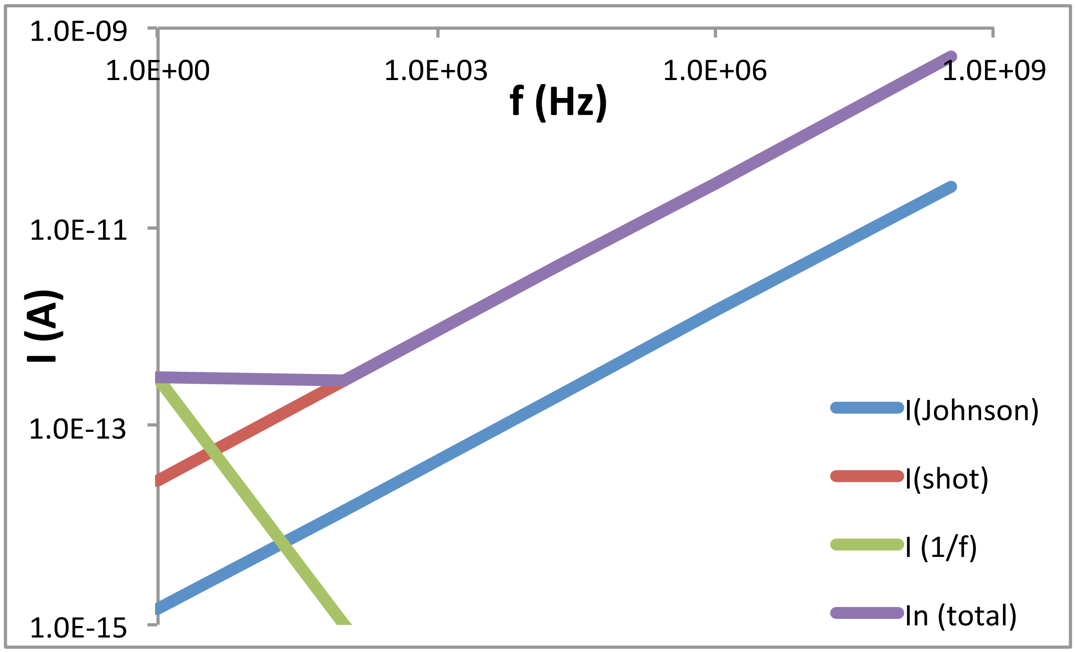
ITEM #	RISE TIME*	ACTIVE AREA	NEP	DARK CURRENT
FDS010	1 ns	0.8 mm² (Ø1 mm)	5×10^{-14} W/√Hz	2.5 nA (20 V)
FDS100	10 ns	13 mm² (3.6 x 3.6 mm)	1.2×10^{-14} W/√Hz	20 nA (20 V)
FDS02	47 ps	Ø0.25 mm	9.3×10^{-15} W/√Hz	35 pA (5 V)
FDS1010	45 ns	94.1 mm² (9.7 x 9.7 mm)	5.0×10^{-13} W/√Hz	0.6 µA (5 V)
FDS100-CAL	10 ns	13 mm² (3.6 x 3.6 mm)	1.2×10^{-14} W/√Hz	20 nA (20 V)
FDS1010-CAL	45 ns	94.1 mm² (9.7 x 9.7 mm)	4×10^{-13} W/√Hz	0.6 µA (5 V)

- The DC resistance is: V_b/I_d=8GΩ, giving Johnson noise: I_{jn}=$1.4{\cdot}10^{-15}$ A·Hz$^{-1/2}$
- The shot noise for bias current of 2.5nA: I_{sn}=$2.8{\cdot}10^{-14}$ A·Hz$^{-1/2}$.
- The photodiode noise current is dominated by shot noise!



Example of LIA use

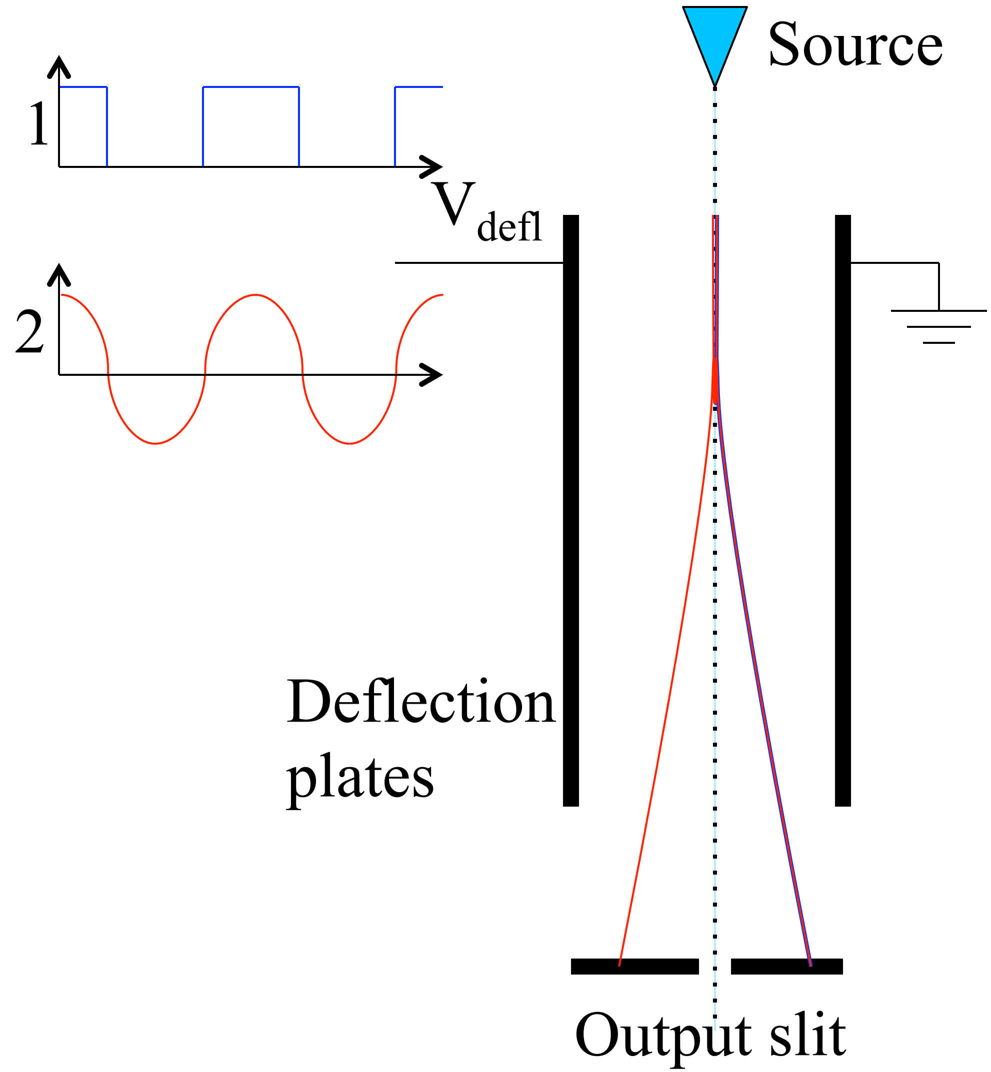
BW (Hz)	I(Johnson)	I(shot)	I (1/f)	In (total)
1.0E+00	1.4E-15	2.8E-14	3.0E-13	3.0E-13
1.0E+02	1.4E-14	2.8E-13	1.0E-15	2.8E-13
2.0E+04	2.0E-13	4.0E-12	5.0E-18	4.0E-12
1.0E+06	1.4E-12	2.8E-11	1.0E-19	2.8E-11
3.5E+08	2.6E-11	5.2E-10	2.9E-22	5.2E-10



- If we keep the nominal BW, total diode noise: I_n=0.5nA, equiv. NEP=1nW, It's a lot!
- Using a low-pass filter with BW=1 Hz, will reduce shot noise to $I_n=2.8\cdot10^{-14}$ A or NEP=$5.6\cdot10^{-14}$ W. However, typical amplifier's 1/f noise could be 2-10 times higher!
- With a lock-in amp, TC=250 ms, we can measure a signal at f=100Hz-100kHz, with 1Hz bandwidth, and keep the reduced noise of the diode ($2.8\cdot10^{-14}$ A)

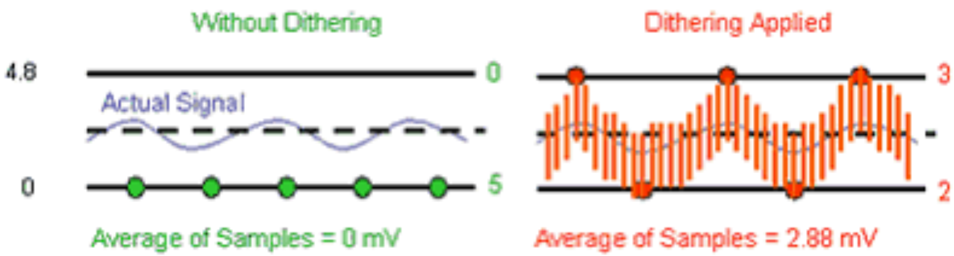
Advanced lock-in techniques

- Sometimes, the signal is a harmonic of the reference.
- Example: particle beam chopper, fed by an AC voltage (reference).
  1. Unipolar voltage: beam is blanked at the reference frequency
  2. Bipolar voltage: beam is blanked at double the reference frequency
- In rare cases, two lock-in or other noise reduction techniques are combined



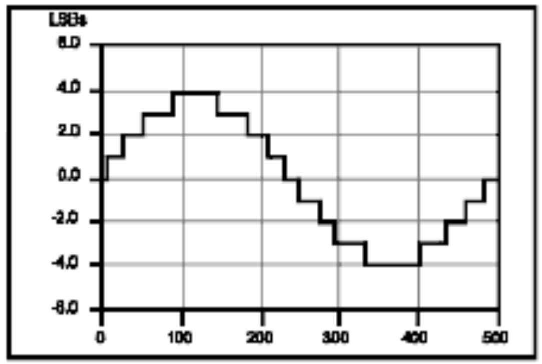
More noise is less: Dithering reduces quantization noise

To reduce quantization noise, Dithering the signal means adding some random (white) noise, equivalent to 0.5 LSB of the DAC. The digitized signal is then averaged to remove the noise. This removes the deterministic nature of the quantization.

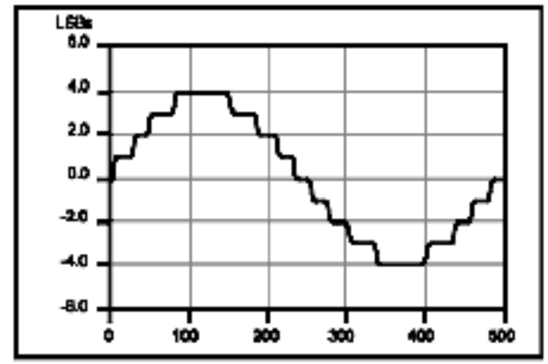


Signal is smaller than 1 LSB: it's always measured as 0 ! (deterministic)

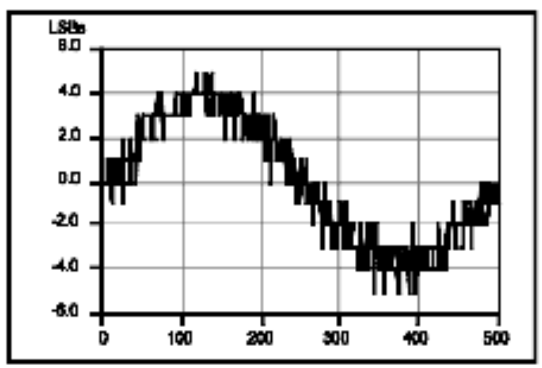
Signal+noise go above the 1 LSB barrier: The average value corresponds to the signal level (fraction of 1 LSB!



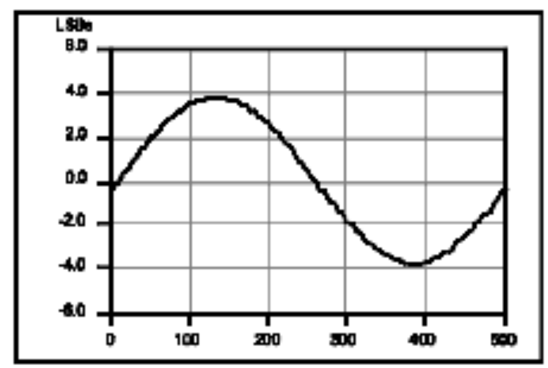
a. Dither disabled; no averaging



b. Dither disabled; average of 50 acquisitions



c. Dither enabled; no averaging



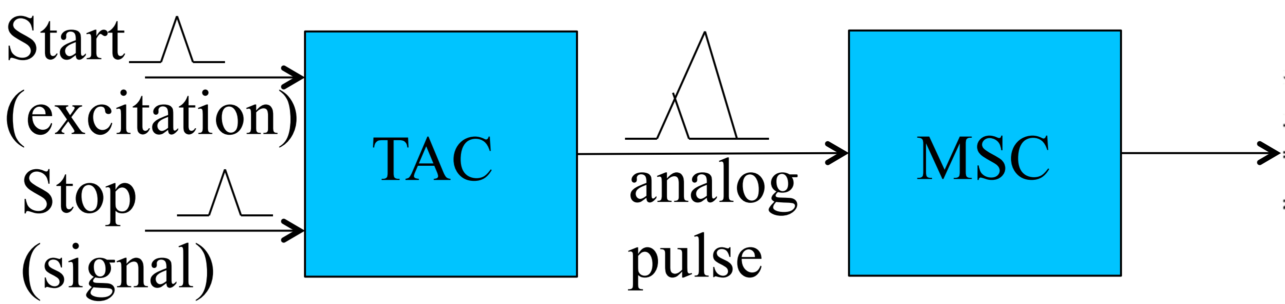
d. Dither enabled; average of 50 acquisitions

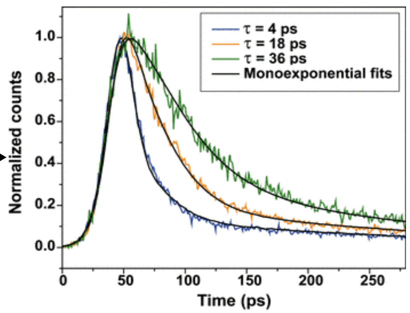
High-frequency (pulse) techniques

- In many experiments, measurements provide short pulses where the frequency, height or area represent the data.
- Examples: Photomultipliers, particle detectors.
- Noise reduction is important because of the high frequencies involved.
- The methods are called multi-channel scaling, gated integration, boxcar averaging

Pulse example: lifetime measurements

- In this experiments, we measure the lifetime by measuring the time lapse between a start pulse (from the excitation source) and a stop pulse (the signal from the experiment). These are repeated many times to get a curve.
- First step: convert time interval to pulse height: Time-Amplitude Converter
- Second step: Scale the pulses into Multiple Channels according to their height, producing a histogram of no. of pulses vs. height (which corresponds to time delay)





Pulse example: noise calculations

- Noise: random pulses mixed with the signal, giving random contribution to all channels. If they were constant – no problem: subtract a fixed number! However, noise has fluctuations, which can blur the signal. For N^2 pulses, fluctuations are ≈N
- Noise reduction: noise pulses are divided equally to all channels, signal pulses accumulate in a specific channel.
- Result: with time, we accumulate more data than noise fluctuations.
- Example: suppose we have 1 signal pulse and 100 noise pulses /sec
- After 1 sec: 101 pulses, fluctuations = 10 pulses, S/N=1/10 (cannot see the signal)
- After 10,000 sec: 1010000 pulses, fluctuations=1000 pulses, S/N=10000/1000 (can see signal).
- This is actually a bandwidth-narrowing technique!

Gated integration (boxcar averaging)

- Another method to reduce noise: the gated integrator
- First step: a gate allows the signal to pass only during the excitation time
- Second step: an integrator accumulates the signal during a long measurement time
- Dual benefit for noise reduction:
  - Noise is acquired only during the signal period (gate on): big improvement for low duty cycles
  - Integration reduces noise.
- Schematic diagram of the system:

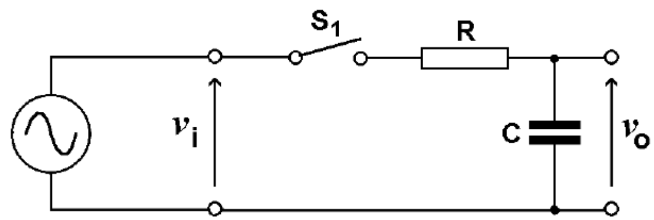
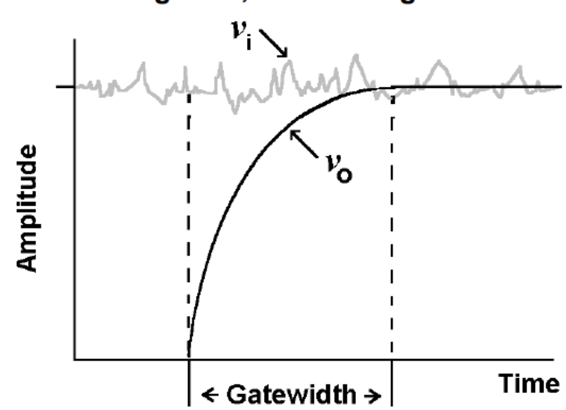
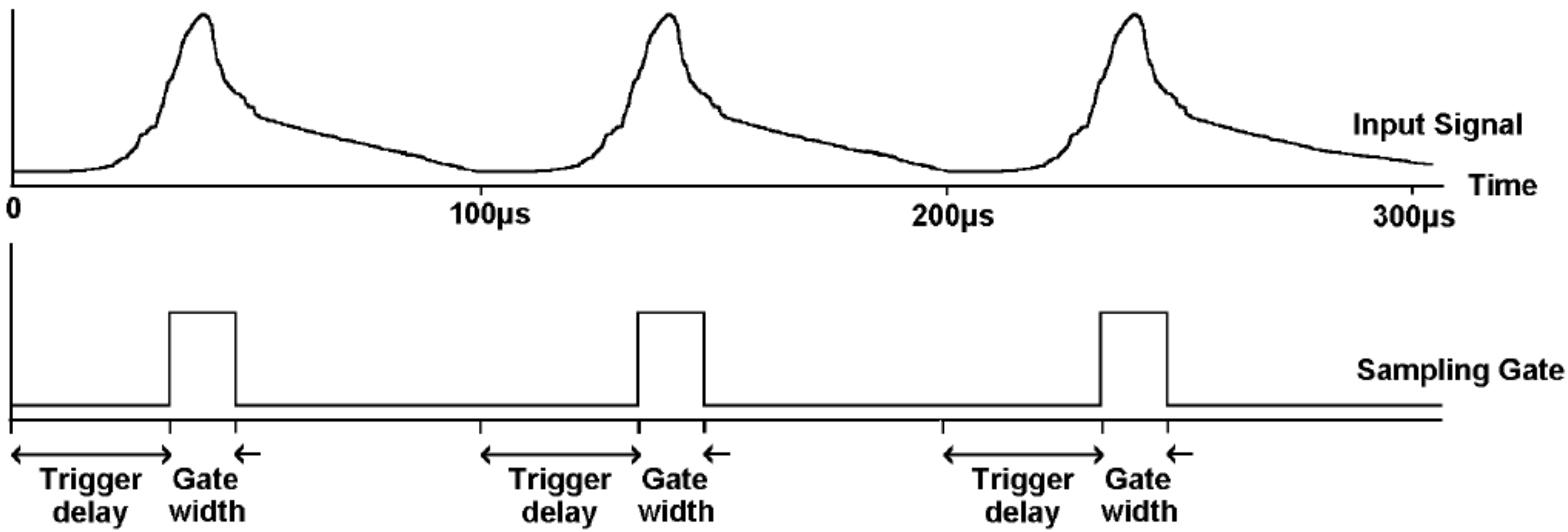


Figure 1, Gated Integrator



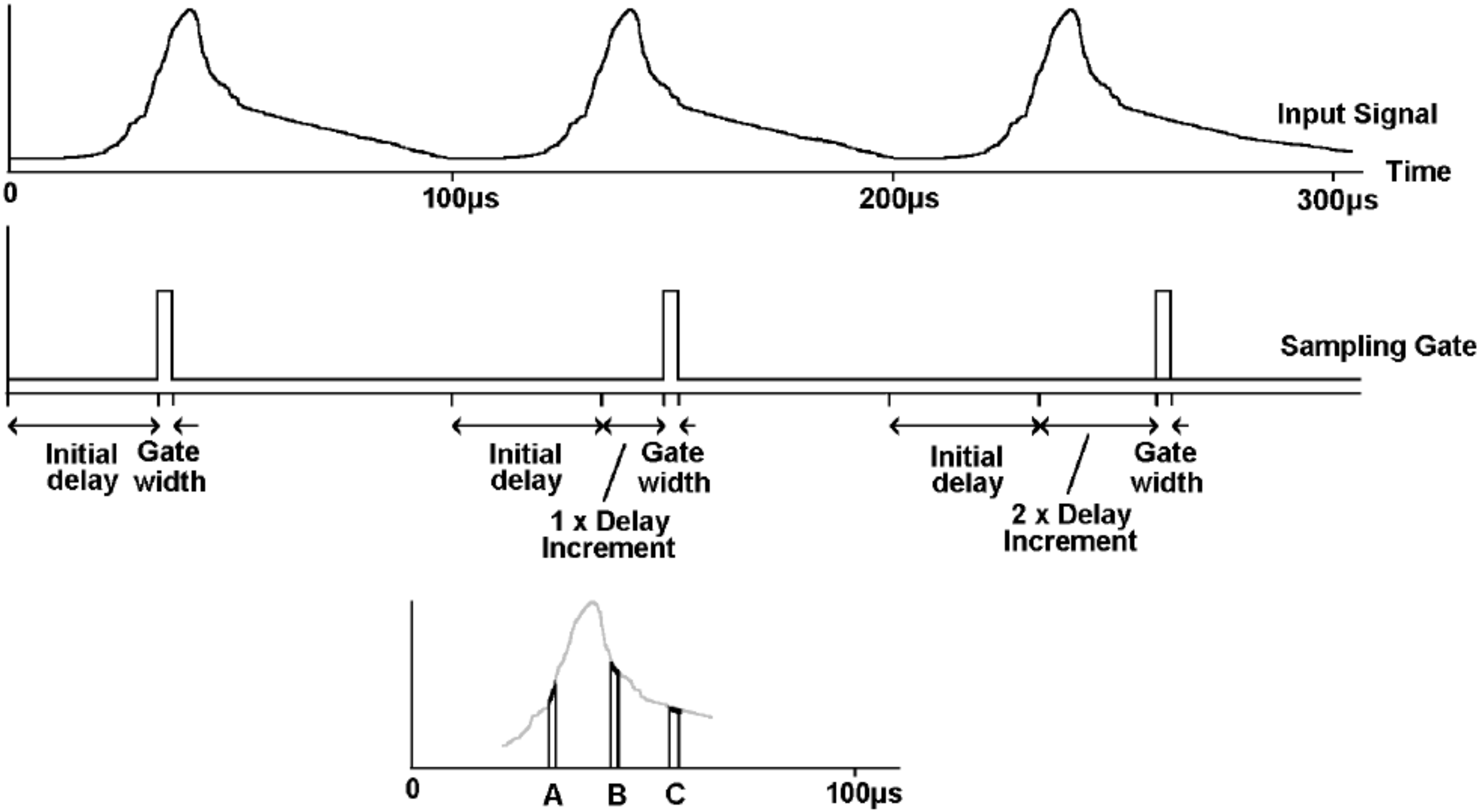
Gated integration: how it works

- In this example, 1μs excitation pulse width and 100μs period.
- Gating the signal at 1μs reduces the noise bandwidth by 100 and noise by 10
- Further integration of many repetitions can reduce the noise bandwidth even more
- The final (DC) output measures the signal at the specific time point (e.g. excitation)
- Final noise reduction is: $\sqrt{T_{\text{int}}/t_g}$



Gated integration: variable delay

- To trace the full waveform, the gate delay is sweeped (slowly!) so that at each point the gate samples one point of the signal.



Why "Boxcar" ?

- The American name of merchandise wagons, because of their box-like shape
- Can imagine a "train" of pulses of the same shape…
- Or the gate pulse as the door!

The Boxcar and the LIA

- The lock-in amplifier can be regarded as a special case of the boxcar integrator, with:
- 50% pulse width
- Phase = delay/period