ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Analyse d'une collision élastique

Mécanique, cours 12.2

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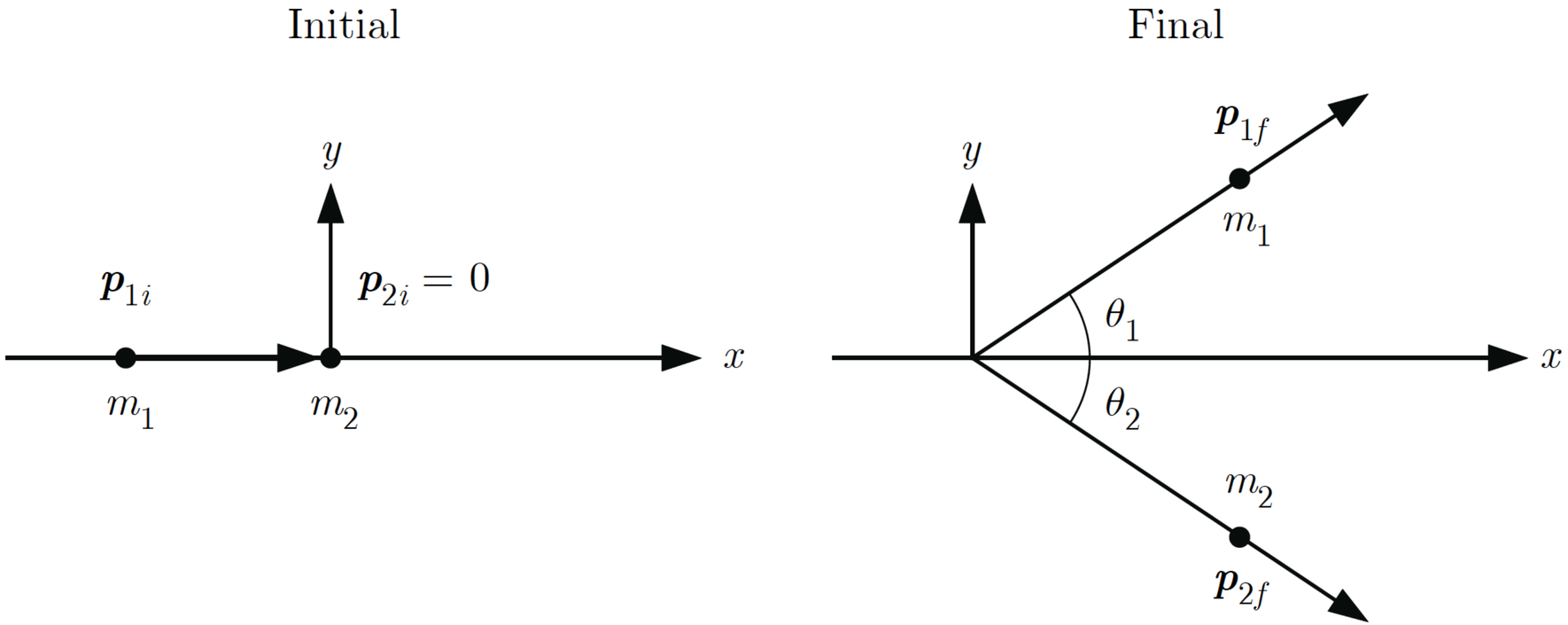
- Principes de conservation
- Dans un plan
- Masses égales
- Sur une ligne droite

Principes de conservation



$$\boldsymbol{p}_{1i} + \boldsymbol{p}_{2i} = \boldsymbol{p}_{1f} + \boldsymbol{p}_{2f}$$

$$T_{1i} + T_{2i} = T_{1f} + T_{2f}$$

Choc élastique : calculs

$$p_{1i} = p_{1f}\cos\theta_1 + p_{2f}\cos\theta_2$$
$$0 = p_{1f}\sin\theta_1 - p_{2f}\sin\theta_2$$
$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2}$$

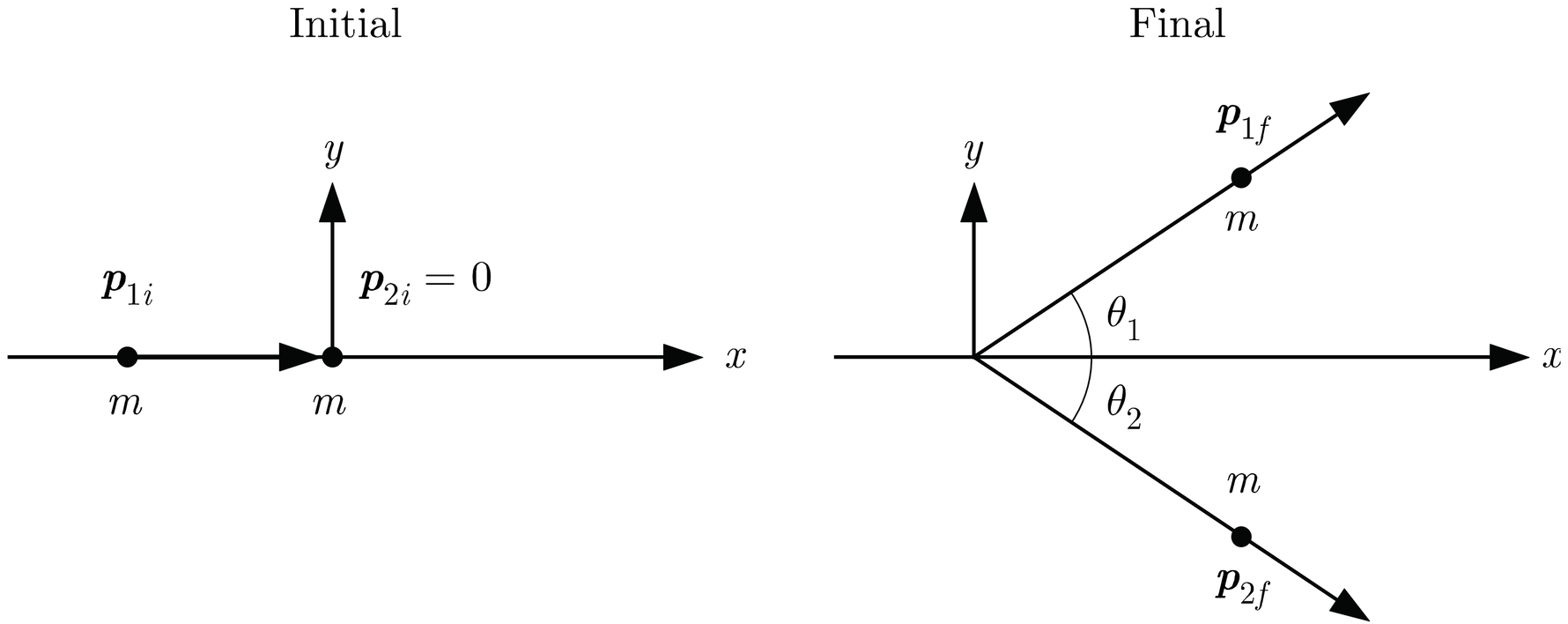
$$(p_{1i} - p_{1f}\cos\theta_1)^2 = p_{2f}^2\cos^2\theta_2$$
$$p_{1f}^2\sin^2\theta_1 = p_{2f}^2\left(1 - \cos^2\theta_2\right)$$

$$(p_{1i} - p_{1f}\cos\theta_1)^2 + p_{1f}^2\sin^2\theta_1 = p_{2f}^2$$

$$p_{1f}^2\left(1 + \frac{m_2}{m_1}\right) - 2p_{1i}p_{1f}\cos\theta_1 + p_{1i}^2\left(1 - \frac{m_2}{m_1}\right) = 0$$

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1 - \frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$

Choc élastique : masses égales



$$\boldsymbol{v}_{1i} = \boldsymbol{v}_{1f} + \boldsymbol{v}_{2f} \longrightarrow \boldsymbol{v}_{1i}^2 = \boldsymbol{v}_{1f}^2 + \boldsymbol{v}_{2f}^2 + 2\boldsymbol{v}_{1f} \cdot \boldsymbol{v}_{2f}$$

$$\boldsymbol{v}_{1i}^2 = \boldsymbol{v}_{1f}^2 + \boldsymbol{v}_{2f}^2$$

$$\boldsymbol{v}_{1f} \cdot \boldsymbol{v}_{2f} = 0$$

Choc élastique : sur une ligne droite

$$\frac{p_{1f}}{p_{1i}} = \frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{\cos\theta_1 \pm \left[\cos^2\theta_1 - \left(1 - \frac{m_2^2}{m_1^2}\right)\right]^{1/2}\right\}$$

$$\theta_1 = 0 \implies$$

$$\frac{v_{1f}}{v_{1i}} = \frac{m_1}{m_1 + m_2}\left\{1 \pm \sqrt{1 - \frac{m_1^2 - m_2^2}{m_1^2}}\right\} = \frac{m_1 \pm m_2}{m_1 + m_2}$$

$$\begin{cases} 1 \\ \dfrac{m_1 - m_2}{m_1 + m_2} \end{cases}$$

$$\frac{p_{1i}^2}{2m_1} = \frac{p_{1f}^2}{2m_1} + \frac{p_{2f}^2}{2m_2} \qquad \frac{1}{2}\,m_1 v_{1i}^2 = \frac{1}{2}\,m_1 v_{1f}^2 + \frac{1}{2}\,m_2 v_{2f}^2$$

$$v_{2f}^2 = \frac{m_1}{m_2}\,v_{1i}^2\left(1 - \frac{v_{1f}^2}{v_{1i}^2}\right) = \frac{m_1}{m_2}\,v_{1i}^2\left(1 - \frac{(m_1 \pm m_2)^2}{(m_1 + m_2)^2}\right)$$

$$\begin{cases} 0 \\ v_{2f} = \dfrac{2m_1}{(m_1 + m_2)}\,v_{1i} \end{cases}$$