ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Référentiels accélérés

Mécanique, cours 15.1

Jean-Philippe Ansermet

Référentiels accélérés

Référentiels accélérés

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

- Cinématique
- Accélération de Coriolis
- Dynamique
- Forces d'inertie

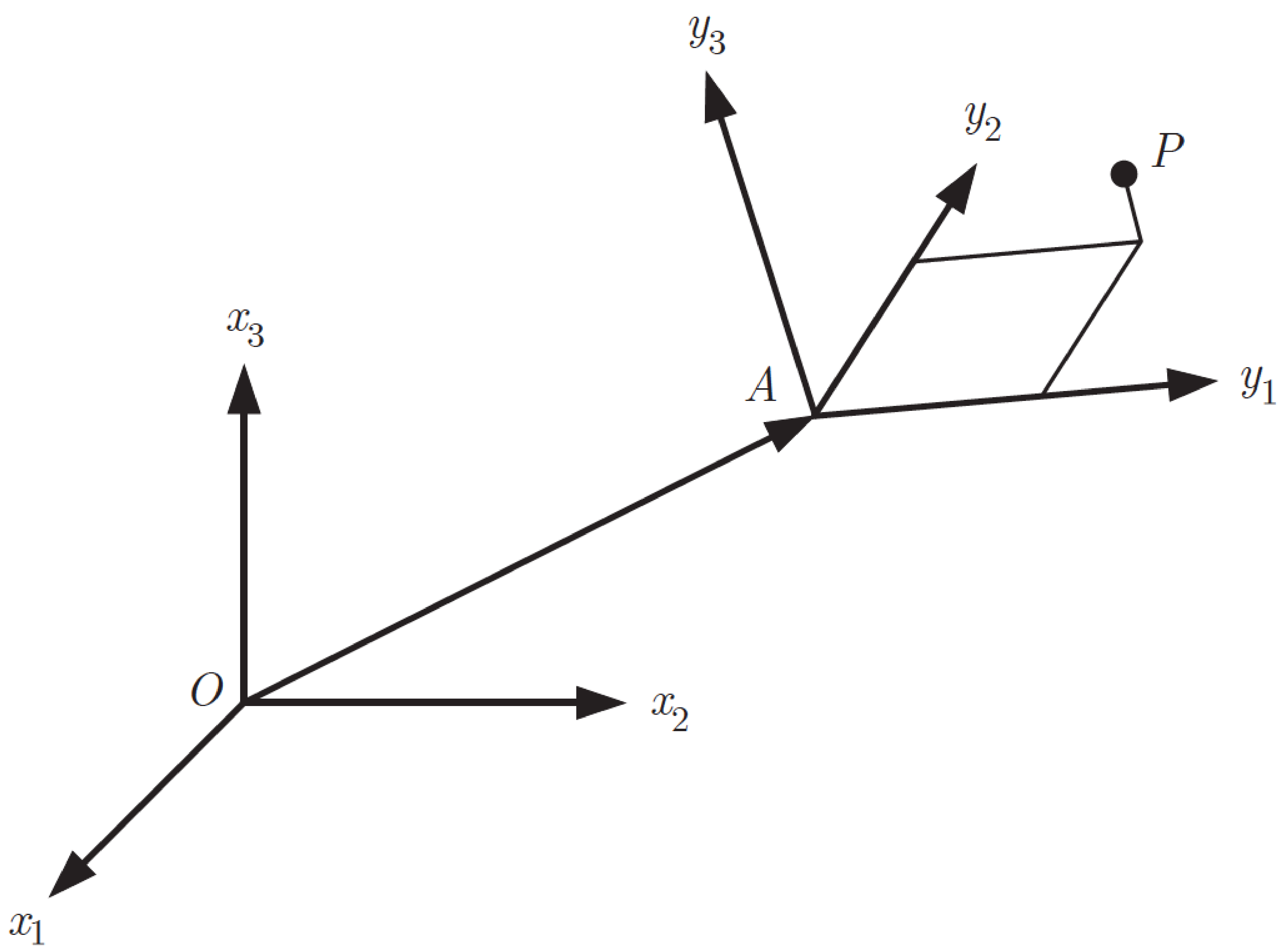
Référentiels ‘absolu’ et ‘relatif’

Equation horaire $\boldsymbol{OA}(t)$

Repère : $\left(A, \hat{\boldsymbol{y}}_1 \hat{\boldsymbol{y}}_2 \hat{\boldsymbol{y}}_3\right)$

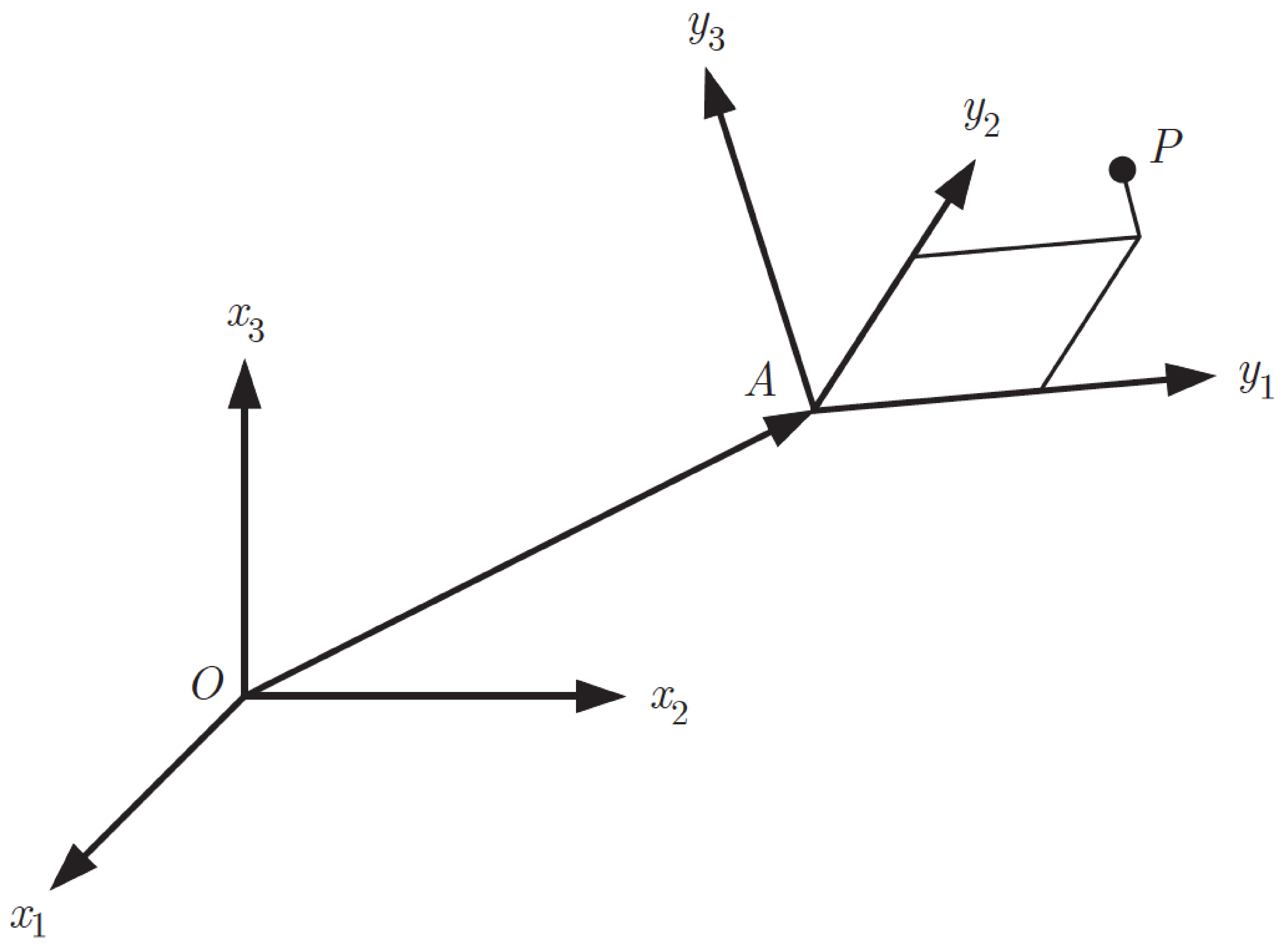
$$\frac{d\hat{\boldsymbol{y}}_i}{dt} = \boldsymbol{\Omega} \wedge \hat{\boldsymbol{y}}_i \qquad (i = 1, 2, 3)$$

‘relatif’ (accéléré) $A\, y_1 y_2 y_3$



‘absolu’ (d’inertie) $O\, x_1 x_2 x_3$

Vitesse et accélération ‘relatives’

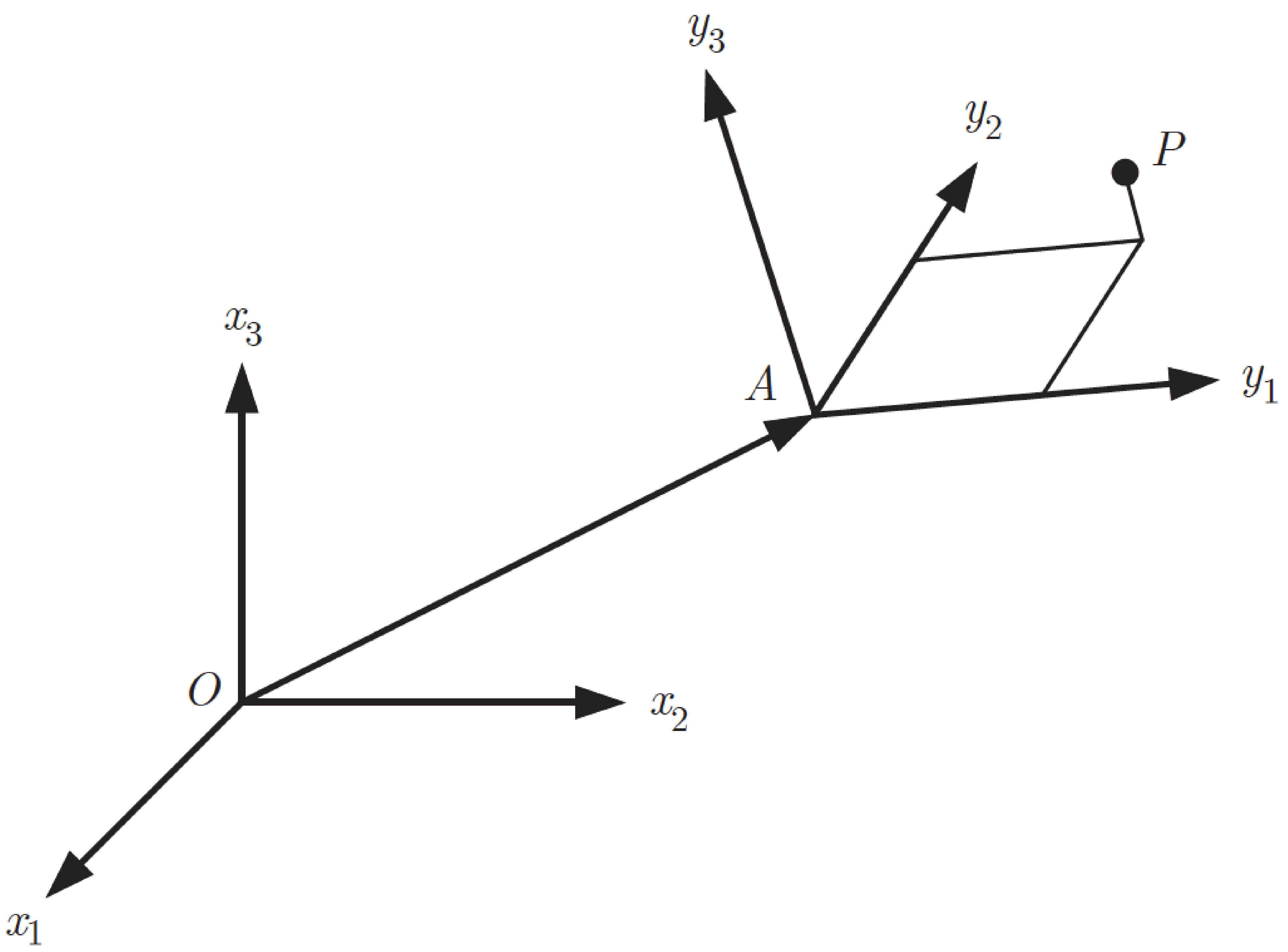


$$\boldsymbol{AP} = \sum_i y_i \hat{\boldsymbol{y}}_i$$

$$\boldsymbol{v}_r(P) = \sum_i \dot{y}_i \, \hat{\boldsymbol{y}}_i$$

$$\boldsymbol{a}_r(P) = \sum_i \ddot{y}_i \, \hat{\boldsymbol{y}}_i$$

Vitesse absolue



$$\boldsymbol{v}_a(P) = \frac{d}{dt}\boldsymbol{OP} \qquad \boldsymbol{OP} = \boldsymbol{OA} + \boldsymbol{AP}$$

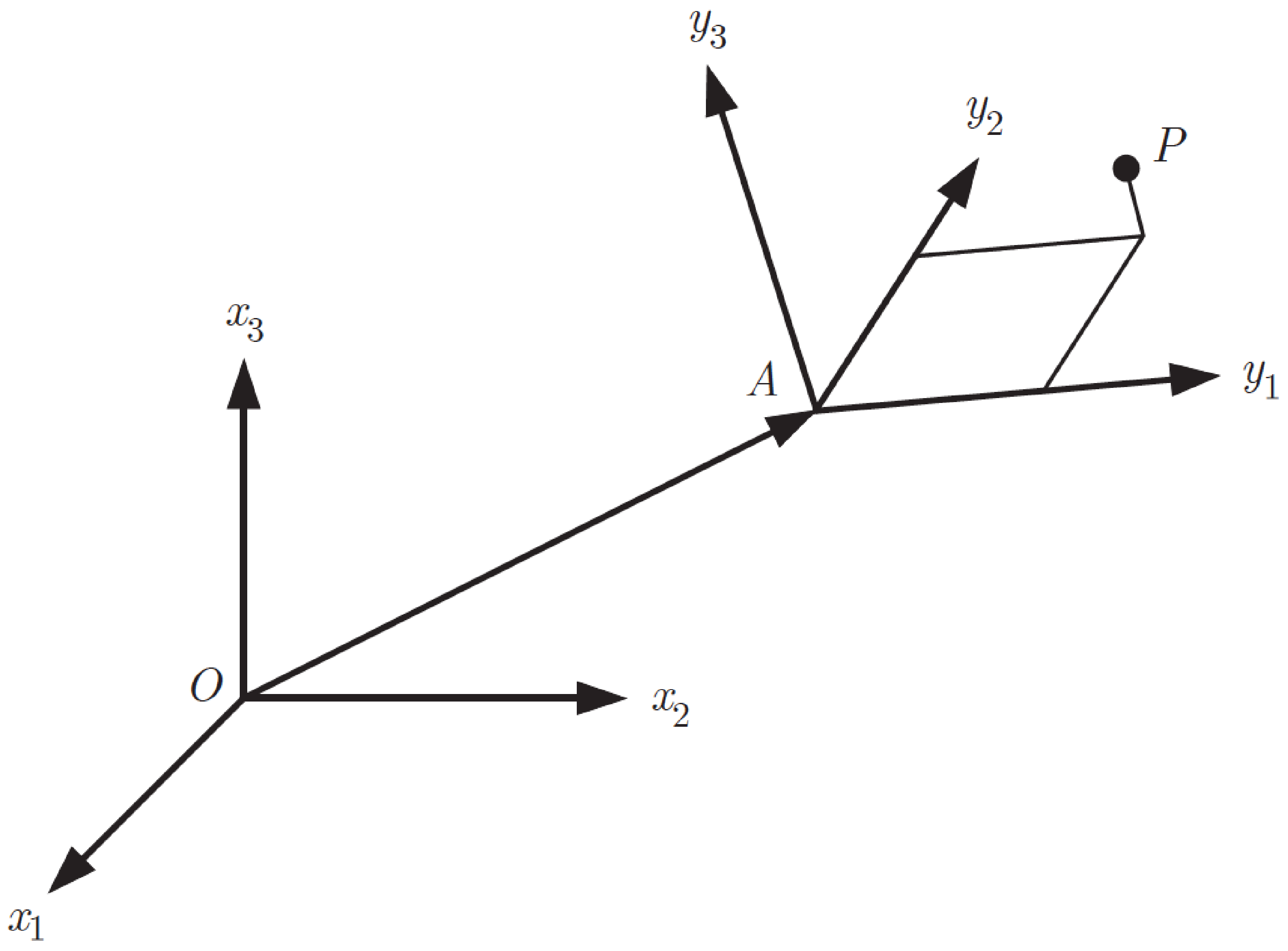
$$\boldsymbol{v}_a(P) = \frac{d}{dt}(\boldsymbol{OA}) + \frac{d}{dt}(\boldsymbol{AP}) = \boldsymbol{v}_a(A) + \frac{d}{dt}\left(\sum_i y_i \hat{\boldsymbol{y}}_i\right)$$

$$= \boldsymbol{v}_a(A) + \sum_i \dot{y}_i \hat{\boldsymbol{y}}_i + \sum_i y_i \dot{\hat{\boldsymbol{y}}}_i$$

$$= \boldsymbol{v}_a(A) + \boldsymbol{v}_r(P) + \sum_i y_i (\boldsymbol{\Omega} \wedge \hat{\boldsymbol{y}}_i)$$

$$\boldsymbol{v}_a(P) = \boldsymbol{v}_a(A) + \boldsymbol{v}_r(P) + \boldsymbol{\Omega} \wedge \boldsymbol{AP}$$

Accélération absolue



$$\boldsymbol{v}_a(P) = \boldsymbol{v}_a(A) + \boldsymbol{v}_r(P) + \boldsymbol{\Omega} \wedge \boldsymbol{AP}$$

$$\frac{d}{dt}\boldsymbol{v}_a(P) = \frac{d}{dt}\boldsymbol{v}_a(A) + \frac{d}{dt}\boldsymbol{v}_r(P) + \frac{d}{dt}\boldsymbol{\Omega} \wedge \boldsymbol{AP} + \boldsymbol{\Omega} \wedge \frac{d}{dt}\boldsymbol{AP}$$

$$\boldsymbol{a}_a(P) = \boldsymbol{a}_a(A) + \frac{d}{dt}\left(\sum_i \dot{y}_i \hat{\boldsymbol{y}}_i\right) + \dot{\boldsymbol{\Omega}} \wedge \boldsymbol{AP} + \boldsymbol{\Omega} \wedge \frac{d}{dt}\left(\sum_i y_i \hat{\boldsymbol{y}}_i\right)$$

$$\sum_i \dot{y}_i \boldsymbol{\Omega} \wedge \hat{\boldsymbol{y}}_i = \boldsymbol{\Omega} \wedge \boldsymbol{v}_r$$

$$\boldsymbol{a}_r \qquad \boldsymbol{v}_r \qquad \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{AP})$$

$$\boldsymbol{a}_a(P) = \boldsymbol{a}_a(A) + \boldsymbol{a}_r(P) + 2\boldsymbol{\Omega} \wedge \boldsymbol{v}_r(P) + \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{AP}) + \dot{\boldsymbol{\Omega}} \wedge \boldsymbol{AP}$$

Définition : accélération de Coriolis

$$\boldsymbol{a}_a(P) = \boldsymbol{a}_a(A) + \boldsymbol{a}_r(P) + 2\boldsymbol{\Omega} \wedge \boldsymbol{v}_r(P) + \boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{AP}) + \dot{\boldsymbol{\Omega}} \wedge \boldsymbol{AP}$$

Accélération de Coriolis ($2\boldsymbol{\Omega} \wedge \boldsymbol{v}_r(P)$)

Accélération centripète ($\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \boldsymbol{AP})$)

Dynamique

Dans le référentiel d'inertie $\boldsymbol{F} = m\boldsymbol{a}_a(P)$

$$\boldsymbol{F} = m\boldsymbol{a}_a(A) + m\boldsymbol{a}_r(P) + m\dot{\boldsymbol{\Omega}} \wedge \boldsymbol{AP} + m\boldsymbol{\Omega} \wedge \left(\boldsymbol{\Omega} \wedge \boldsymbol{AP}\right) + 2m\boldsymbol{\Omega} \wedge \boldsymbol{v}_r(\boldsymbol{P})$$

$$m\boldsymbol{a}_r(P) = \boldsymbol{F}_{effectif}$$

$$m\boldsymbol{a}_r(P) = \boldsymbol{F} - m\boldsymbol{a}_a(A) - m\boldsymbol{\Omega} \wedge \left(\boldsymbol{\Omega} \wedge \boldsymbol{AP}\right) - m\dot{\boldsymbol{\Omega}} \wedge \boldsymbol{AP} - 2m\boldsymbol{\Omega} \wedge \boldsymbol{v}_r(P)$$

Force centrifuge

Force de Coriolis