ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Dynamique terrestre : exemples

Mécanique, cours 16.2

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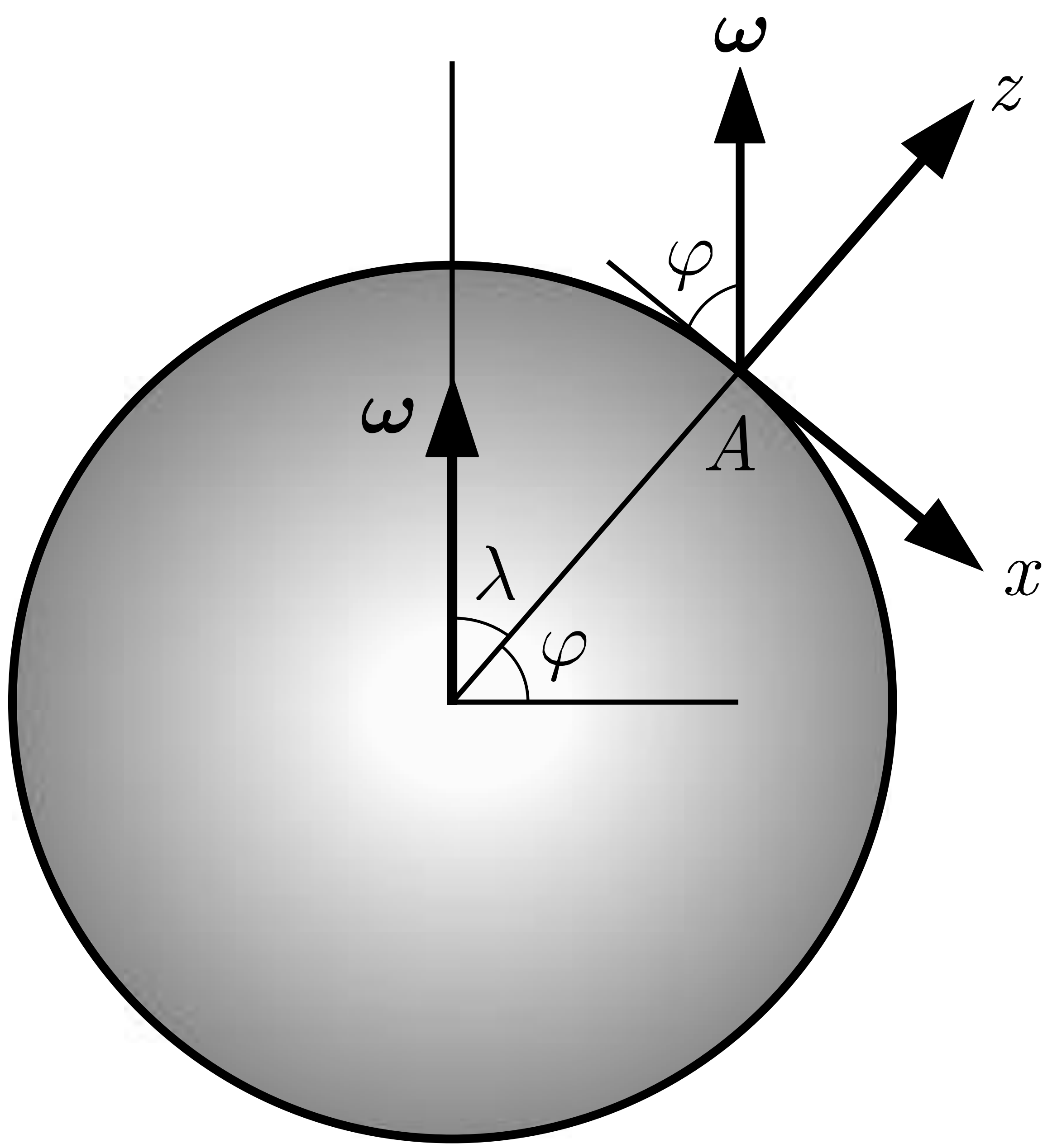
- Mouvement vertical
- Mouvement horizontal
- Pendule de Foucault

Equations du mouvement en projections



$$\boldsymbol{a}_r(P) = \boldsymbol{g} + \frac{\boldsymbol{F}}{m} - 2\boldsymbol{\omega} \wedge \boldsymbol{v}_r(P)$$

$$\boldsymbol{\omega} = \begin{pmatrix} -\omega\cos\varphi \\ 0 \\ \omega\sin\varphi \end{pmatrix}$$

$$\boldsymbol{v}_{rel}(P) = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

$$\boldsymbol{g} = \begin{pmatrix} 0 \\ 0 \\ -g \end{pmatrix}$$

$$2\boldsymbol{\omega} \wedge \boldsymbol{v}_{rel}(P) = 2\begin{vmatrix} \hat{\boldsymbol{x}} & -\omega\cos\varphi & \dot{x} \\ \hat{\boldsymbol{y}} & 0 & \dot{y} \\ \hat{\boldsymbol{z}} & \omega\sin\varphi & \dot{z} \end{vmatrix} = \begin{pmatrix} -2\dot{y}\omega\sin\varphi \\ 2\dot{x}\omega\sin\varphi + 2\dot{z}\omega\cos\varphi \\ -2\omega\cos\varphi\dot{y} \end{pmatrix}$$

Mouvement vertical

Equation du mouvement
avec seulement la pesanteur

$$\ddot{x} = +2\dot{y}\omega\sin\varphi$$
$$\ddot{y} = -2\dot{z}\omega\cos\varphi - 2\dot{x}\omega\sin\varphi$$
$$\ddot{z} = +2\omega\cos\varphi\dot{y} - g$$

Conditions initiales

$$t = 0;\ x(0) = y(0) = 0,\ z(0) = z_0, \quad \dot{x}(0) = \dot{y}(0) = 0,\ \dot{z}(0) = v_0$$

$$\dot{x}(t) - \underbrace{\dot{x}(0)}_{=0} = +2\big(y(t) - \underbrace{y(0)}_{=0}\big)\omega\sin\varphi \qquad \dot{x} = +2\omega\sin\varphi y$$

$$\dot{z}(t) - \underbrace{\dot{z}(0)}_{v_0} = +2\omega\cos\varphi\big(y(t) - \underbrace{y(0)}_{=0}\big) - gt \qquad \dot{z} = v_0 - gt + 2\omega\cos\varphi y$$

Mouvement vertical

$$\dot{x} = +2\omega \sin\varphi y \qquad \ddot{y} = -2\dot{z}\omega\cos\varphi - 2\dot{x}\omega\sin\varphi$$

$$\dot{z} = v_0 - gt + 2\omega\cos\varphi y$$

$$\ddot{y} = -2[v_0 - gt + 2\omega\cos\varphi y]\,\omega\cos\varphi - 2[2\omega\sin\varphi y]\,\omega\sin\varphi$$

$$\ddot{y} \simeq -2\omega\cos\varphi(v_0 - gt)$$

$$y(t) = -2\omega\cos\varphi\left(\frac{1}{2}\,v_0 t^2 - \frac{1}{6}\,gt^3\right)$$

Déviation dans la direction est-ouest !

$$\dot{z} = v_0 - gt \underbrace{- 4\omega^2\cos^2\varphi\left(\frac{1}{2}\,v_0 t^2 - \frac{1}{6}\,gt^3\right)}_{\text{négligé}} \qquad z(t) = z_0 + v_0 t - \frac{1}{2}\,gt^2$$

Valeurs numériques

$$y(t) = -2\omega \cos\varphi \left(\frac{1}{2}\, v_0 t^2 - \frac{1}{6}\, g t^3 \right)$$

$$z(t) = z_0 + v_0 t - \frac{1}{2}\, g t^2$$

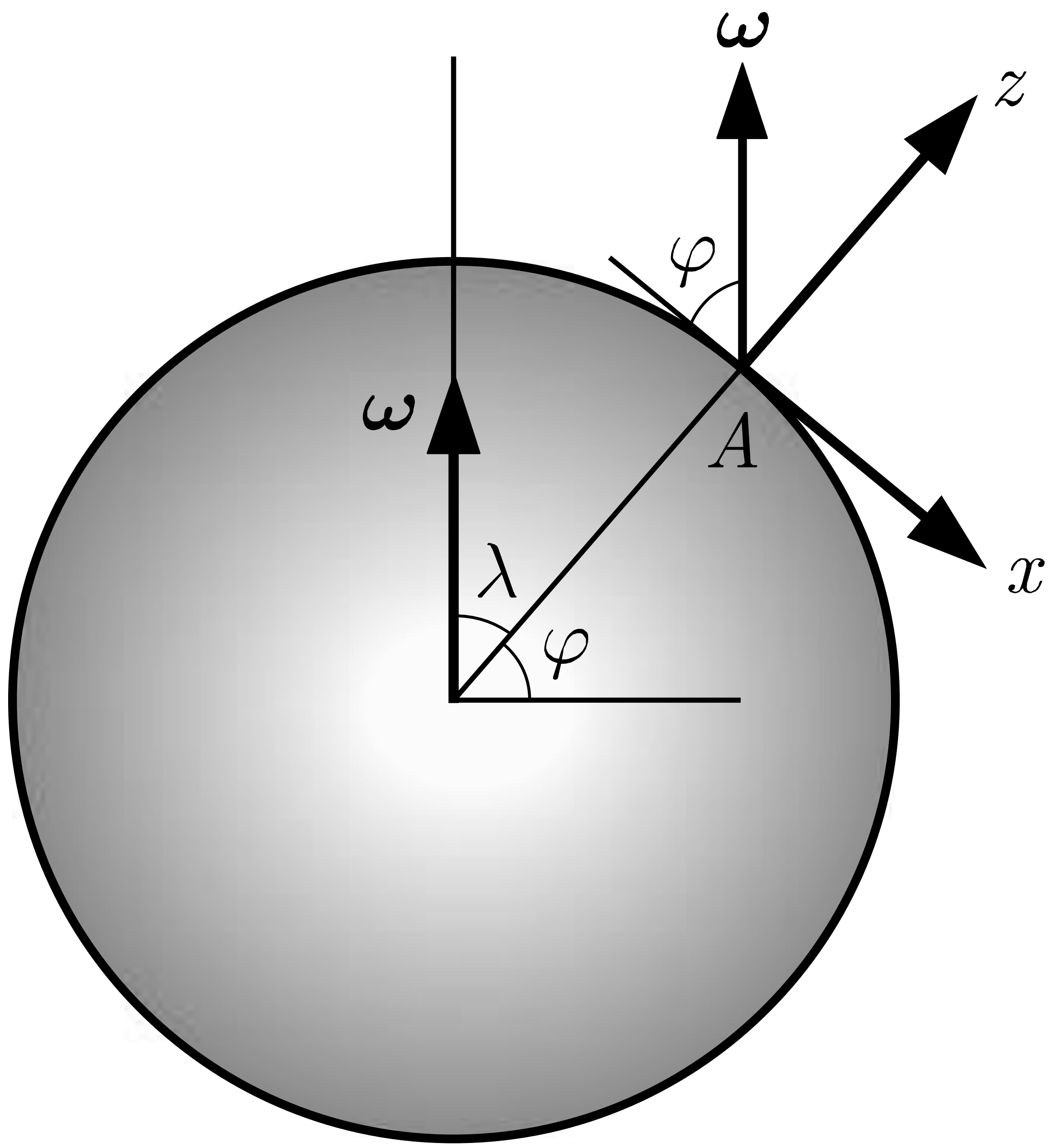
$$v_0 = 0$$
$$z_0 = 158\, m$$
$$\varphi = 51\, \mathrm{deg}$$

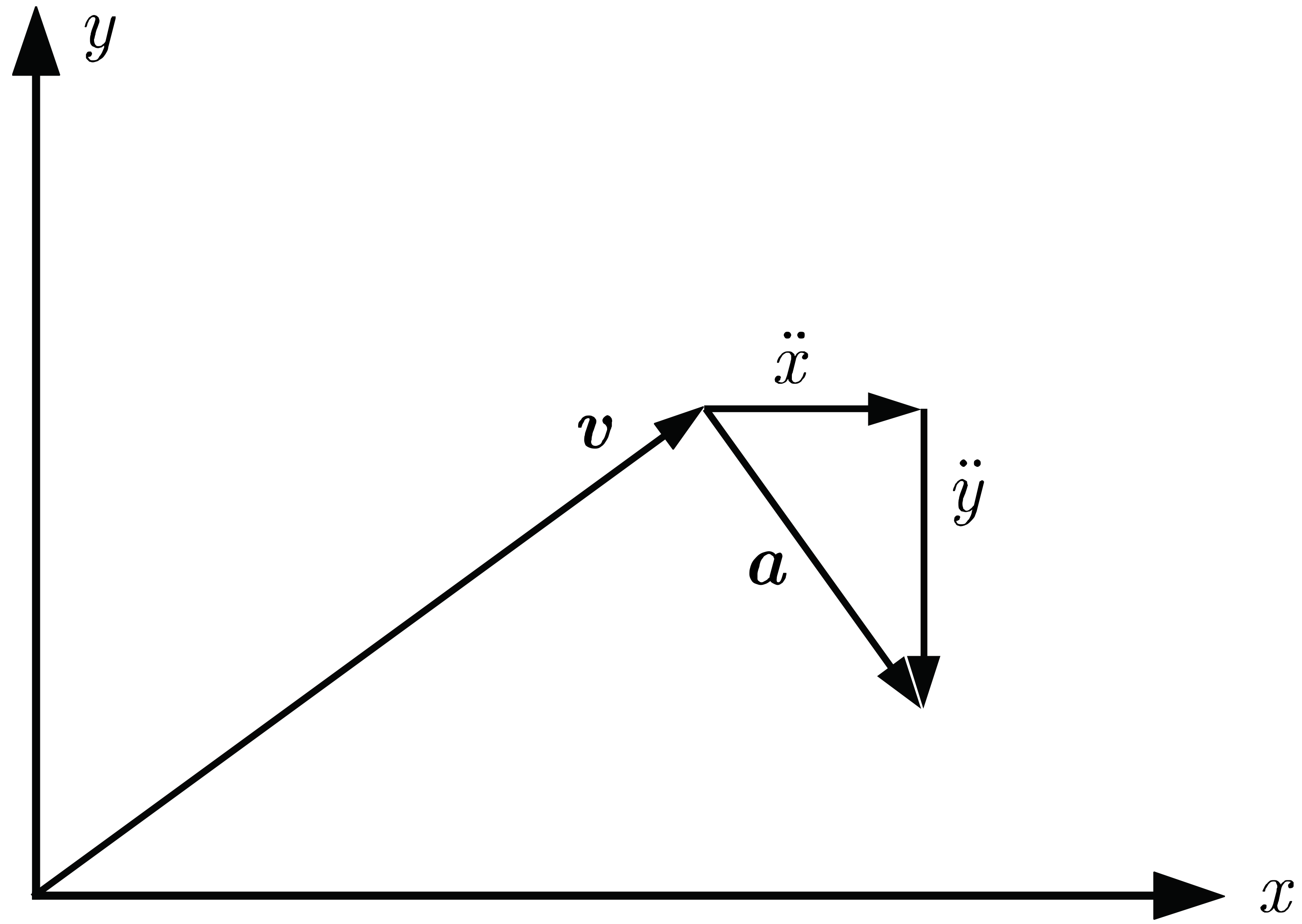
$$z(T) = 0$$
$$y(T) = 2.8\, cm$$

Mouvement horizontal

Equations du mouvement sous contrainte : $z = 0$

$$\ddot{x} = +2\omega\dot{y}\sin\varphi$$
$$\ddot{y} = -2\omega\dot{x}\sin\varphi$$





Intégration des équations du mouvement

$$\ddot{x} = +2\omega\dot{y}\sin\varphi$$
$$\ddot{y} = -2\omega\dot{x}\sin\varphi$$

$$\dot{x}(t) - \dot{x}(0) = 2\omega\sin\varphi\big(y(t) - y(0)\big)$$

$$\dot{x}(t) = 2\omega\sin\varphi[y(t)] + \dot{x}(0)$$

$$\dot{y}(t) = -2\omega\sin\varphi\left[\dot{x}(0)t\right] + \dot{y}(0)$$

$$\dot{x}(t) = 2\omega\sin\varphi\left[\dot{y}(0)t\right] + \dot{x}(0)$$

Déviation horizontale

$$\dot{x}(t) = 2\omega \sin\varphi \left[\dot{y}(0)t\right] + \dot{x}(0) \qquad x(t) = \omega \sin\varphi \dot{y}(0)t^2 + \dot{x}(0)t$$

$$\dot{y}(t) = -2\omega \sin\varphi \left[\dot{x}(0)t\right] + \dot{y}(0) \qquad y(t) = -\omega \sin\varphi \dot{x}(0)t^2 + \dot{y}(0)t$$

$$s = \sqrt{\left(x - \dot{x}(0)t\right)^2 + \left(y - \dot{y}(0)t\right)^2} = \omega \sin\varphi v t^2$$

$$v = \sqrt{\dot{x}(0)^2 + \dot{y}(0)^2}$$

Pendule de Foucault, heuristique

$$s = \omega \sin\varphi \, v \, t^2 \qquad\qquad t \to 0 \implies v \approx \text{constante}$$

$$\Delta\phi = \frac{s}{vt} = \omega \, t \sin\varphi \qquad\qquad \varphi\text{: latitutde}$$

Vitesse angulaire de rotation du plan d'oscillation : $\dfrac{\Delta\phi}{t} = \dot{\phi} = \omega \sin\varphi$

Déviation en 10 minutes :

$$\Delta\phi = \sin\varphi \cdot 7 \times 10^{-5} \times 10 \times 60 = \sin\varphi(0.04 \text{ radian}) = \sin\varphi(2.4 \text{ degrés})$$