Principes de conservation, applications

Mécanique, cours 17.2

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Principes de conservation, applications

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

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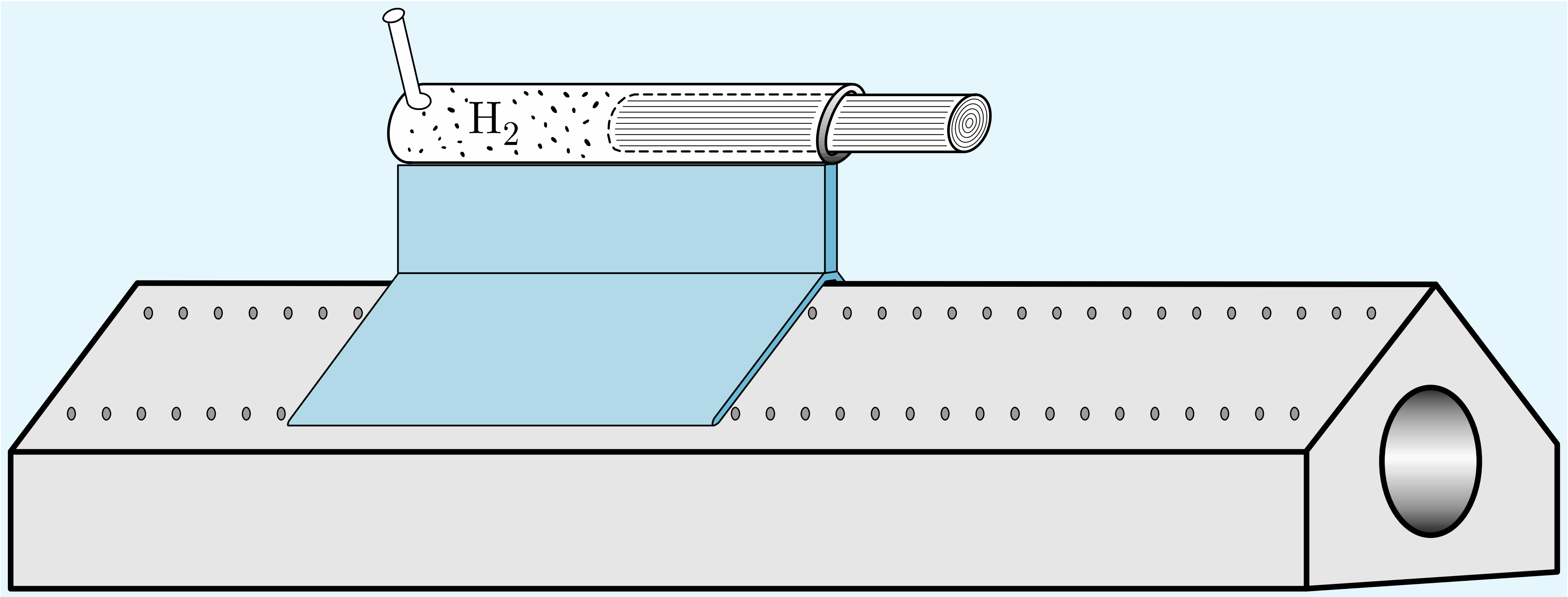
- Recul du canon
- Poussée de la fusée
- Tabouret tournant
- Force centrale

Recul du canon



Donnée technique : $\boldsymbol{u}$

vitesse d'éjection mesurée relativement au canon

Avant : $\boldsymbol{P} = (M_0 + \Delta m)\boldsymbol{v}$

Après : $\boldsymbol{P} = M_0(\boldsymbol{v} + \Delta\boldsymbol{v}) + \Delta m(\boldsymbol{u} + \boldsymbol{v} + \Delta\boldsymbol{v})$

$$(M_0 + \Delta m)\,\Delta\boldsymbol{v} = -\Delta m\,\boldsymbol{u}$$

Poussée de la fusée

Données techniques

taux d'éjection de masse : $m(t+\delta t) = m(t) + \frac{dm}{dt}\delta t$

vitesse de cette masse, relative à la fusée : $\boldsymbol{u}$

Au temps t : $\boldsymbol{p}(t) = m\boldsymbol{v}$

Au temps $t+\delta t$: $\delta m = -\frac{dm}{dt}\,\delta t$ masse éjectée

$$\boldsymbol{p}(t+\delta t) = m(t+\delta t)(\boldsymbol{v}+\delta\boldsymbol{v}) + \delta m(\boldsymbol{u}+\boldsymbol{v})$$

$$= \left(m + \frac{dm}{dt}\,\delta t\right)(\boldsymbol{v}+\delta\boldsymbol{v}) - \frac{dm}{dt}\,\delta t(\boldsymbol{u}+\boldsymbol{v})$$

Poussée de la fusée

Pesanteur et force sur carburant éjecté : $\boldsymbol{p}(t+\delta t)-\boldsymbol{p}(t)=\left(\boldsymbol{F}+\boldsymbol{F}^c\right)\delta t$

$$\left(\boldsymbol{F}+\boldsymbol{F}^c\right)\delta t=\left(m+\frac{dm}{dt}\,\delta t\right)(\boldsymbol{v}+\delta\boldsymbol{v})-\frac{dm}{dt}\,\delta t(\boldsymbol{u}+\boldsymbol{v})-m\boldsymbol{v}$$

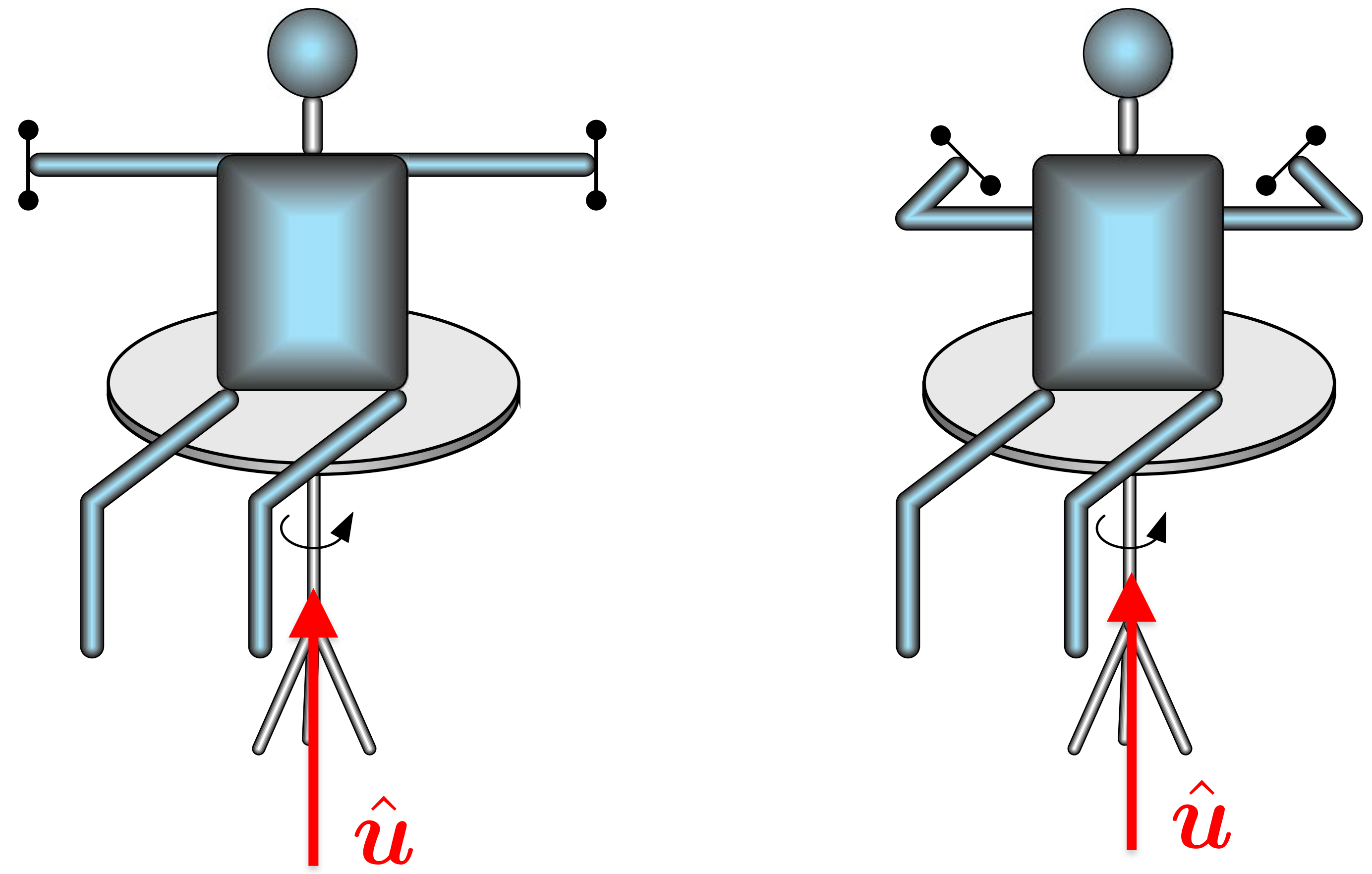
$$\delta t\to 0 \implies \boldsymbol{F}^c\,\delta t\cong 0$$

$$\delta\boldsymbol{v}\,\delta\,t\cong 0$$

$$m\frac{d\boldsymbol{v}}{dt}=\underset{\uparrow}{\frac{dm}{dt}}\,\boldsymbol{u}+\boldsymbol{F}$$

poussée

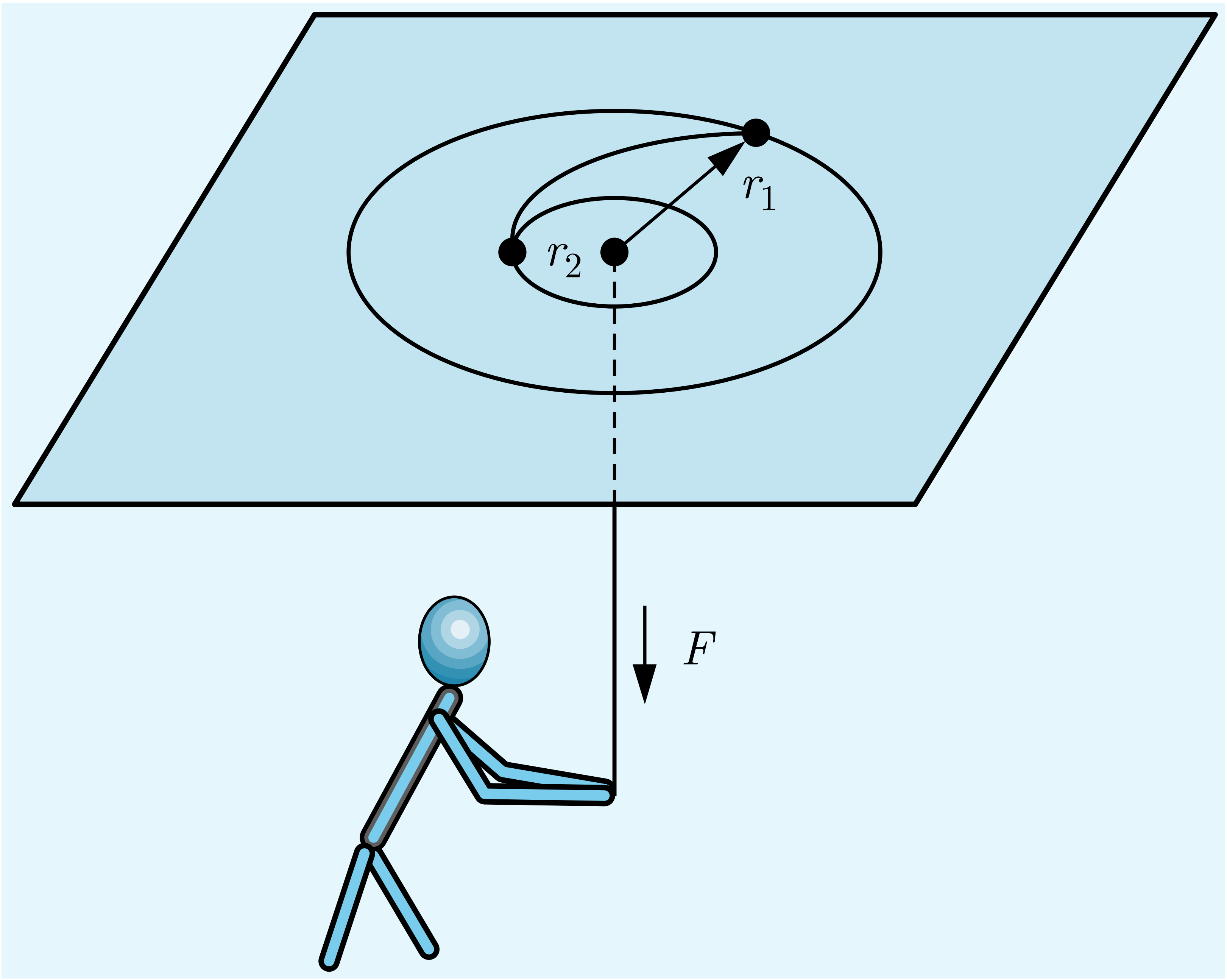
Tabouret tournant



$$\boldsymbol{M}_O^{ext} \cdot \hat{\boldsymbol{u}} = 0 \qquad \boldsymbol{L}_O \cdot \hat{\boldsymbol{u}} = \text{constante}$$

$$(\boldsymbol{r} \wedge m\boldsymbol{v}) \cdot \hat{\boldsymbol{u}} = m|\boldsymbol{r}||\boldsymbol{v}| = m|\boldsymbol{r}|^2\omega$$

Force centrale



$$\boldsymbol{L}_0 = \boldsymbol{r} \wedge m\boldsymbol{v}$$

$$\boldsymbol{v} = \boldsymbol{\omega} \wedge \boldsymbol{r}$$

$$\boldsymbol{L}_0 = \boldsymbol{r} \wedge m\left(\boldsymbol{\omega} \wedge \boldsymbol{r}\right) = mr^2\boldsymbol{\omega}$$

$$mr_1^2\omega_1 = mr_2^2\omega_2$$