ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Point de référence du moment cinétique

Mécanique, cours 19.2

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- Moment cinétique en G, en O
- Moment cinétique en A quelconque
- Théorème du moment cinétique
- Point du solide C fixe

Propriété : moment cinétique en O ou en G

$$\boldsymbol{L}_G = \sum_\alpha \{\boldsymbol{GP}_\alpha \wedge m_\alpha (\boldsymbol{\omega} \wedge \boldsymbol{GP}_\alpha)\}$$

$$\boldsymbol{L}_O = \sum_\alpha \{m_\alpha \boldsymbol{OP}_\alpha \wedge (\boldsymbol{V}_G + (\boldsymbol{\omega} \wedge \boldsymbol{GP}_\alpha))\}$$

$$\boldsymbol{L}_O = M\boldsymbol{OG} \wedge \boldsymbol{V}_G + \boldsymbol{L}_G$$

Propriété : moment cinétique en A

Soit A un point quelconque : $\sum_\alpha m_\alpha \boldsymbol{GP}_\alpha = 0 \qquad \sum_\alpha m_\alpha \boldsymbol{v}'_\alpha = 0$

$$\boldsymbol{L}_A = \sum_\alpha \{\boldsymbol{AP}_\alpha \wedge m_\alpha \boldsymbol{v}_\alpha\} \qquad \boldsymbol{L}_G = \sum_\alpha \{\boldsymbol{GP}_\alpha \wedge m_\alpha (\boldsymbol{\omega} \wedge \boldsymbol{GP}_\alpha)\}$$

$$= \sum_\alpha \{(\boldsymbol{AG} + \boldsymbol{GP}_\alpha) \wedge m_\alpha (\boldsymbol{V}_G + \boldsymbol{v}'_\alpha)\}$$

$$= \boldsymbol{AG} \wedge M\boldsymbol{V}_G + \sum_\alpha \{\boldsymbol{GP}_\alpha \wedge m_\alpha \boldsymbol{v}'_\alpha\} = \boldsymbol{AG} \wedge M\boldsymbol{V}_G + \boldsymbol{L}_G$$

$$\frac{d\boldsymbol{L}_A}{dt} = (\boldsymbol{V}_G - \boldsymbol{V}_A) \wedge M\boldsymbol{V}_G + \boldsymbol{AG} \wedge M\frac{d\boldsymbol{V}_G}{dt} + \frac{d\boldsymbol{L}_G}{dt} = -\boldsymbol{V}_A \wedge M\boldsymbol{V}_G + \boldsymbol{M}_A^{ext}$$

$$\boldsymbol{AG} \wedge \sum_\alpha \boldsymbol{F}_\alpha^{ext} + \sum_\alpha \boldsymbol{GP}_\alpha \wedge \boldsymbol{F}_\alpha^{ext} = \sum_\alpha \boldsymbol{AP}_\alpha \wedge \boldsymbol{F}_\alpha^{ext}$$

$$\frac{d\boldsymbol{L}_A}{dt} = -\boldsymbol{V}_A \wedge M\boldsymbol{V}_G + \boldsymbol{M}_A^{ext}$$

Propriété : point C du solide fixe

$$\frac{d\boldsymbol{L}_A}{dt} = -\boldsymbol{V}_A \wedge M\boldsymbol{V}_G + \boldsymbol{M}_A^{ext}$$

C, un point du solide, fixe : $\dfrac{d\boldsymbol{L}_C}{dt} = \boldsymbol{M}_C^{ext}$

$$\boldsymbol{L}_C = \sum_\alpha \{\boldsymbol{CP}_\alpha \wedge m_\alpha(\boldsymbol{\omega} \wedge \boldsymbol{CP}_\alpha)\} = I_C\,\boldsymbol{\omega}$$

$$I_{Cij} = \sum_\alpha m_\alpha\left[\boldsymbol{CP}_\alpha^2\,\delta_{ij} - CP_{\alpha,j}CP_{\alpha,i}\right]$$