Mouvement quelconque d’un solide

Mécanique, cours 21.1

Jean-Philippe Ansermet

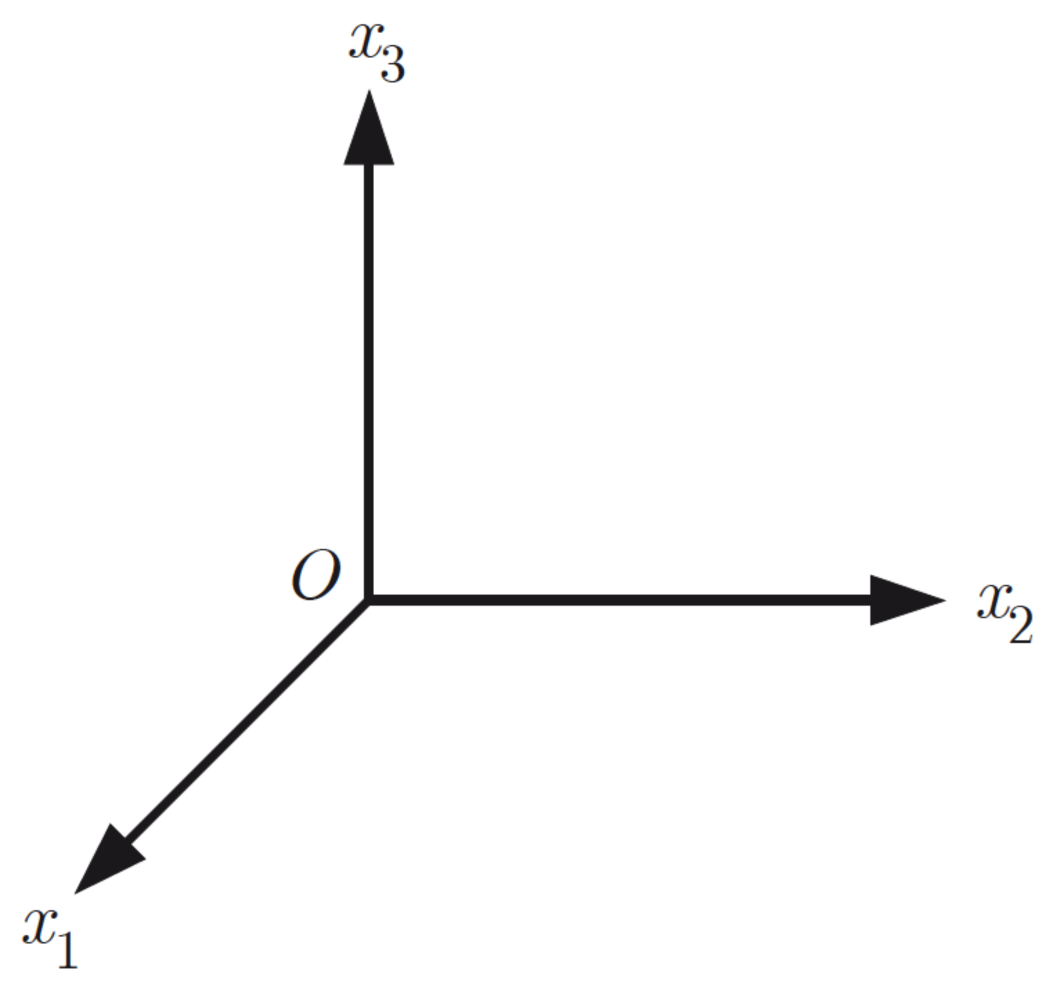
ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

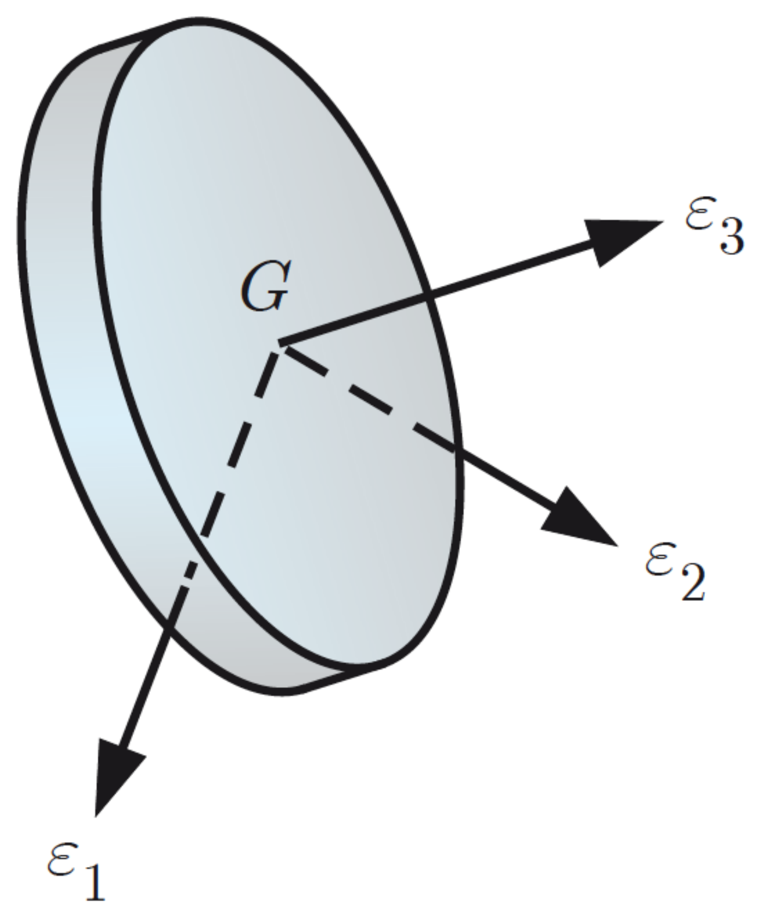
Mouvement quelconque d’un solide

Mouvement quelconque d’un solide

- Equations d’Euler
- Moment exercé sur un axe fixe

Equations d’Euler



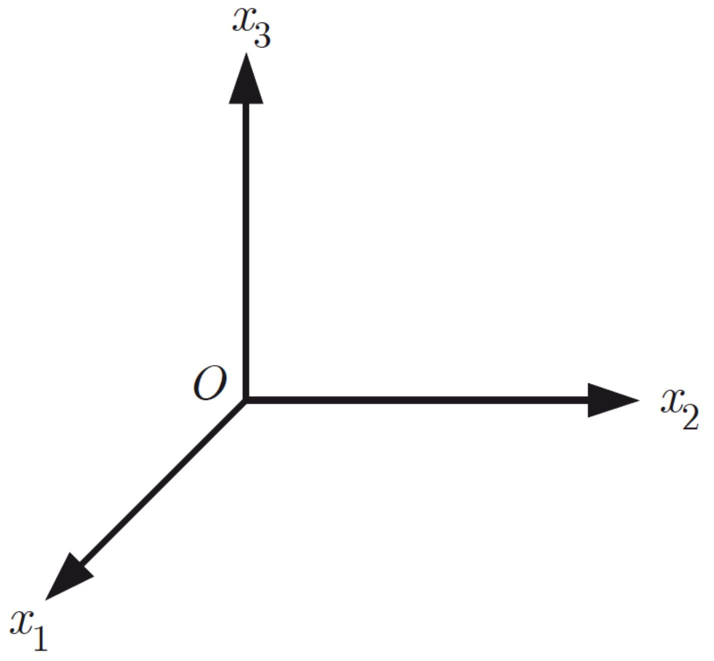


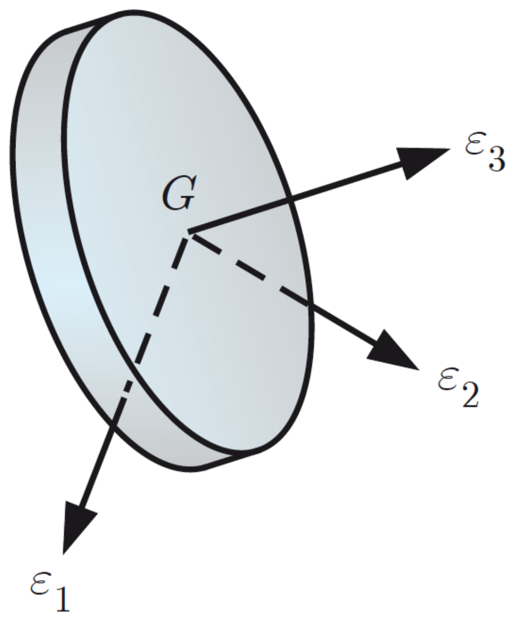
$$\frac{d\boldsymbol{L}_G}{dt} = \boldsymbol{M}_G^{ext}$$

$$\begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} = \begin{pmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \begin{pmatrix} I_1\omega_1 \\ I_2\omega_2 \\ I_3\omega_3 \end{pmatrix}$$

$$\boldsymbol{L}_G = I_1\omega_1\hat{\boldsymbol{\varepsilon}}_1 + I_2\omega_2\hat{\boldsymbol{\varepsilon}}_2 + I_3\omega_3\hat{\boldsymbol{\varepsilon}}_3$$

Equations d'Euler





$$\boldsymbol{L}_G = I_1\omega_1\hat{\boldsymbol{\varepsilon}}_1 + I_2\omega_2\hat{\boldsymbol{\varepsilon}}_2 + I_3\omega_3\hat{\boldsymbol{\varepsilon}}_3$$

$$\frac{d\boldsymbol{L}_G}{dt} = I_1\dot{\omega}_1\hat{\boldsymbol{\varepsilon}}_1 + I_2\dot{\omega}_2\hat{\boldsymbol{\varepsilon}}_2 + I_3\dot{\omega}_3\hat{\boldsymbol{\varepsilon}}_3 + I_1\omega_1\dot{\hat{\boldsymbol{\varepsilon}}}_1 + I_2\omega_2\dot{\hat{\boldsymbol{\varepsilon}}}_2 + I_3\omega_3\dot{\hat{\boldsymbol{\varepsilon}}}_3$$

$$\begin{aligned}
\dot{\hat{\boldsymbol{\varepsilon}}}_1 &= -\omega_2\hat{\boldsymbol{\varepsilon}}_3 + \omega_3\hat{\boldsymbol{\varepsilon}}_2 \\
\dot{\hat{\boldsymbol{\varepsilon}}}_2 &= -\omega_3\hat{\boldsymbol{\varepsilon}}_1 + \omega_1\hat{\boldsymbol{\varepsilon}}_3 \\
\dot{\hat{\boldsymbol{\varepsilon}}}_3 &= -\omega_1\hat{\boldsymbol{\varepsilon}}_2 + \omega_2\hat{\boldsymbol{\varepsilon}}_1
\end{aligned}$$

$$\begin{aligned}
\frac{d\boldsymbol{L}_G}{dt} = I_1\dot{\omega}_1\hat{\boldsymbol{\varepsilon}}_1 + I_2\dot{\omega}_2\hat{\boldsymbol{\varepsilon}}_2 + I_3\dot{\omega}_3\hat{\boldsymbol{\varepsilon}}_3 \\
+(I_3 - I_2)\omega_2\omega_3\hat{\boldsymbol{\varepsilon}}_1 \\
+(I_1 - I_3)\omega_1\omega_3\hat{\boldsymbol{\varepsilon}}_2 \\
+(I_2 - I_1)\omega_2\omega_1\hat{\boldsymbol{\varepsilon}}_3
\end{aligned}$$

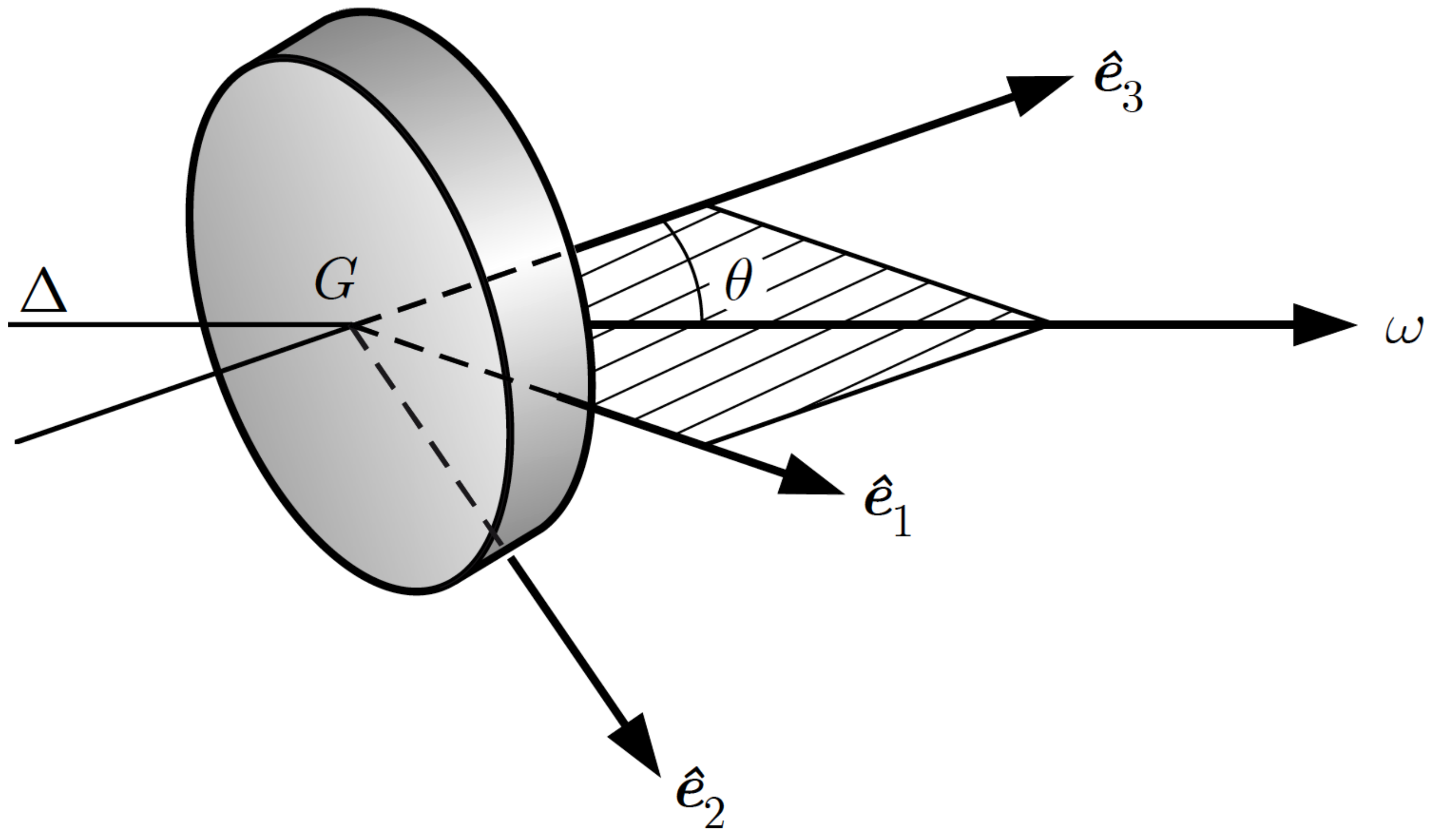
Equations du mouvement :

$$\begin{aligned}
M_1 &= I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 \\
M_2 &= I_2\dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1 \\
M_3 &= I_3\dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2
\end{aligned}$$

Cas particulier : $\dot{\omega}_1 = \dot{\omega}_2 = \dot{\omega}_3 = 0$

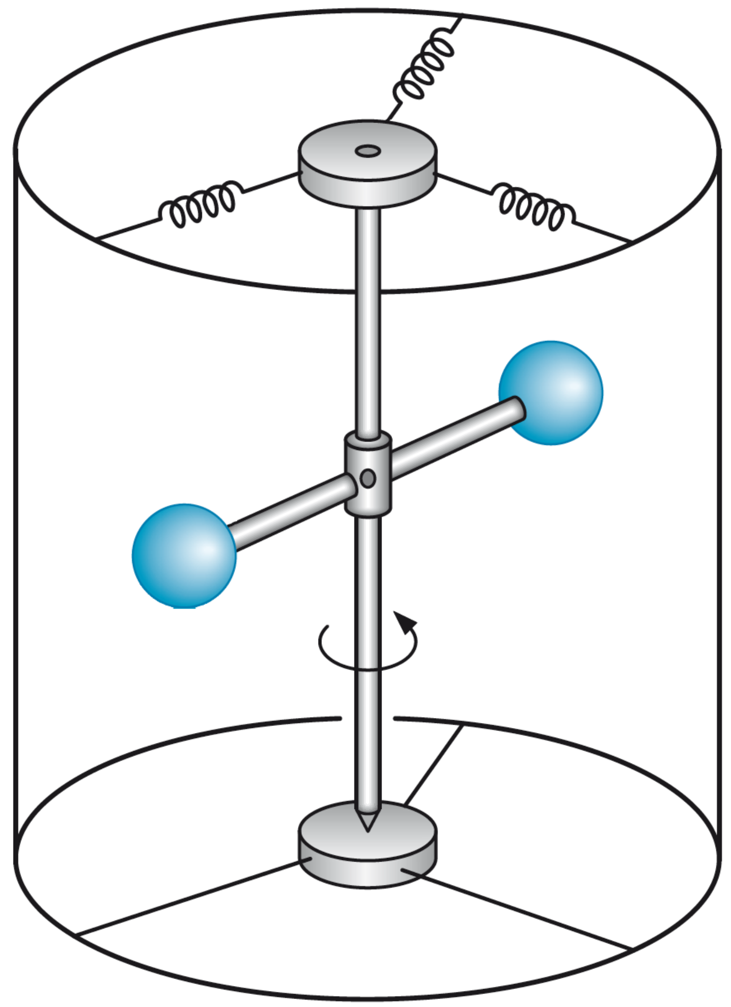
$$\frac{d\boldsymbol{L}_G}{dt} = \boldsymbol{\omega} \wedge \boldsymbol{L}_G$$

Moment des forces exercées sur un axe fixe



Choix du repère d'inertie:

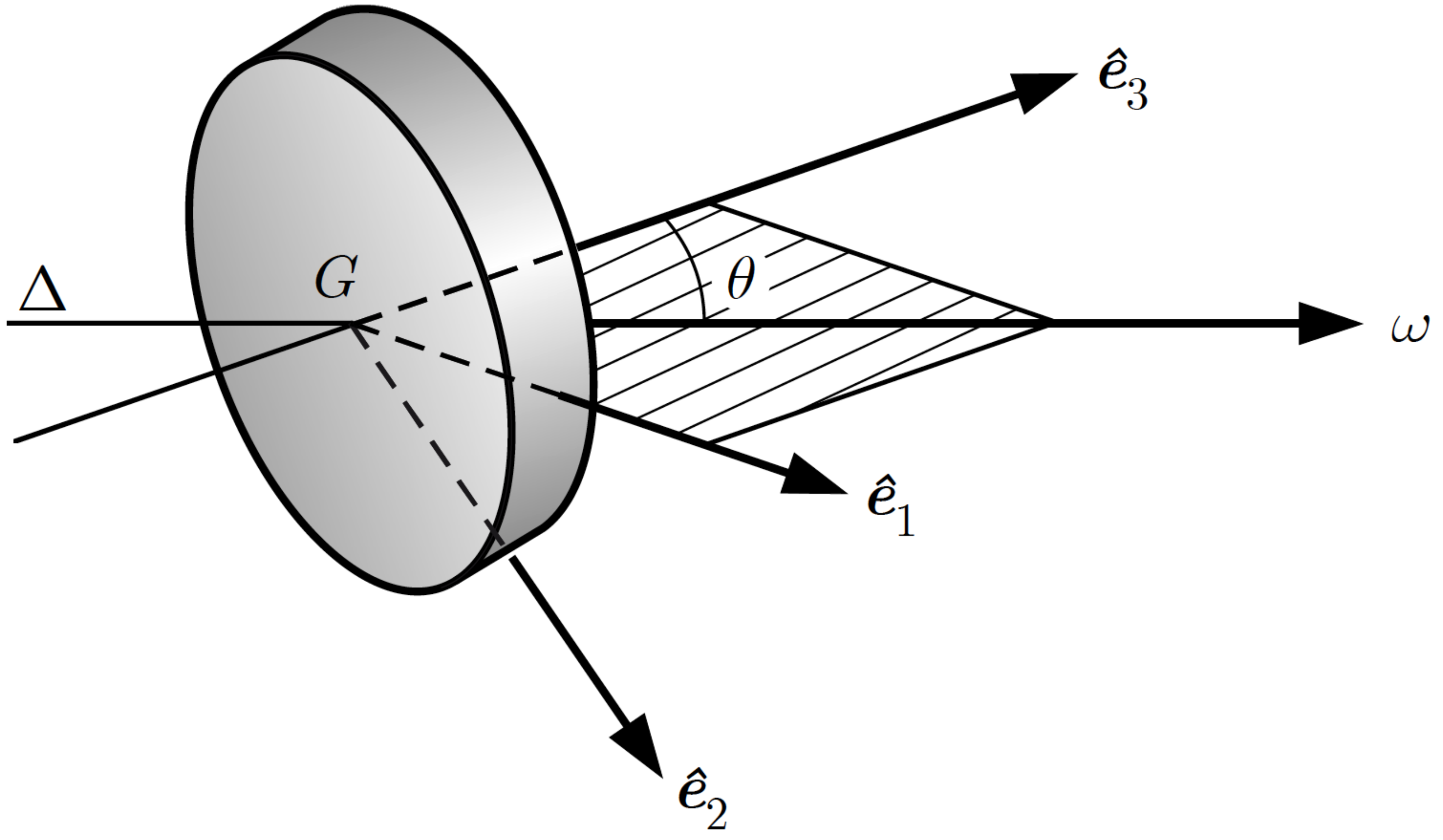
$$(G, \hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3)$$



$$I_G = \begin{pmatrix} I_\perp & 0 & 0 \\ 0 & I_\perp & 0 \\ 0 & 0 & I_\parallel \end{pmatrix} \qquad \boldsymbol{\omega} = \begin{pmatrix} \omega \sin\theta \\ 0 \\ \omega\cos\theta \end{pmatrix}$$

$$\boldsymbol{L}_G = I_\perp \omega \sin\theta \hat{\boldsymbol{e}}_1 + I_\parallel \omega \cos\theta \hat{\boldsymbol{e}}_3$$

Moment des forces exercées sur un axe fixe



$$\boldsymbol{L}_G = I_\perp \omega \sin\theta \hat{\boldsymbol{e}}_1 + I_\parallel \omega \cos\theta \hat{\boldsymbol{e}}_3$$

$$\frac{d\boldsymbol{L}_G}{dt} = I_\perp \omega \sin\theta (\boldsymbol{\omega} \wedge \hat{\boldsymbol{e}}_1) + I_\parallel \omega \cos\theta (\boldsymbol{\omega} \wedge \hat{\boldsymbol{e}}_3)$$

$$\boldsymbol{\omega} \wedge \hat{\boldsymbol{e}}_1 = \begin{vmatrix} \hat{\boldsymbol{e}}_1 & \omega\sin\theta & 1 \\ \hat{\boldsymbol{e}}_2 & 0 & 0 \\ \hat{\boldsymbol{e}}_3 & \omega\cos\theta & 0 \end{vmatrix} = \omega\cos\theta\,\hat{\boldsymbol{e}}_2$$

$$\boldsymbol{\omega} \wedge \hat{\boldsymbol{e}}_3 = \begin{vmatrix} \hat{\boldsymbol{e}}_1 & \omega\sin\theta & 0 \\ \hat{\boldsymbol{e}}_2 & 0 & 0 \\ \hat{\boldsymbol{e}}_3 & \omega\cos\theta & 1 \end{vmatrix} = -\omega\sin\theta\,\hat{\boldsymbol{e}}_2$$

$$\frac{d\boldsymbol{L}_G}{dt} = \boldsymbol{M}_G = (I_\perp - I_\parallel)\frac{\omega^2}{2} \sin 2\theta\,\hat{\boldsymbol{e}}_2$$

$\boldsymbol{M}_G$ tourne avec le solide