ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Méthode de Lagrange, applications

Mécanique, cours 25.2

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- Mouvement rectiligne
- Pendule mathématique
- Exemple à 2 degrés de liberté
- Cylindre roulant sans glisser

Mouvement rectiligne, force conservative

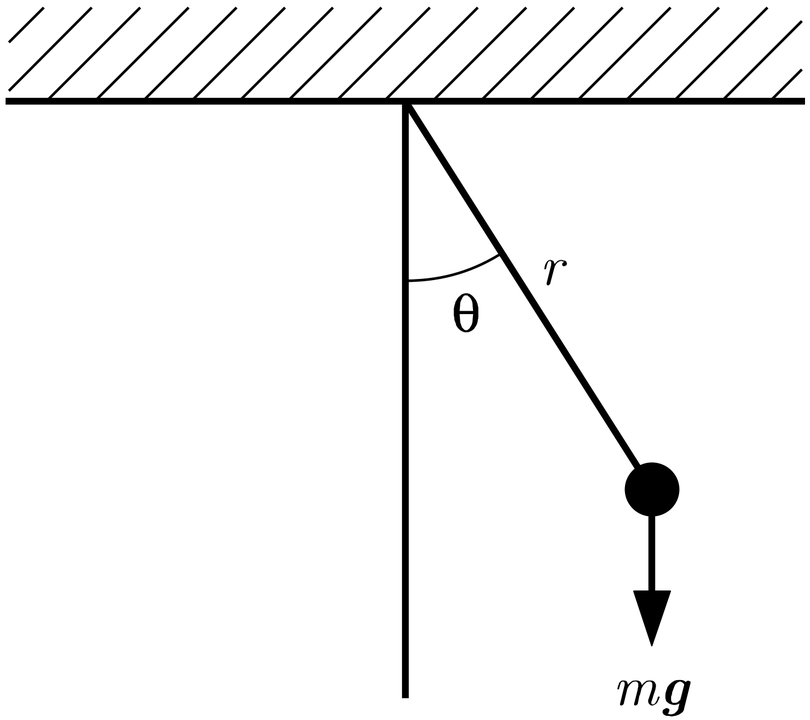
$$L = \frac{1}{2} m\dot{x}^2 - V(x)$$

$$\frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \text{la quantité de mouvement}$$

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x} = F \qquad \text{la force}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = m\ddot{x} = F$$

Pendule mathématique



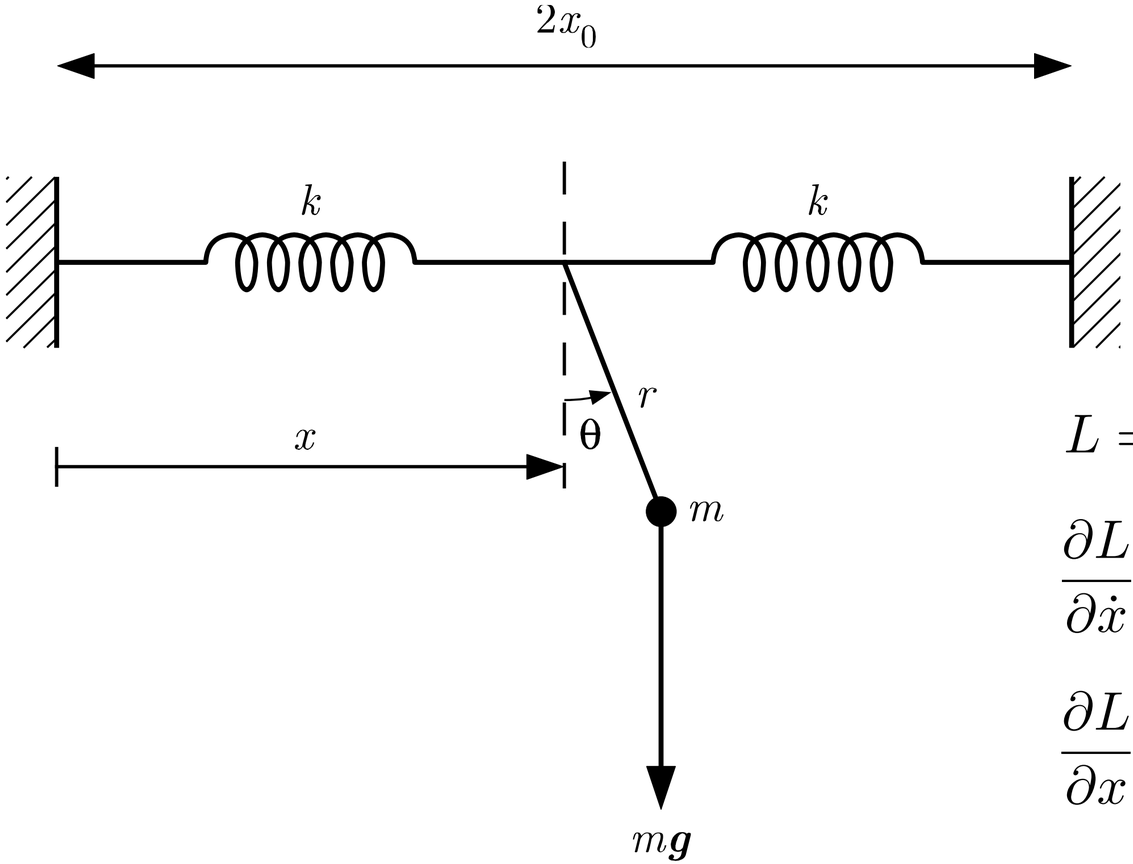
$$L = T - V = \frac{1}{2} mr^2\dot{\theta}^2 + mgr\cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} \qquad \text{le moment cinétique}$$

$$\frac{\partial L}{\partial \theta} = -mgr\sin\theta \qquad \text{le moment de force}$$

$$\ddot{\theta} = -\frac{g}{r}\sin\theta$$

Exemple à 2 degrés de liberté



$$V = \frac{1}{2}\,k(x-x_0)^2 + \frac{1}{2}\,k(x-x_0)^2 - mgr\cos\theta$$

$$\boldsymbol{r} = \begin{pmatrix} x + r\sin\theta \\ -r\cos\theta \end{pmatrix} \qquad \boldsymbol{v} = \begin{pmatrix} \dot{x} + r\dot{\theta}\cos\theta \\ +r\dot{\theta}\sin\theta \end{pmatrix}$$

$$L = \frac{1}{2}\,m\dot{x}^2 + \frac{1}{2}\,mr^2\dot{\theta}^2 + mr\dot{x}\dot{\theta}\cos\theta + mgr\cos\theta - k(x-x_0)^2$$

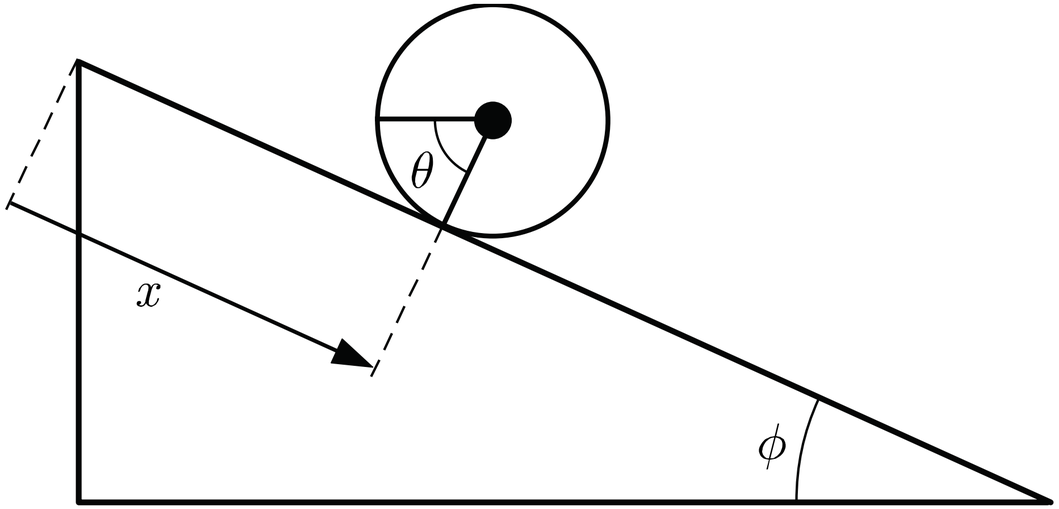
$$\frac{\partial L}{\partial \dot{x}} = m\big(\dot{x} + r\dot{\theta}\cos\theta\big) \qquad \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} + mr\dot{x}\cos\theta$$

$$\frac{\partial L}{\partial x} = -2k(x-x_0) \qquad \frac{\partial L}{\partial \theta} = -m\dot{x}r\dot{\theta}\sin\theta - mgr\sin\theta$$

$$m\ddot{x} + mr\ddot{\theta}\cos\theta - mr\dot{\theta}^2\sin\theta + 2k(x-x_0) = 0$$

$$mr^2\ddot{\theta} + mr\ddot{x}\cos\theta + mgr\sin\theta = 0$$

Cylindre roulant sans glisser



$$L = \frac{1}{2} I_\Delta \dot{\theta}^2 + \frac{1}{2} M \dot{x}^2 + Mgx \sin\phi$$

Roulement sans glissement : $x = R\theta$

1 degré de liberté

$$L = \frac{1}{2} M \left(1 + \frac{I_\Delta}{MR^2}\right) \dot{x}^2 + Mgx \sin\phi$$

$$\ddot{x} = \frac{g \sin\phi}{\left(1 + \frac{I_\Delta}{MR^2}\right)}$$

$$\frac{\partial L}{\partial \dot{x}} = M \left(1 + \frac{I_\Delta}{MR^2}\right) \dot{x} \qquad \frac{\partial L}{\partial x} = Mg \sin\phi$$