

Vitesse en coord. cylindriques et sphériques

Mécanique, cours 7.2

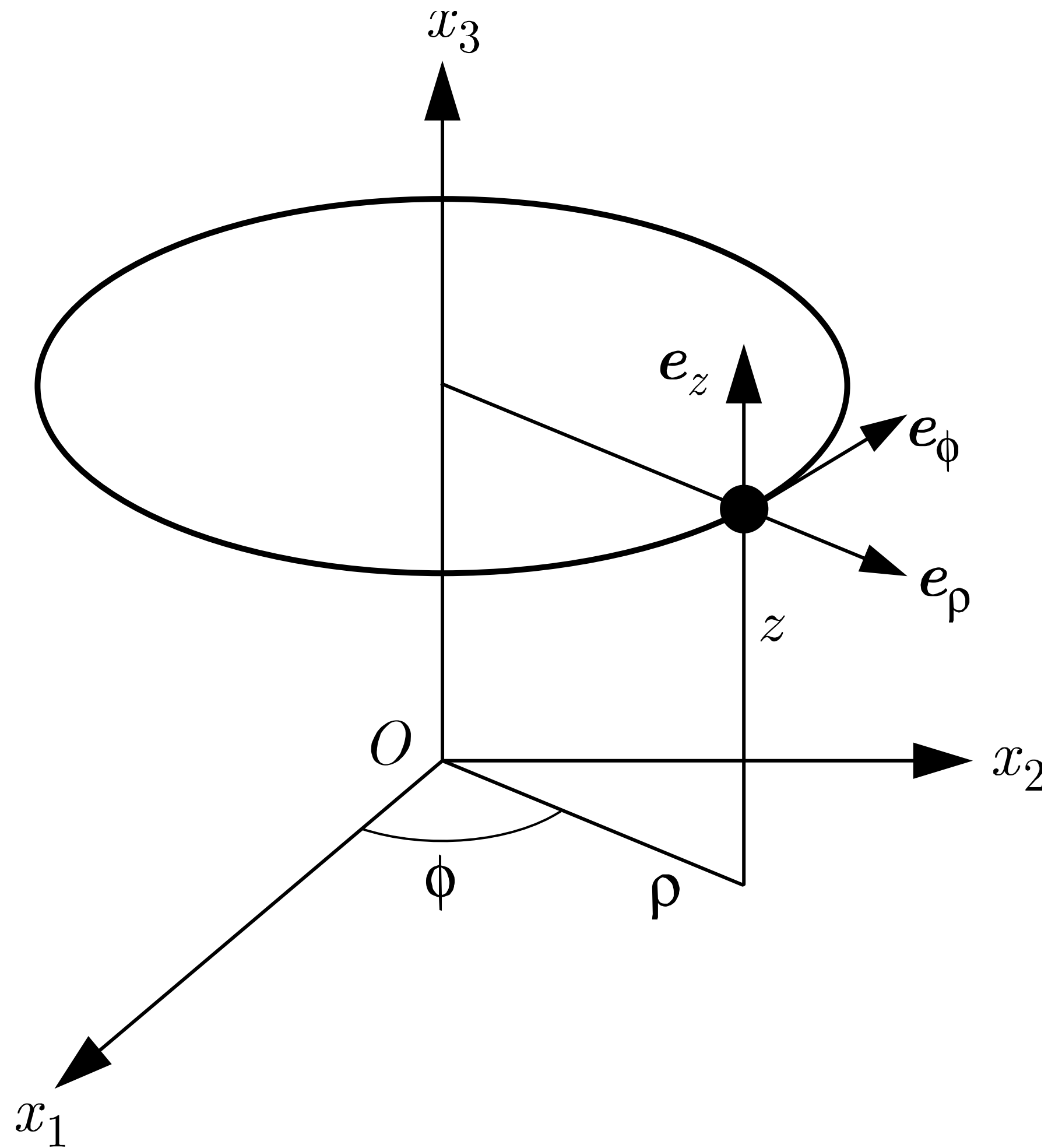
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Vitesse en coord. cylindriques et sphériques

Vitesse vectorielle, projection sur le repère des coordonnées :

- cylindriques
- sphériques

Vitesse projetée sur le repère des c. cylindriques



$$x_1 = \rho \cos \phi$$

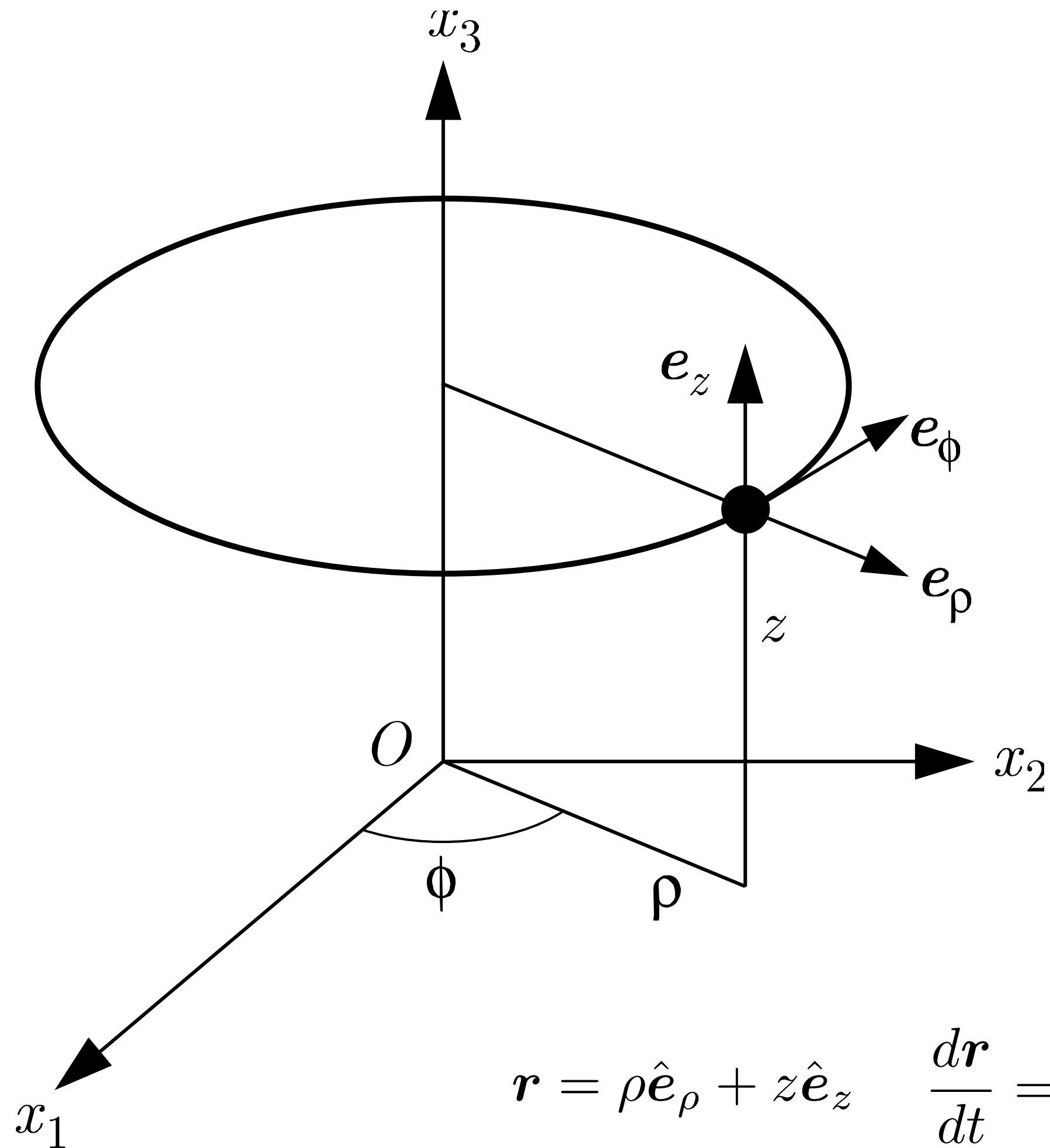
$$x_2 = \rho \sin \phi$$

$$x_3 = z$$

$$\mathbf{r} = \rho \hat{\mathbf{e}}_\rho + z \hat{\mathbf{e}}_z$$

$$\mathbf{v} = \dot{\rho} \mathbf{e}_\rho + \rho \dot{\phi} \mathbf{e}_\phi + \dot{z} \mathbf{e}_z$$

Dérivée temporelle des vecteurs unité



$$\mathbf{r} = \rho \hat{\mathbf{e}}_\rho + z \hat{\mathbf{e}}_z \quad \frac{d\mathbf{r}}{dt} = \dot{\rho} \hat{\mathbf{e}}_\rho + \rho \frac{d}{dt} \hat{\mathbf{e}}_\rho + \dot{z} \hat{\mathbf{e}}_z = \dot{\rho} \mathbf{e}_\rho + \rho \dot{\phi} \mathbf{e}_\phi + \dot{z} \mathbf{e}_z$$

$$\hat{\mathbf{e}}_\rho = \cos \phi \hat{\mathbf{x}}_1 + \sin \phi \hat{\mathbf{x}}_2$$

$$\hat{\mathbf{e}}_\phi = -\sin \phi \hat{\mathbf{x}}_1 + \cos \phi \hat{\mathbf{x}}_2$$

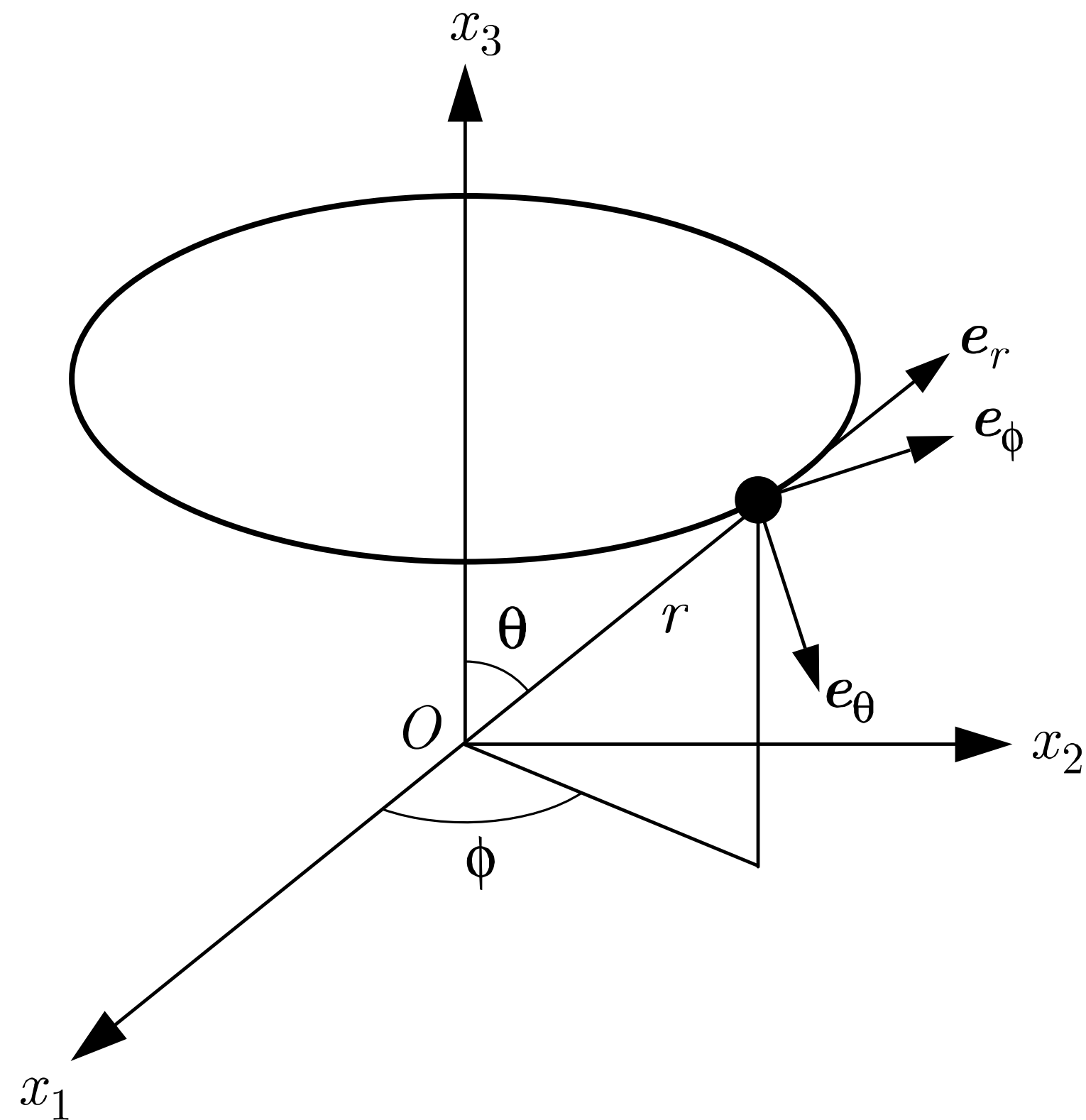
$$\hat{\mathbf{e}}_z = \hat{\mathbf{x}}_3$$

$$\frac{d\hat{\mathbf{e}}_\rho}{dt} = -\dot{\phi} \sin \phi \hat{\mathbf{x}}_1 + \dot{\phi} \cos \phi \hat{\mathbf{x}}_2$$

$$\frac{d\hat{\mathbf{e}}_\phi}{dt} = -\dot{\phi} \cos \phi \hat{\mathbf{x}}_1 - \dot{\phi} \sin \phi \hat{\mathbf{x}}_2$$

$$\frac{d\hat{\mathbf{e}}_\rho}{dt} = \dot{\phi} \hat{\mathbf{e}}_\phi \quad \frac{d\hat{\mathbf{e}}_\phi}{dt} = -\dot{\phi} \hat{\mathbf{e}}_\rho$$

Vitesse projetée sur le repère des c. sphériques



$$x_1 = r \sin \theta \cos \phi$$

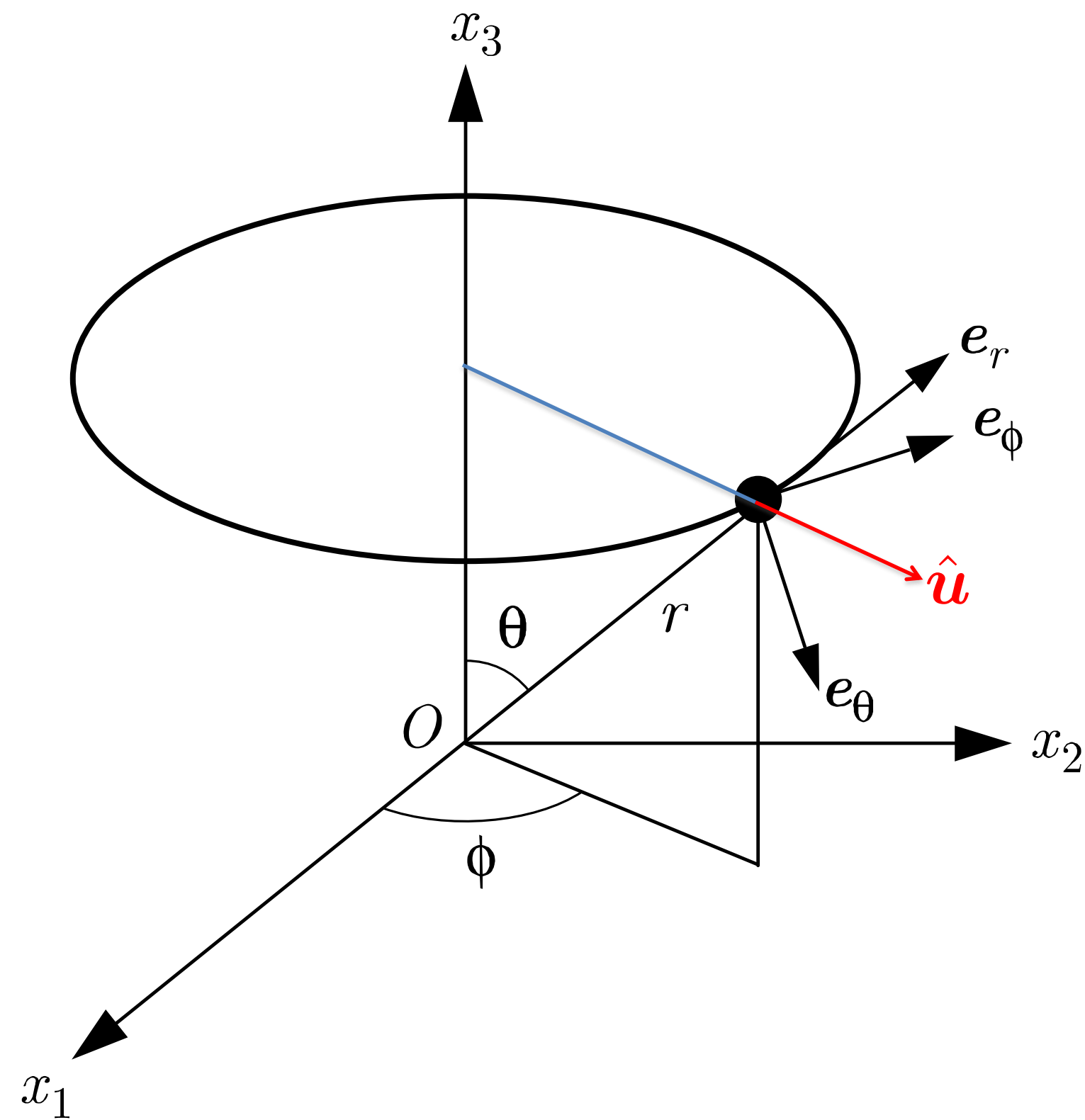
$$x_2 = r \sin \theta \sin \phi$$

$$x_3 = r \cos \theta$$

$$\mathbf{r} = r \hat{\mathbf{e}}_r$$

$$\mathbf{v} = \dot{r} \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta + r \dot{\phi} \sin \theta \mathbf{e}_\phi$$

Dérivée temporelle des vecteurs unité



$$\frac{d\mathbf{e}_r}{dt} = \dot{\theta}\mathbf{e}_\theta + \dot{\phi}\sin\theta\mathbf{e}_\phi$$

$$\frac{d\mathbf{e}_\phi}{dt} = -\dot{\phi}\hat{\mathbf{u}}$$

$$\hat{\mathbf{u}} = \sin\theta\mathbf{e}_r + \cos\theta\mathbf{e}_\theta$$

$$\frac{d\mathbf{e}_\theta}{dt} = -\dot{\theta}\mathbf{e}_r + \dot{\phi}\cos\theta\mathbf{e}_\phi$$

$$\frac{d}{dt}\mathbf{r} = \dot{r}\hat{\mathbf{e}}_r + r\frac{d}{dt}\hat{\mathbf{e}}_r$$

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + r\dot{\phi}\sin\theta\mathbf{e}_\phi$$

Coordonnées cylindriques :

$$\mathbf{r} = \rho \hat{\mathbf{e}}_\rho + z \hat{\mathbf{e}}_z$$

$$\mathbf{v} = \dot{\rho} \mathbf{e}_\rho + \rho \dot{\phi} \mathbf{e}_\phi + \dot{z} \mathbf{e}_z$$

Coordonnées sphériques :

$$\mathbf{r} = r \hat{\mathbf{e}}_r$$

$$\mathbf{v} = \dot{r} \mathbf{e}_r + r \dot{\theta} \mathbf{e}_\theta + r \dot{\phi} \sin \theta \mathbf{e}_\phi$$