Provenance Semirings

Todd Green   Grigoris Karvounarakis   Val Tannen

presented by Clemens Ley

“place of origin”

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algebraic structure, e.g $(\mathbb{N},*,+,0,1)$

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Outline

- Data provenance by example
- Relational algebra for data provenance
- Datalog for data provenance

Data Provenance

Data Provenance

Data provenance aims to explain how a particular query result was obtained.

Data Provenance

R:

A	B	C
	...	
a	b	c
	...	

S:

D	B	E
	...	
d	b	e
	...	

join on **B**

R⋈S:

A	B	C	D	E
		...		
a	b	c	d	e
		...		

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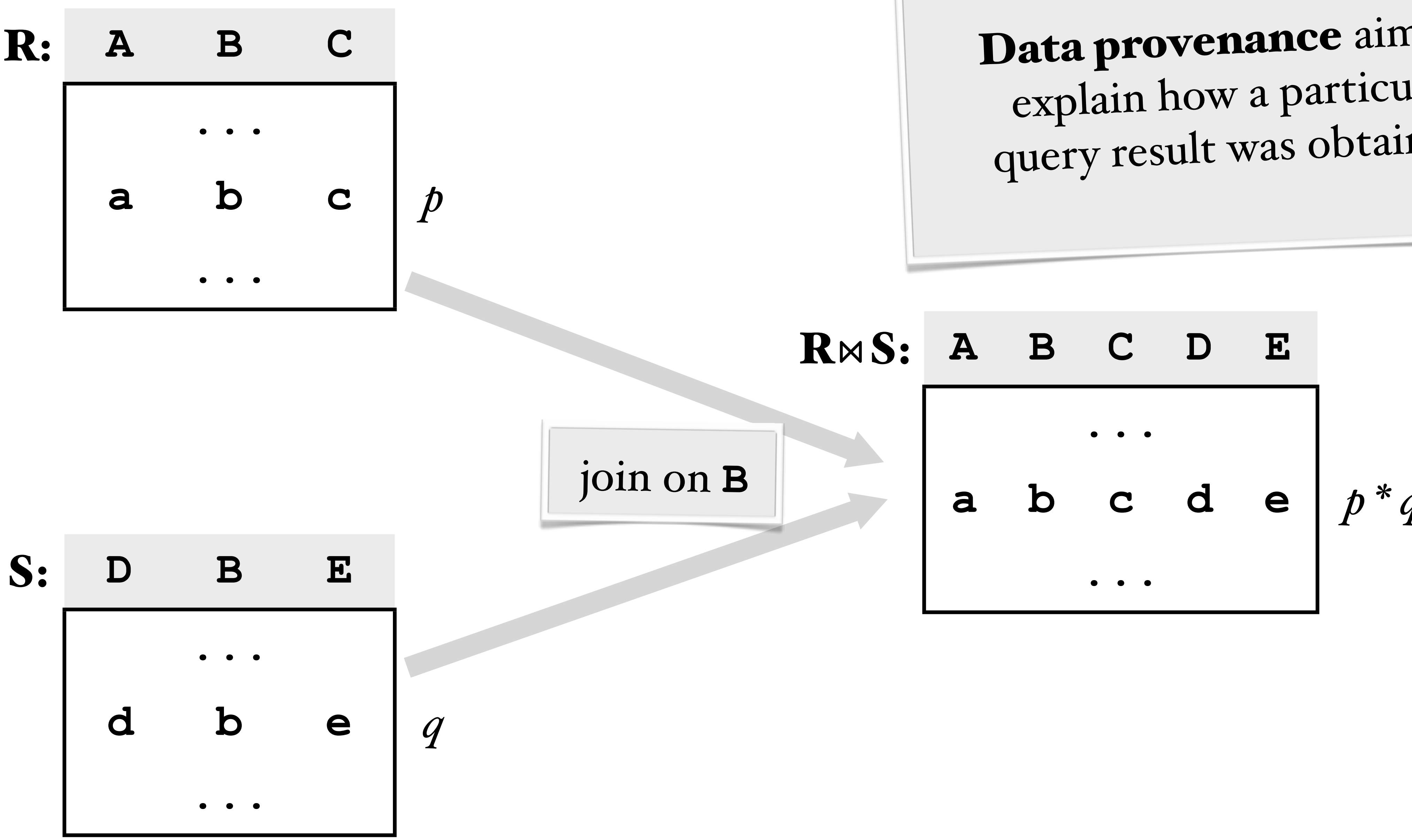
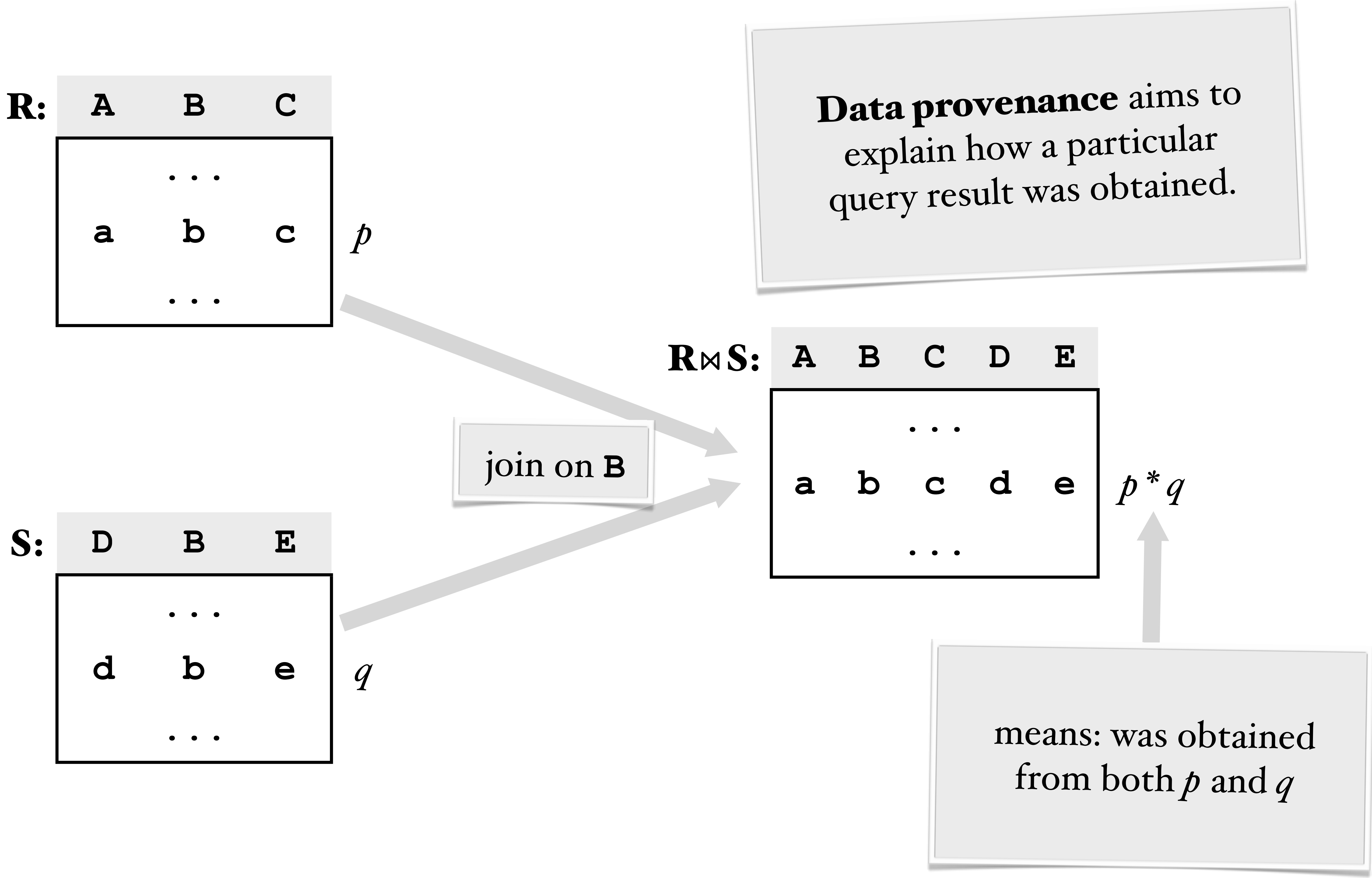
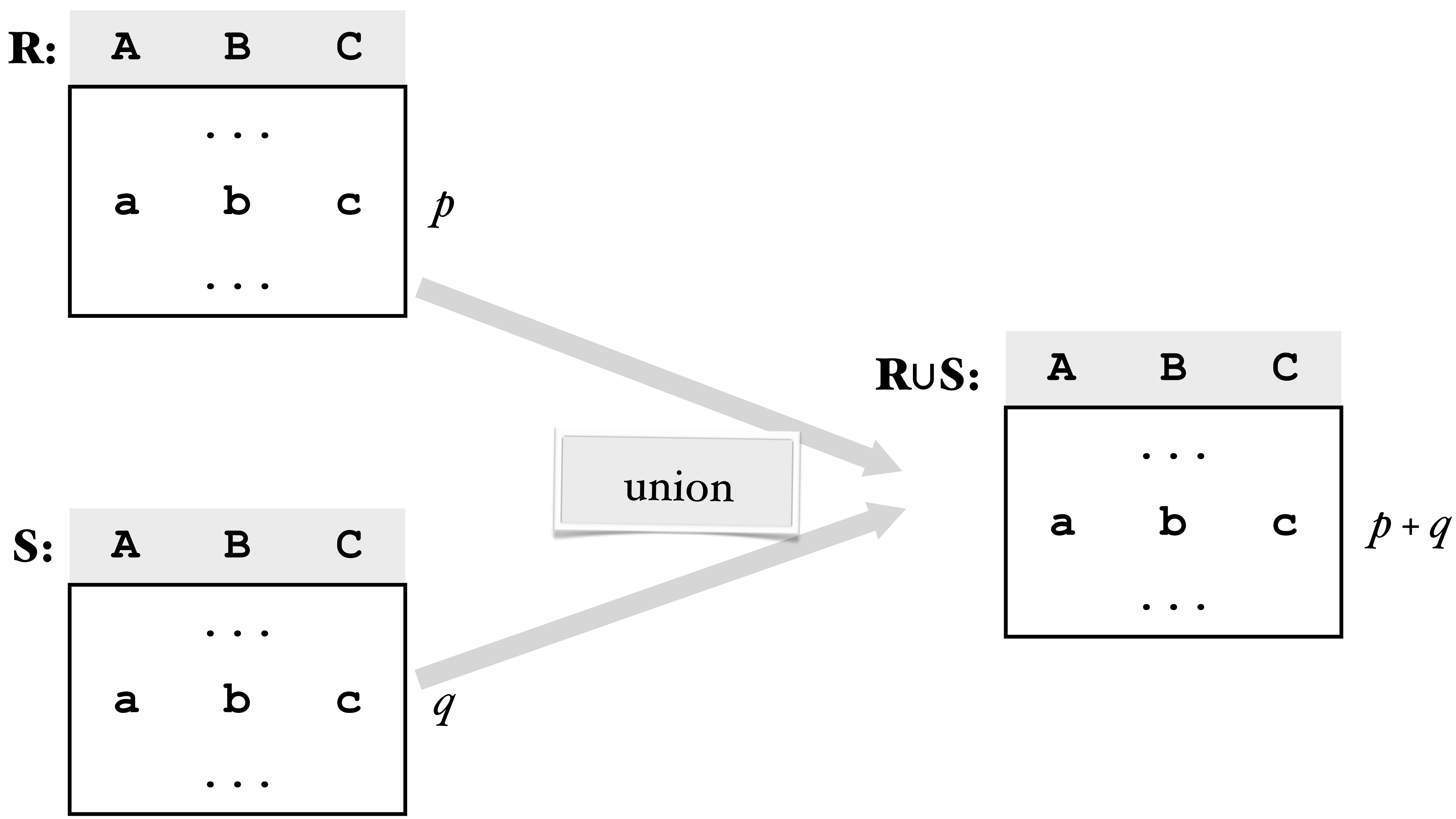


Data Provenance



Data Provenance (2)



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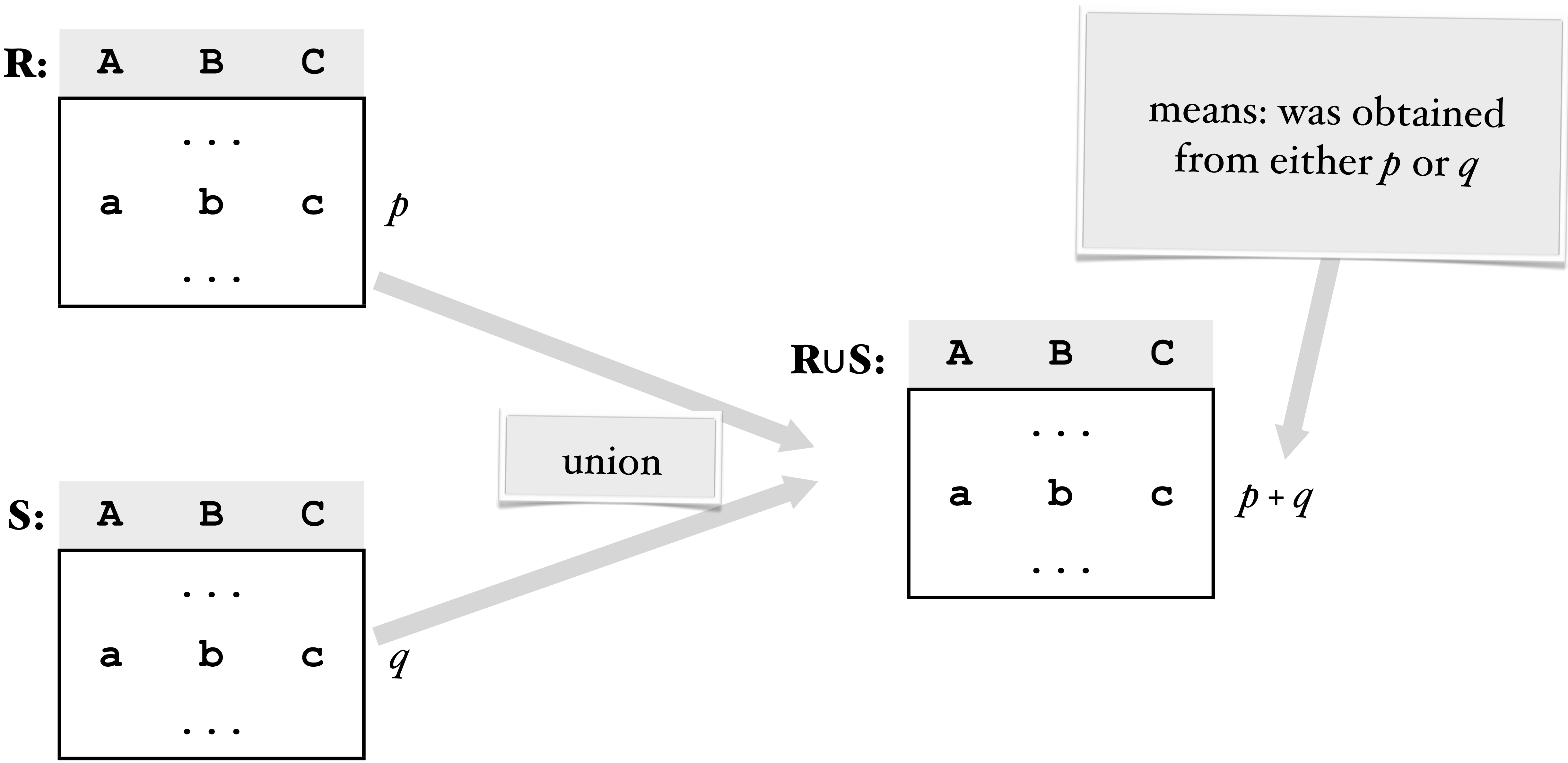
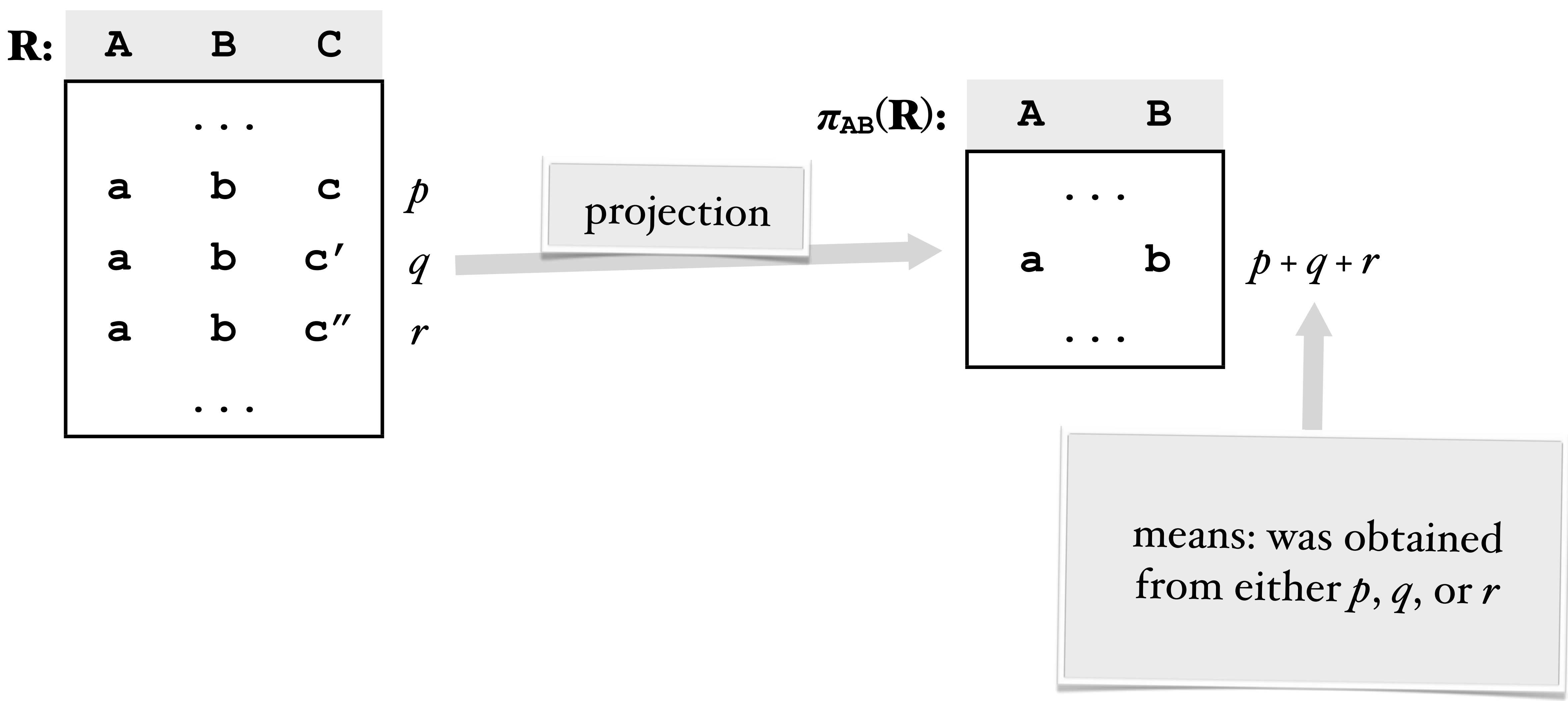


Data Provenance (3)



Data Provenance (4)

$$\mathbf{Q} = \sigma_{C=e}\, \pi_{AC}(\, \pi_{AC}\mathbf{R} \bowtie \pi_{BC}\mathbf{R} \cup \pi_{AB}\mathbf{R} \bowtie \pi_{BC}\mathbf{R})$$

R:

A	B	C	
a	b	c	*p*
d	b	e	*r*
f	g	e	*s*

Q:

A	C	
a	c	$(p^2 + p^2) * 0$
a	e	$(pr) * 1$
d	c	$(rp) * 0$
d	e	$(r^2 + rs + r^2) * 1$
f	e	$(s^2 + rs + s^2) * 1$

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for selection, multiply by *1* or *0*.

Why would this be useful?

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Q:

A	C	
a	c	$(p^2 + p^2) * 0 = 0$
a	e	$(pr) * 1 = 10$
d	c	$(rp) * 0 = 0$
d	e	$(r^2 + rs + r^2) * 1 = 55$
f	e	$(s^2 + rs + s^2) * 1 = 7$

R:

A	B	C	
a	b	c	$p := 2$
d	b	e	$r := 5$
f	g	e	$s := 1$

for **bag semantics**, consider annotations as multiplicities

Why would this be useful?

Q:

A	C	
a	c	$(p^2 + p^2) * 0 = ((b_1 \wedge b_1) \vee (b_1 \wedge b_1)) \wedge \text{false}$
a	e	$(pr) * 1 = (b_1 \wedge b_2) \wedge \text{true}$
d	c	$(rp) * 0 = (b_2 \wedge b_1) \wedge \text{false}$
d	e	$(r^2 + rs + r^2) * 1 = (b_2 \vee (b_2 \wedge b_3) \vee b_2) \wedge \text{true}$
f	e	$(s^2 + rs + s^2) * 1 = (b_3 \vee (b_2 \wedge b_3) \vee b_3) \wedge \text{true}$

R:

A	B	C	
a	b	c	$p := b_1$
d	b	e	$r := b_2$
f	g	e	$s := b_3$

for **incomplete databases**, consider annotations as boolean values,
* as $\wedge$, + as $\vee$, *1* as true, and *0* as false

Data Structure

- Relations are mappings from tuples to annotations in K; we require that $R(t) \neq 0$ for only finitely many tuples t.
- intuitively, "+" means "alternative use" corresponds to union
- "*" means "joint use" and corresponds to join
- "0" and "1" are special annotations

- But what is a query languages for such relations?

Data Structure

- Relations are **mappings from tuples to annotations in *K***; we require that $R(t) \neq 0$ for only finitely many tuples t.
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- But what is $(K,+,*,0,1)$ and how are annotations computed?

Positive Algebra

Positive Algebra

DEFINITION 3.2. *Let* $(K, +, \cdot, 0, 1)$ *be an algebraic structure with two binary operations and two distinguished elements. The operations of the* **positive algebra** *are defined as follows:*

empty relation *For any set of attributes* U*, there is* $\emptyset : U\text{-}\mathsf{Tup} \to K$ *such that* $\emptyset(t) = 0$*.*

union *If* $R_1, R_2 : U\text{-}\mathsf{Tup} \to K$ *then* $R_1 \cup R_2 : U\text{-}\mathsf{Tup} \to K$ *is defined by*

$$(R_1 \cup R_2)(t) \stackrel{\mathtt{def}}{=} R_1(t) + R_2(t)$$

projection *If* $R : U\text{-}\mathsf{Tup} \to K$ *and* $V \subseteq U$ *then* $\pi_V R : V\text{-}\mathsf{Tup} \to K$ *is defined by*

$$(\pi_V R)(t) \stackrel{\mathtt{def}}{=} \sum_{t=t' \text{ on } V \text{ and } R(t') \neq 0} R(t')$$

(here $t = t'$ *on* V *means* t' *is a* U*-tuple whose restriction to* V *is the same as the* V*-tuple* t*; note also that the sum is finite since* R *has finite support)*

Positive Algebra (2)

selection *If* $R : U\text{-}\mathsf{Tup} \to K$ *and the selection predicate* $\mathbf{P}$ *maps each* U*-tuple to either* 0 *or* 1 *then* $\sigma_{\mathbf{P}} R : U\text{-}\mathsf{Tup} \to K$ *is defined by*

$$(\sigma_{\mathbf{P}} R)(t) \stackrel{\text{def}}{=} R(t) \cdot \mathbf{P}(t)$$

Which $\{0, 1\}$*-valued functions are used as selection predicates is left unspecified, except that we assume that* false*—the constantly* 0 *predicate, and* true*—the constantly* 1 *predicate, are always available.*

natural join *If* $R_i : U_i\text{-}\mathsf{Tup} \to K \quad i = 1, 2$ *then* $R_1 \bowtie R_2$ *is the* K*-relation over* $U_1 \cup U_2$ *defined by*

$$(R_1 \bowtie R_2)(t) \stackrel{\text{def}}{=} R_1(t_1) \cdot R_2(t_2)$$

where $t_1 = t$ on U_1 *and* $t_2 = t$ on U_2 *(recall that* t *is a* $U_1 \cup U_2$*-tuple).*

renaming *If* $R : U\text{-}\mathsf{Tup} \to K$ *and* $\beta : U \to U'$ *is a bijection then* $\rho_\beta R$ *is a* K*-relation over* U' *defined by*

$$(\rho_\beta R)(t) \stackrel{\text{def}}{=} R(t \circ \beta)$$

What is K?

PROPOSITION 3.4. *The following $\mathcal{RA}$ identities:*

- *union is associative, commutative and has identity* $\emptyset$*;*
- *join is associative, commutative and distributive over union;*
- *projections and selections commute with each other as well as with unions and joins (when applicable);*
- $\sigma_{\mathsf{false}}(R) = \emptyset$ *and* $\sigma_{\mathsf{true}}(R) = R$.

hold for the positive algebra on K*-relations if and only if* $(K, +, \cdot, 0, 1)$ *is a commutative semiring.*

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Def. A **commutative semiri**

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- * is associative with identity
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- $a * 0 = 0 * a = 0$

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Def. A **commutative semiring** is a structure (K,+,*,0,1) where

- + is commutative, associative, with identity 0
- * is associative with identity 1
- * distributes over +
- $a * 0 = 0 * a = 0$

What is K?

Def. A **commutative semiring** is a structure (K,+,*,0,1) where

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- * is associative with identity 1
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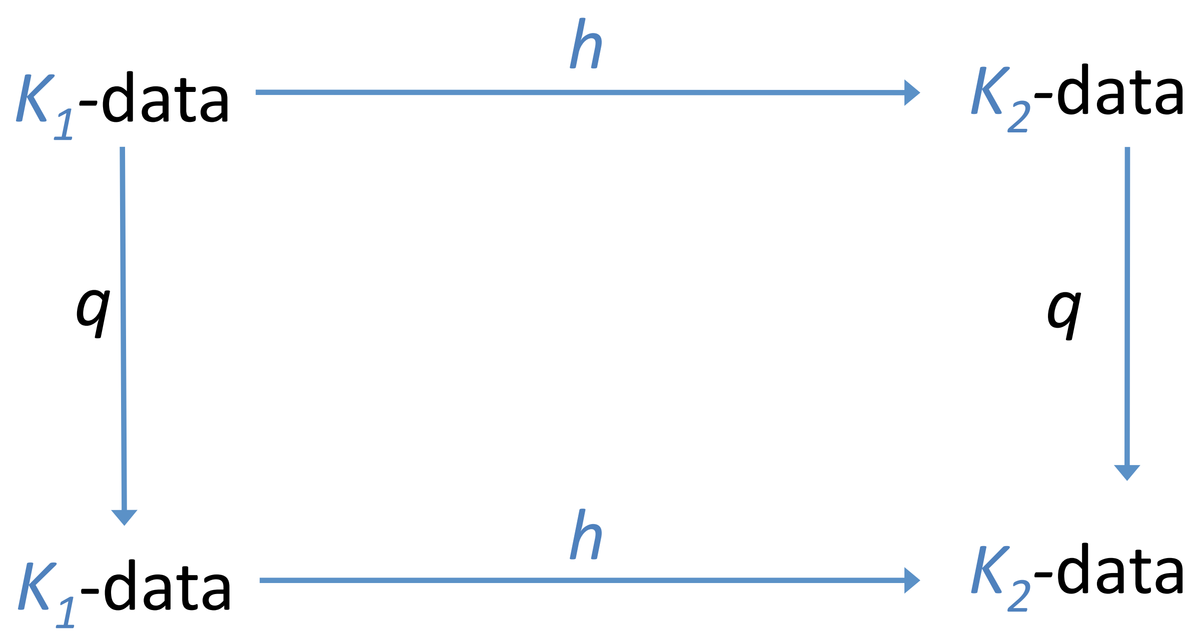
- $a * 0 = 0 * a = 0$

Examples:

- the natural numbers: $(\mathbb{N}, +, *, 0, 1)$
- the booleans: $(\mathbb{B}, \wedge, \vee, \mathit{true}, \mathit{false})$
- subsets of a set: $(\mathscr{P}(\Omega), \cup, \cap, \emptyset, \Omega)$
- the naturals with infinity: $(\mathbb{N}^{\infty}, +, *, 0, 1)$
- polynomials in X: $(\mathbb{N}[X], +, *, 0, 1)$

The fundamental property of RA

For every query q and every homomorphism of commutative semirings $h : K_1 \rightarrow K_2$ the following "commutes":



Recall, semiring homomorphism is mapping h:$K_1 \rightarrow K_2$ such that

$h(1_{K_1}) = 1_{K_2}$ $\quad$ $h(0_{K_1}) = 0_{K_2}$

$h(a +_{K_1} b) = h(a) +_{K_2} h(b)$ $\quad$ $h(a *_{K_1} b) = h(a) *_{K_2} h(b)$

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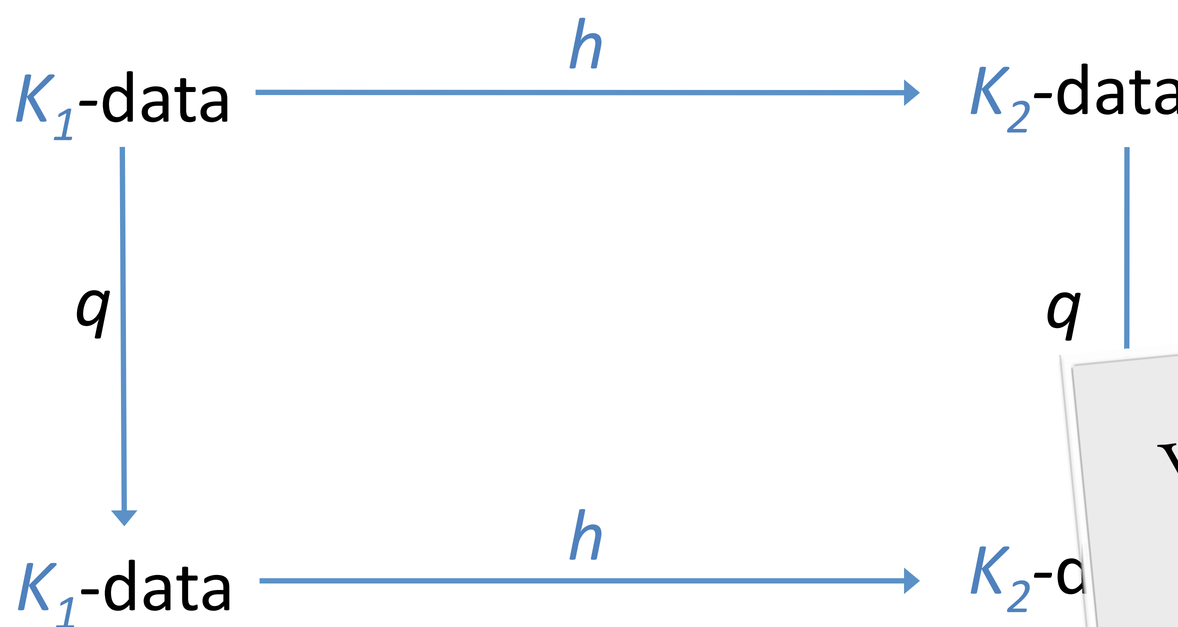

Works only if q in RA⁺.
Does not generalize
e.g. to negation.

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Which semiring do we choose?

Definition 4.1. *Let X be the set of tuple ids of a (usual) database instance I. The* **positive algebra provenance semiring** *for I is the semiring of polynomials with variables (a.k.a. indeterminates) from X and coefficients from $\mathbb{N}$, with the operations defined as usual*[4]*:* $(\mathbb{N}[X], +, \cdot, 0, 1)$.

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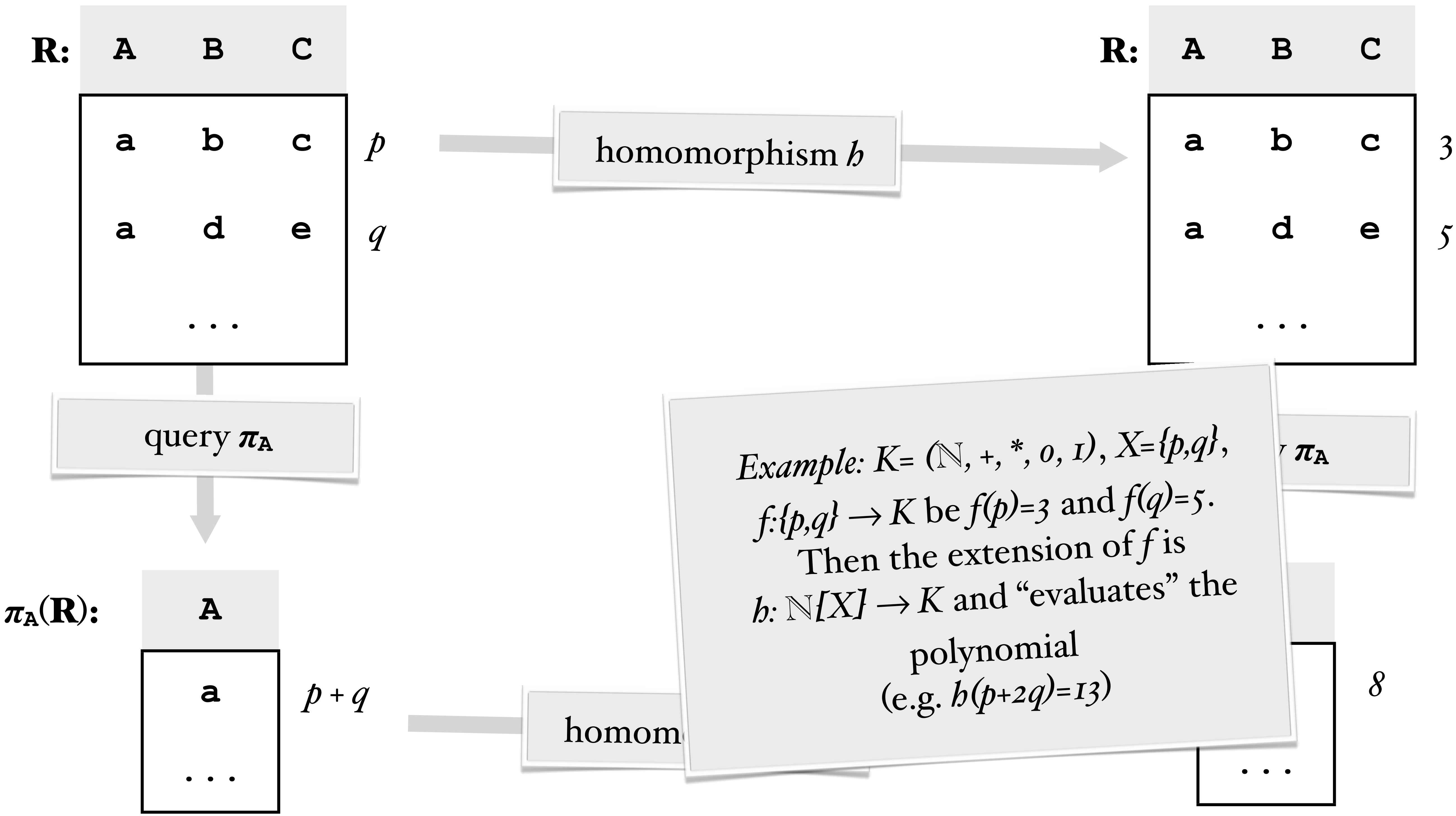
But why?

A nice property of $\mathbb{N}[X]$

If K is a commutative semiring, then any function on tokens, $f : X \to K$ extends uniquely to a homomorphism $h : \mathbb{N}[X] \to K$.

Example: $K = (\mathbb{N}, +, *, 0, 1)$, $X = \{p,q\}$, $f:\{p,q\} \to K$ be $f(p)=3$ and $f(q)=5$. Then the extension of f is $h: \mathbb{N}[X] \to K$ and "evaluates" the polynomial (e.g. $h(p+2q)=13$)

Nice + Fundamental



Nice + Fundamental

R:

A	B	C	
a	b	c	p
a	d	e	q
	. . .		

homomorphism h →

R:

A	B	C	
a	b	c	*3*
a	d	e	*5*
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query π_A ↓

π_A(**R**):

A	
a	$p + q$
. . .	

homomorphism h →

π_A(**R**):

A	
a	*8*
. . .	

Free the semiring!

"Nice" implies: For every commutative semiring K, and every K-relation **R**, there is abstractly tagged N[X]-relation $\overline{\mathbf{R}}$ and a homomorphism $Eval_v$ from $\overline{\mathbf{R}}$ to **R**.

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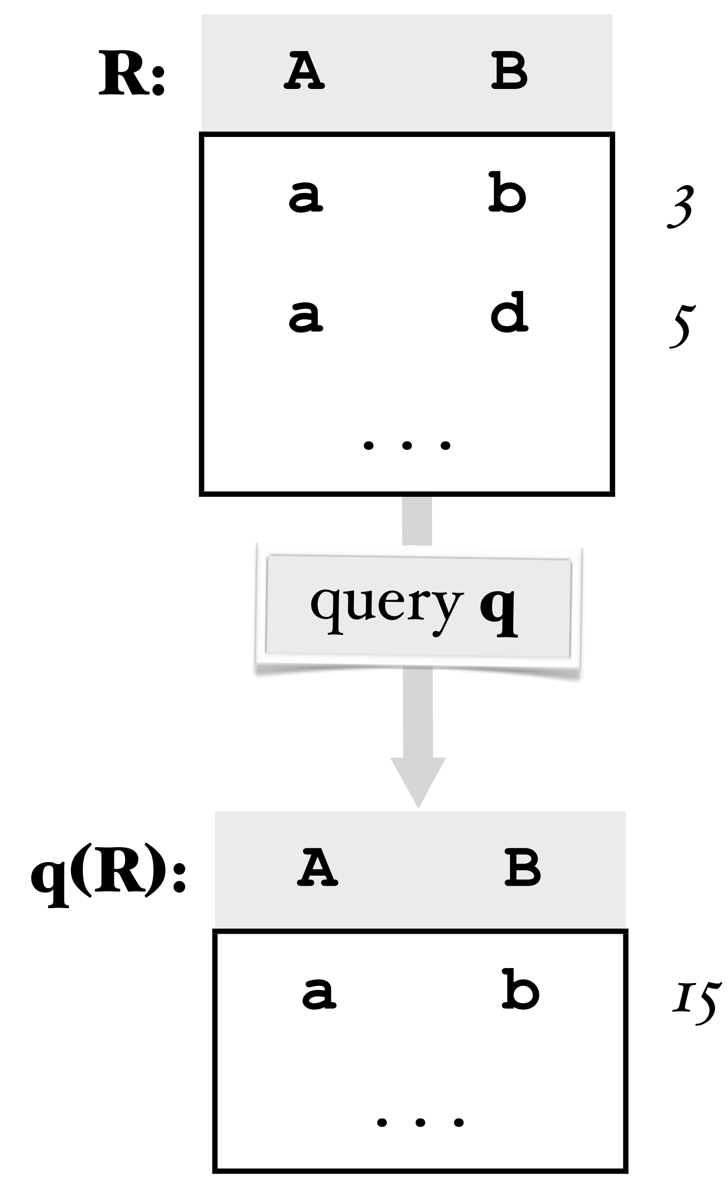

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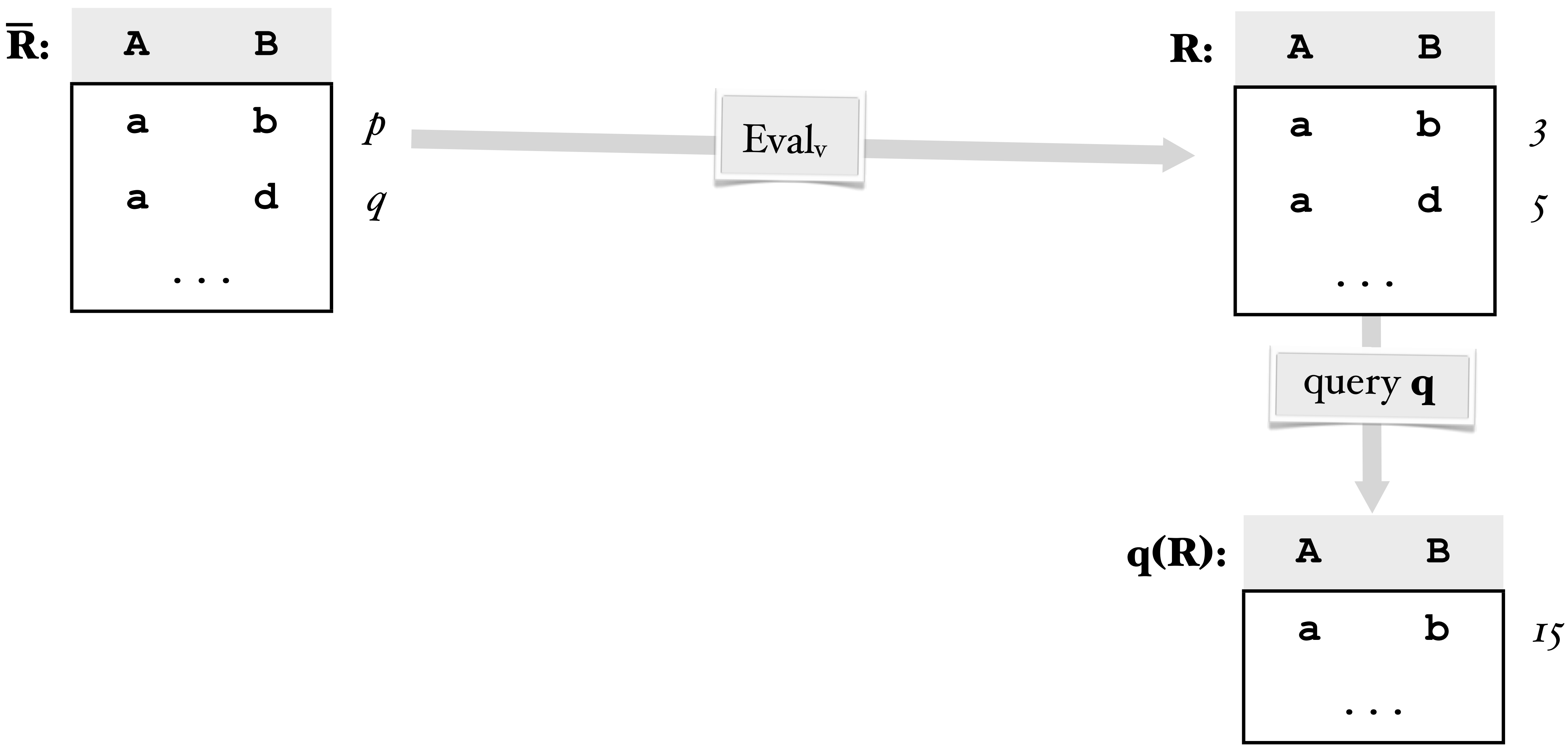

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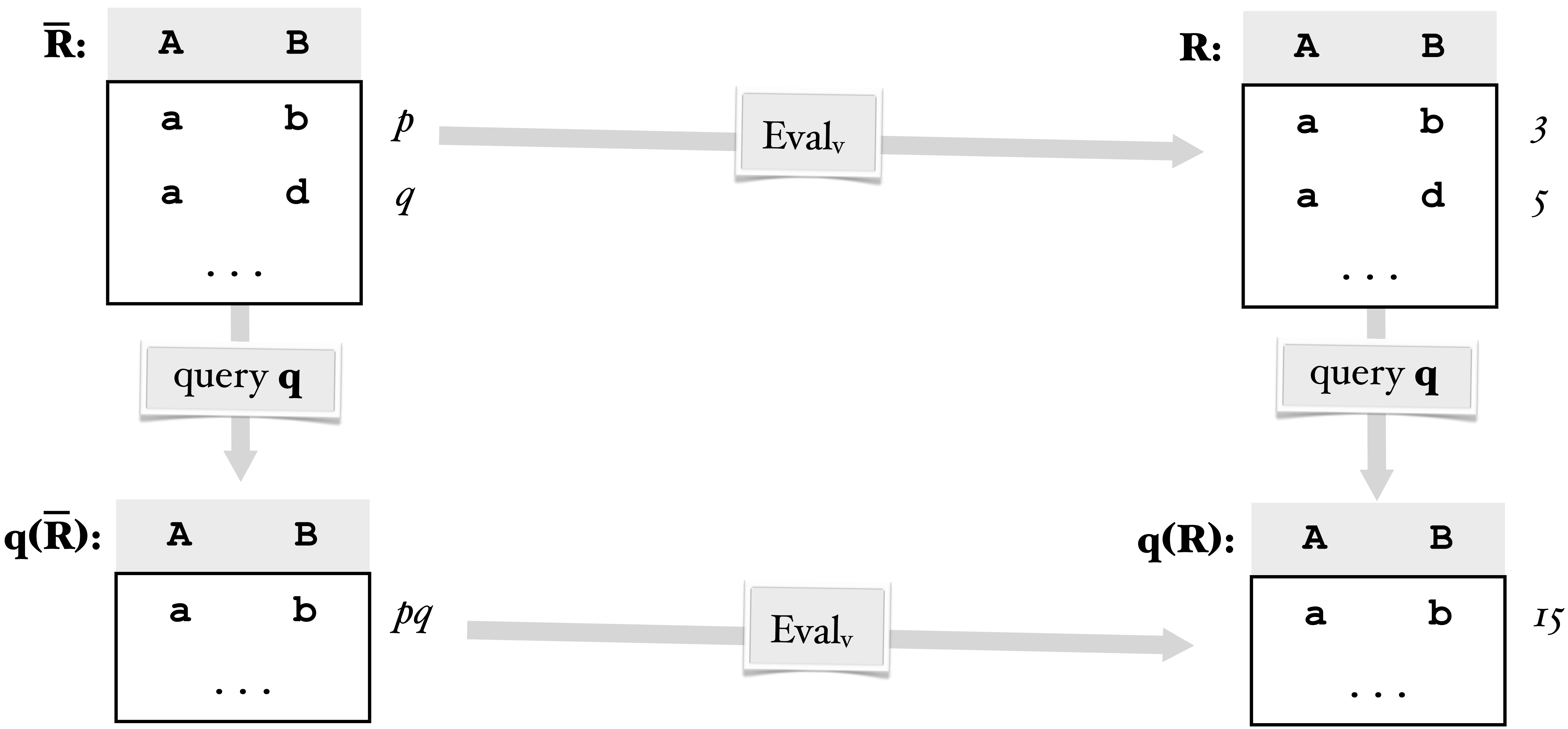

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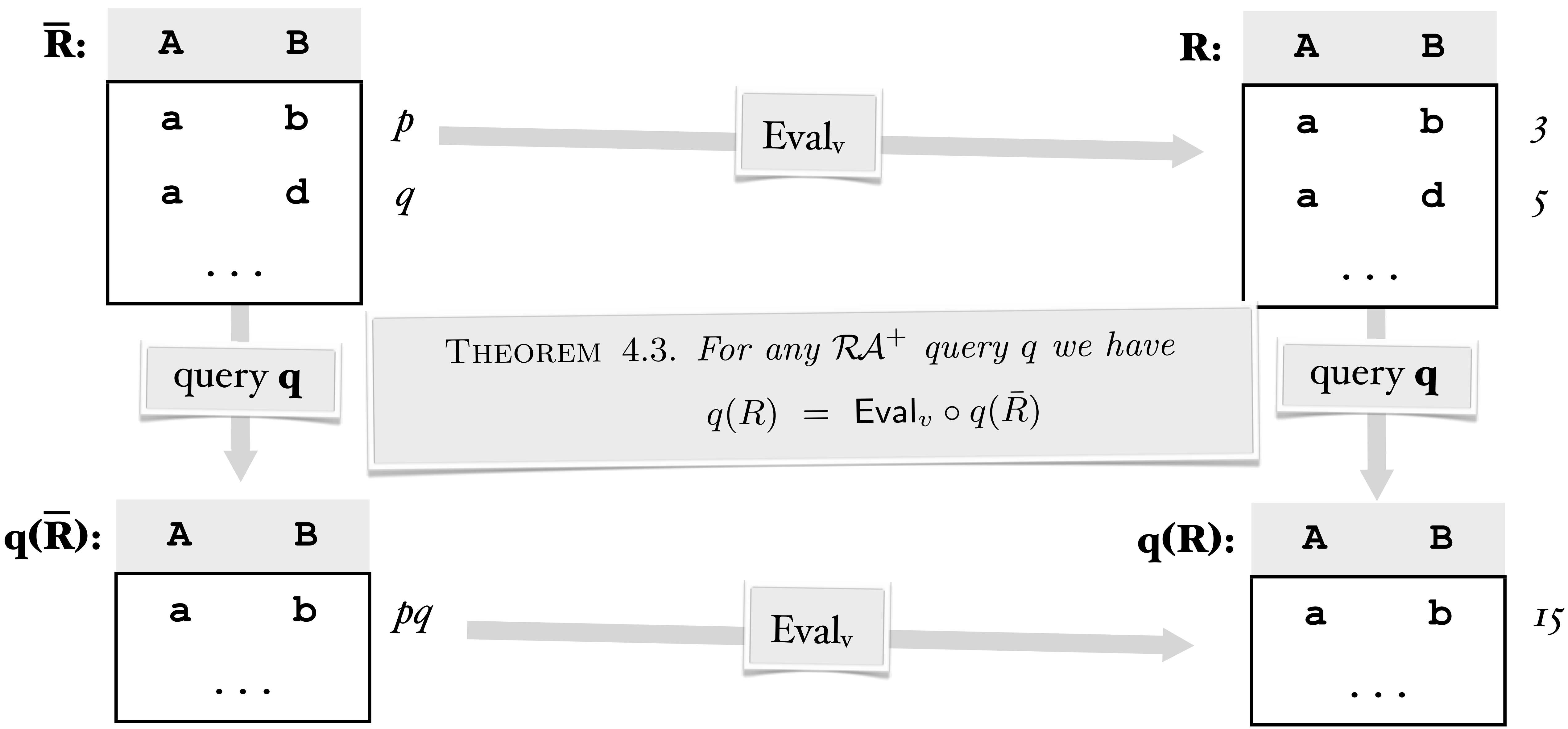
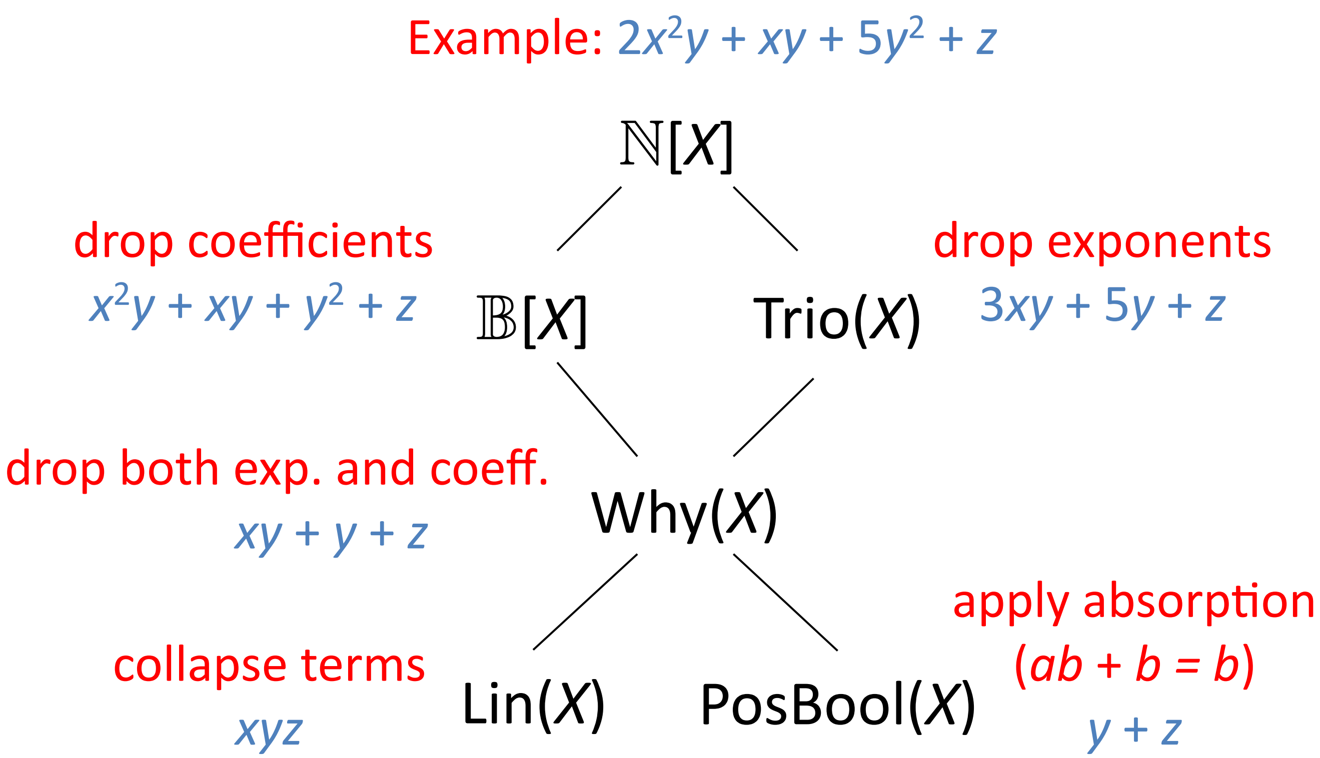


Instantiation of Positive Algebra

$(\mathbb{B}, \wedge, \vee, \text{true}, \text{false})$	Set semantics
$(\mathbb{N}, +, *, 0, 1)$	Bag semantics
$(\mathscr{P}(\Omega), \cup, \cap, \emptyset, \Omega)$	Probabilistic events
$(\text{BoolExp}(P), \vee, \wedge, \text{true}, \text{false})$	Conditional tables
$(A, \min, \max, 0, P)$ where $A = \mathbb{P} < \mathbb{C} < \mathbb{S} < \mathbb{T} < \mathbb{0}$	Access control levels

More nice…



A path downward from K_1 to K_2 indicates that there exists an **onto (surjective) semiring homomorphism** $h : K_1 \rightarrow K_2$

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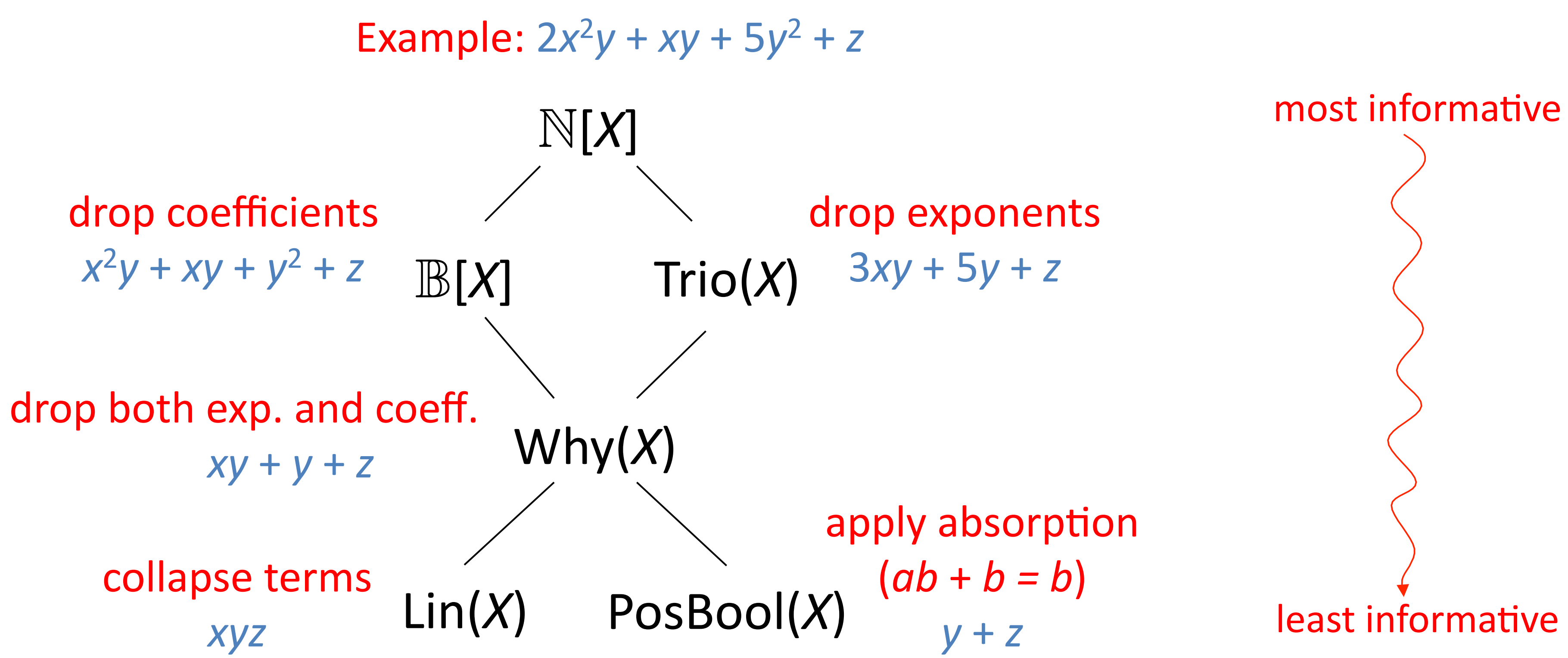


A path downward from K_1 to K_2 indicates that there exists an **onto (surjective) semiring homomorphism** $h : K_1 \rightarrow K_2$

Datalog

Syntax and Semantics

$$Q(x, y) \text{ :- } R(x, z), R(z, y)$$

edb

R:

a	a
a	b
b	b

Q:

a	a
a	b
b	b

idb

Syntax and Semantics

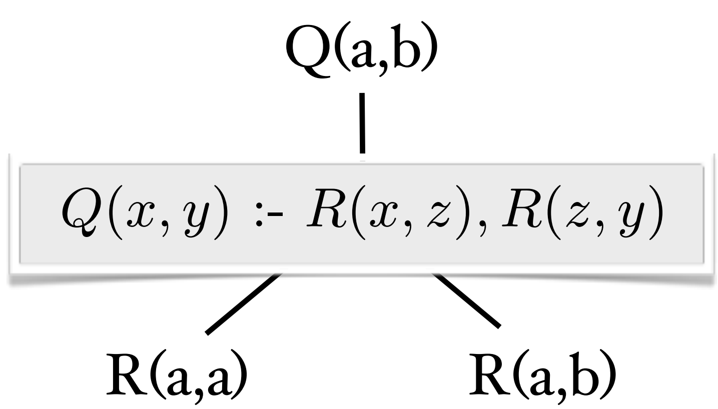
$$Q(x, y) \text{ :- } R(x, z), R(z, y)$$

edb → R:

a	a
a	b
b	b

Q: ← idb

a	a
a	b
b	b



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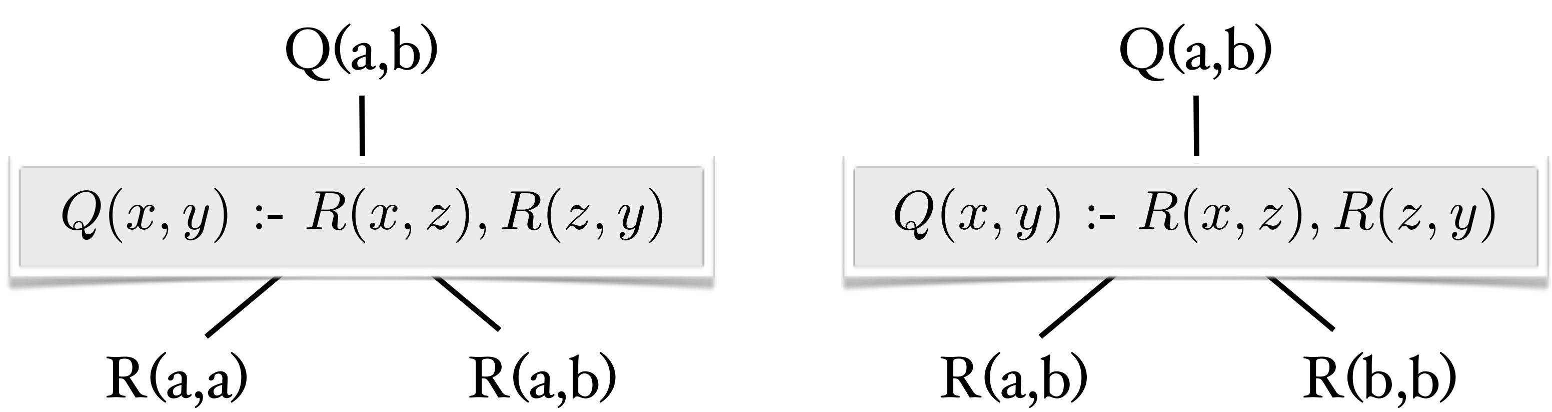

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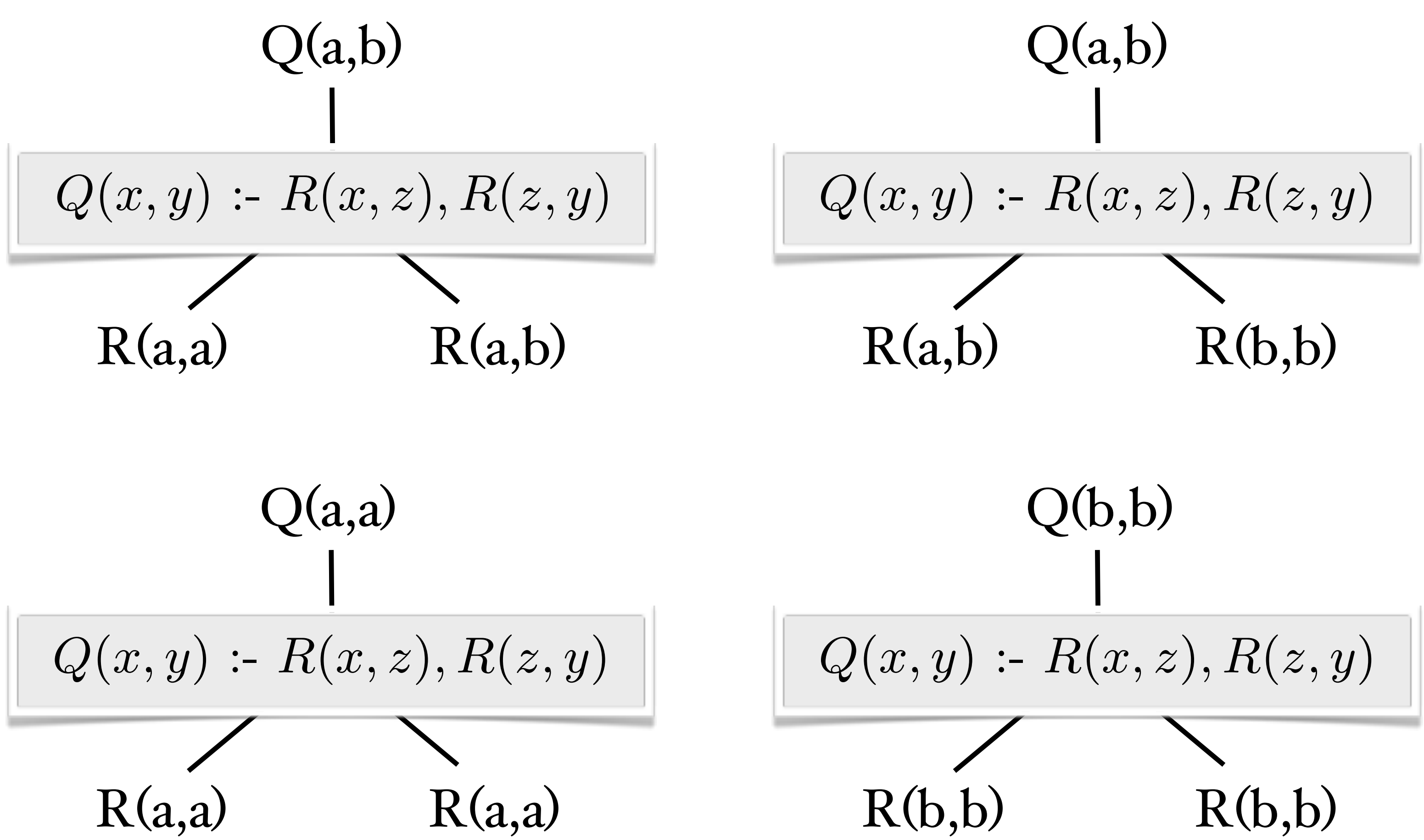


Datalog with Bag Semantics

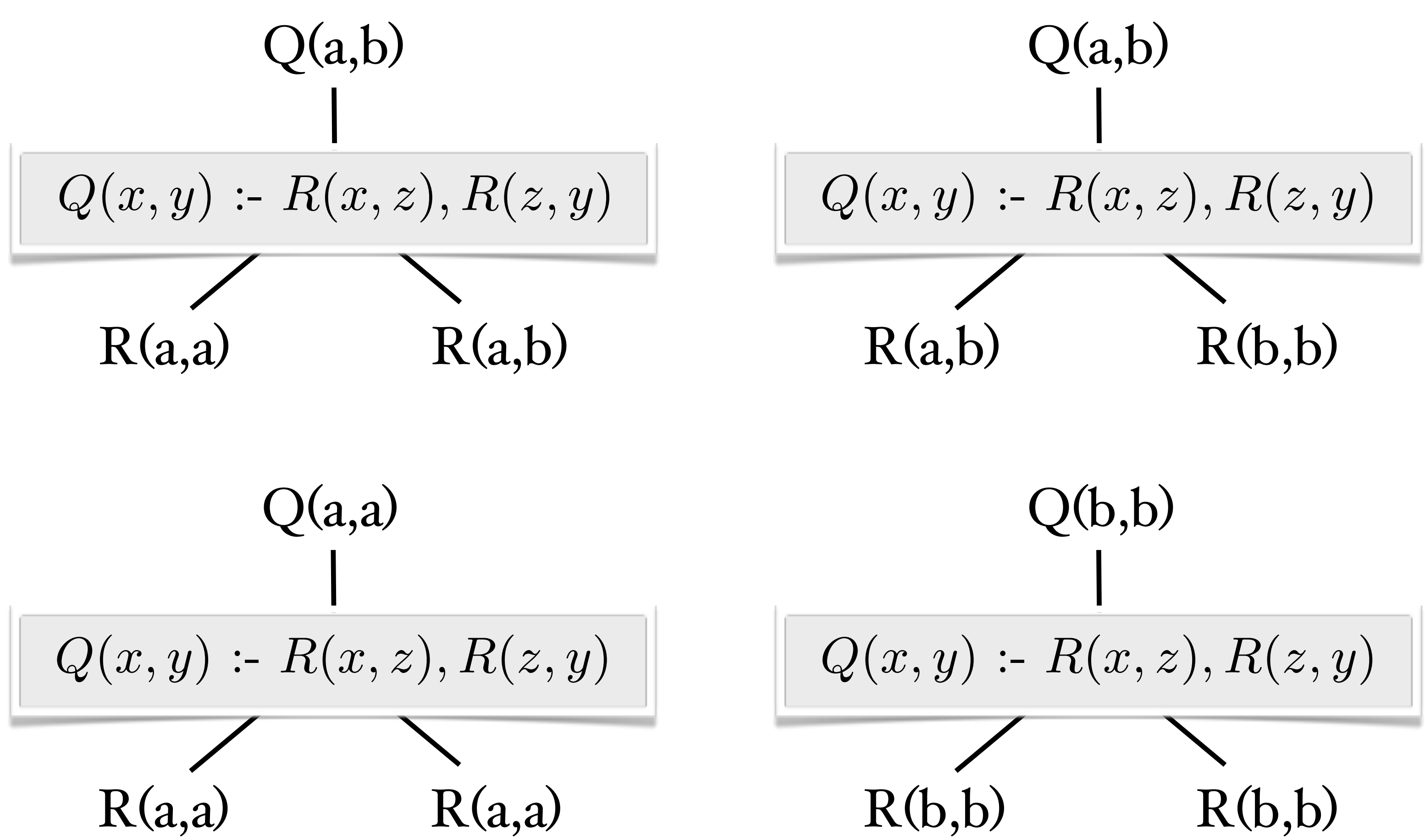
$$Q(x, y) \text{ :- } R(x, z), R(z, y)$$

R:

a	a
a	b
b	b

Q:

a	a
a	b
b	b



Datalog with Bag Semantics

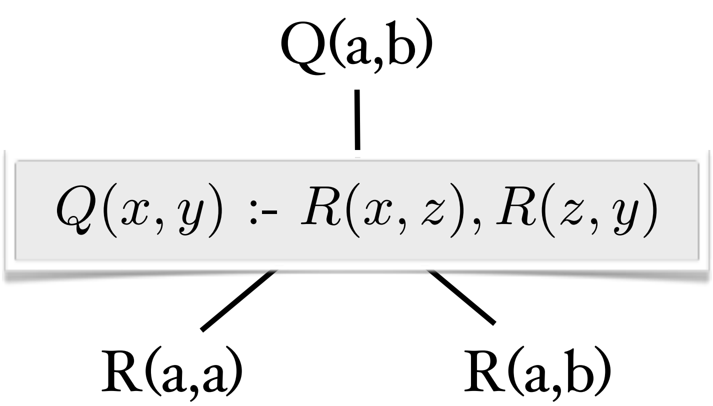
$$Q(x,y) \text{ :- } R(x,z), R(z,y)$$

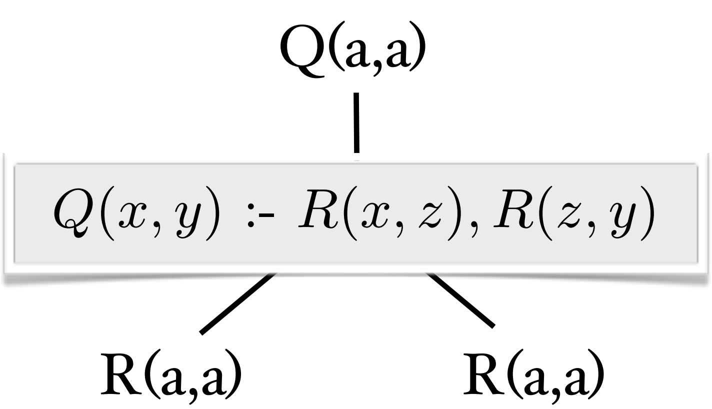
R:

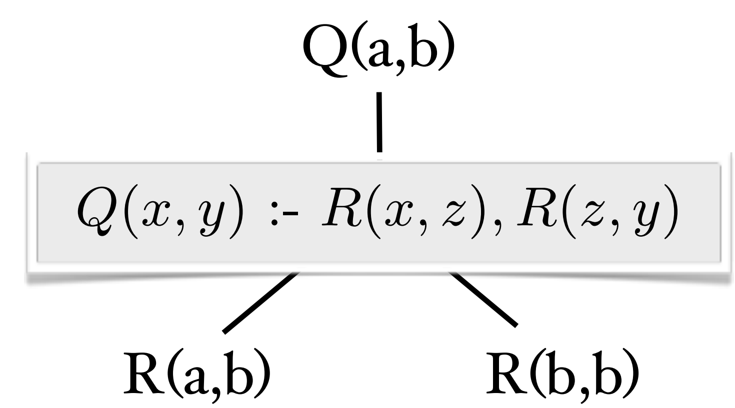
$a\ a$	2
$a\ b$	3
$b\ b$	4

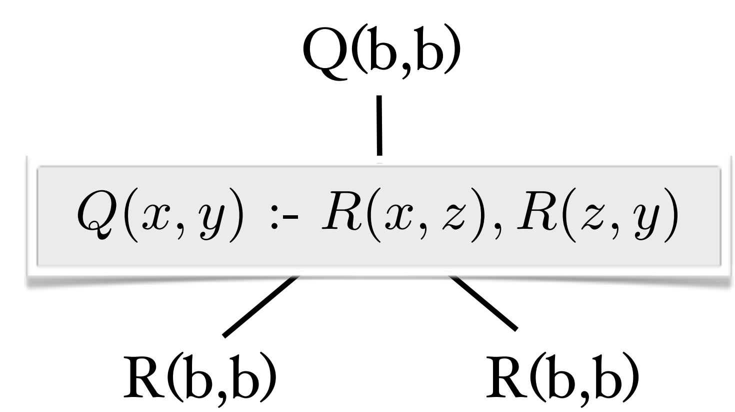
Q:

$a\ a$
$a\ b$
$b\ b$









Datalog with Bag Semantics

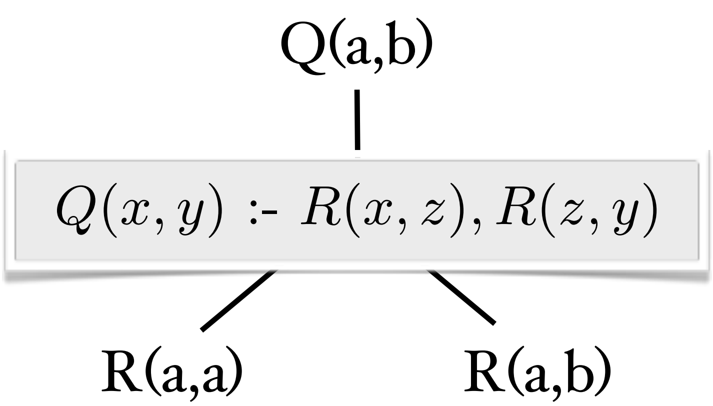
$$Q(x,y) \text{ :- } R(x,z), R(z,y)$$

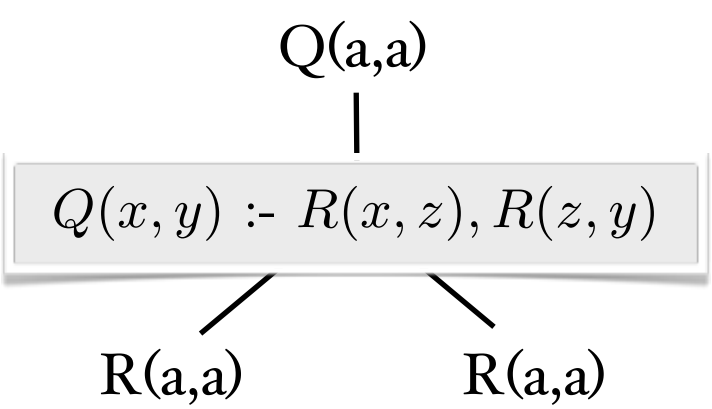
R:

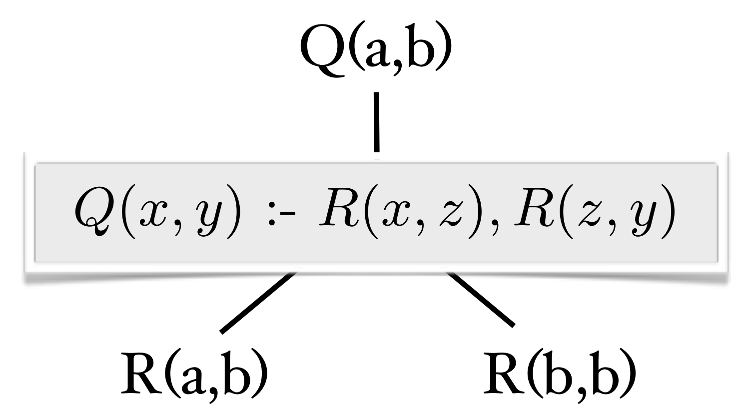
$a\ a$	2
$a\ b$	3
$b\ b$	4

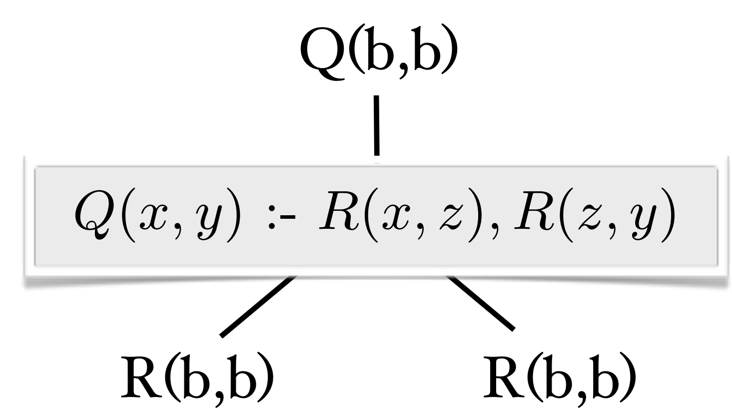
Q:

$a\ a$	$2 \cdot 2 = 4$
$a\ b$	$2 \cdot 3 + 3 \cdot 4 = 18$
$b\ b$	$4 \cdot 4 = 16$









What annotations do we need?

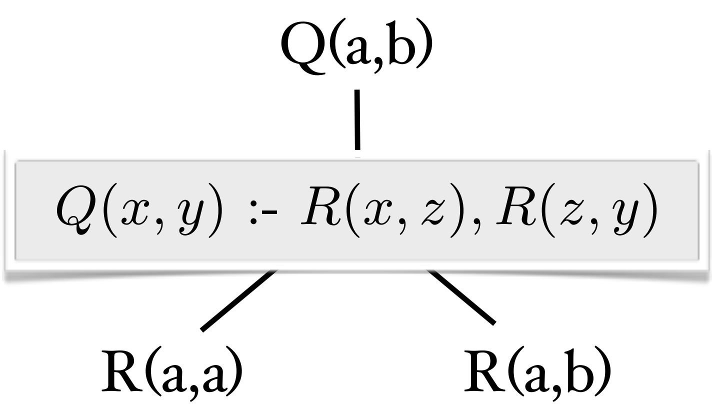
$$Q(x, y) \text{ :- } R(x, z), R(z, y)$$

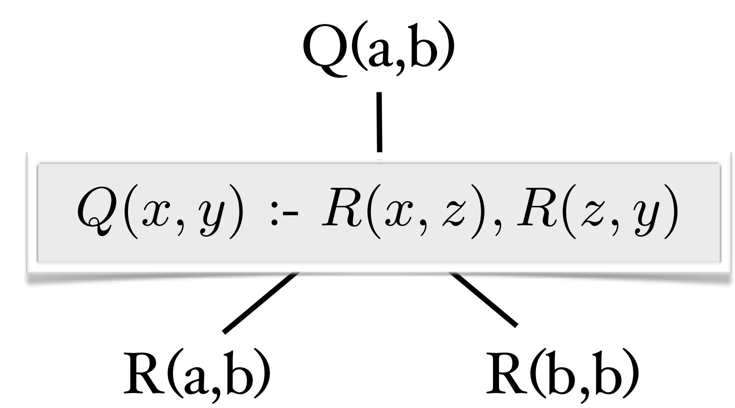
R:

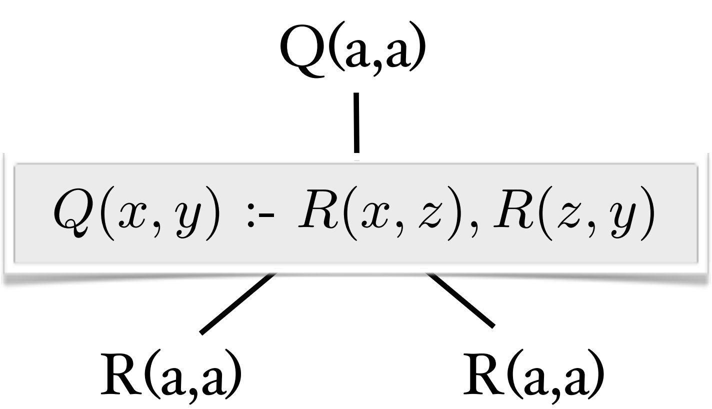
$a\ a$	2
$a\ b$	3
$b\ b$	4

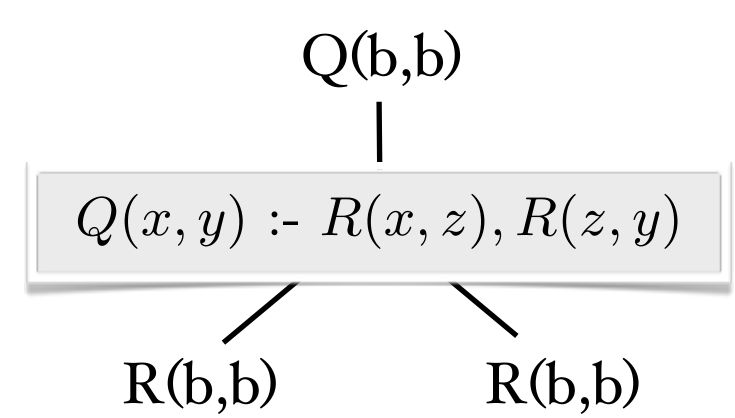
Q:

$a\ a$	$2 \cdot 2 = 4$
$a\ b$	$2 \cdot 3 + 3 \cdot 4 = 18$
$b\ b$	$4 \cdot 4 = 16$









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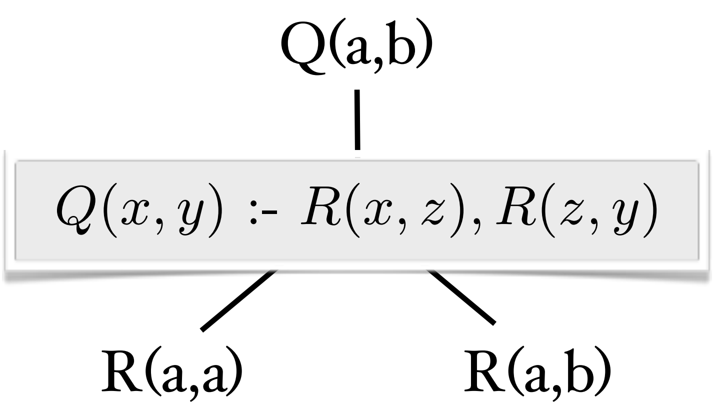

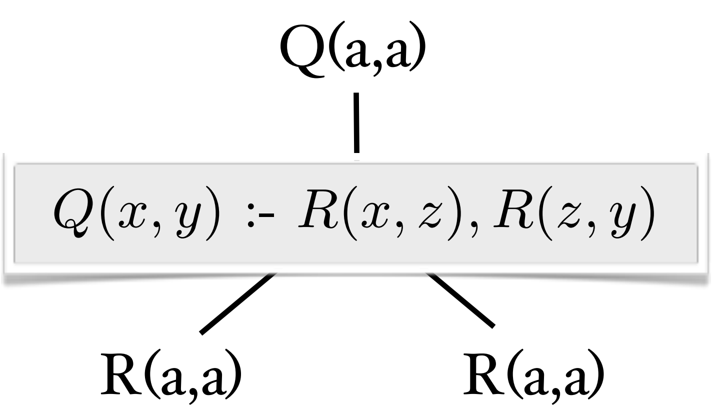

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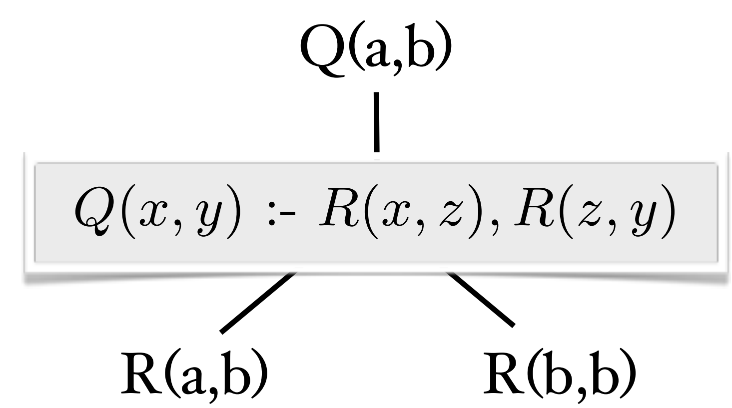

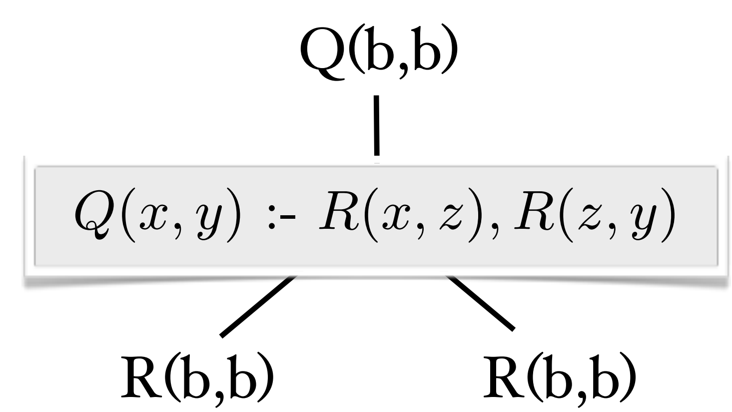

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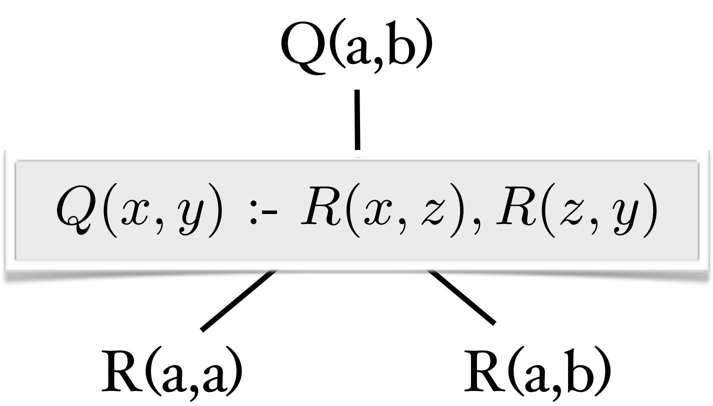

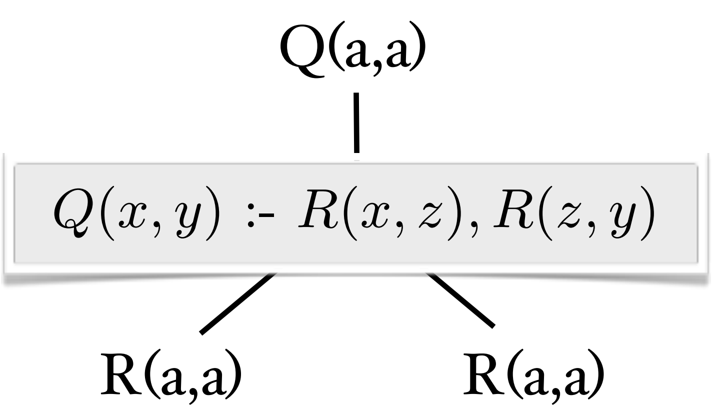

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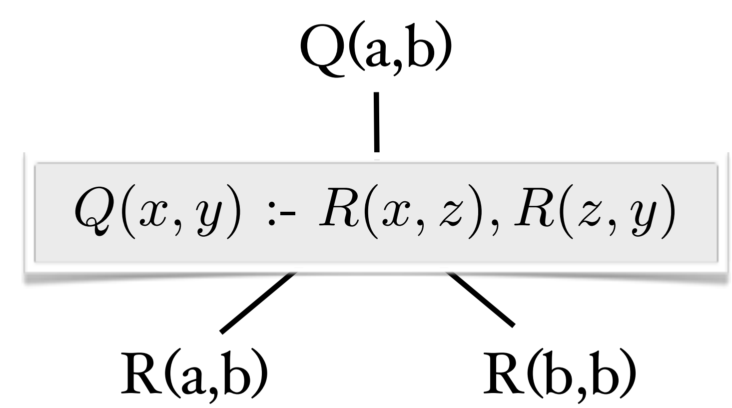
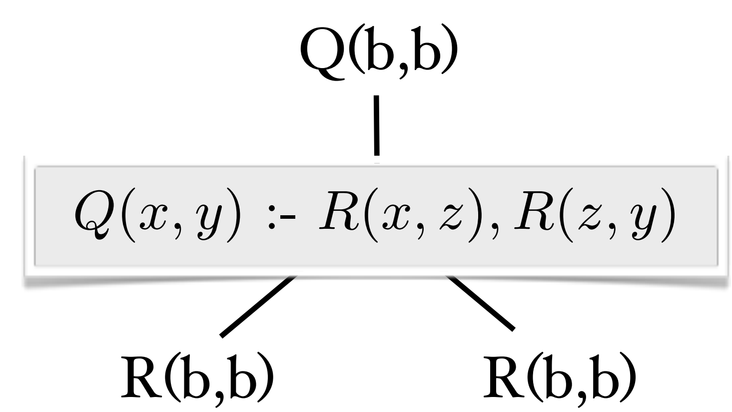
Problem: A tuple may have infinitely many derivation trees. Hence we need to work in semirings in which infinite sums are defined.





ω-continuos semirings

Def. ***(Natural preorder)*** $x \leq y$ iff there is z such that $x+z=y$.

Def. ***(Naturally ordered semiring)*** if the natural pre-order is an order.

Def. ***(ω-complete)*** when $x_1 \leq x_2 \leq x_2 \leq \ldots$ have suprema.

In naturally ordered semirings, we can make sense of infinite sums:

$$\sum_{n \in \mathbb{N}} a_n \stackrel{\text{def}}{=} \sup_{m \in \mathbb{N}} (\sum_{i=0}^{m} a_i)$$

Def. ***(ω-continous)*** when * and + preserve suprema.
(e.g. $\sup(a_i + b_i) = \sup(a_i) + b_i$).

Lemma. Over ω-continuos semirings, functions defined by polynomials have least fixed points.

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Preorder: reflexive and transitive.
Not necessarily anti-symmetric
($x \leq y$ and $y \leq x$ implies $x=y$)

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E.g. $\mathbb{Z}$ is not naturally ordered because $-5 \leq 5$, $5 \leq -5$, but $-5 \neq 5$

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Semantics of annotated Datalog

Definition 5.1. *Let* $(K, +, \cdot, 0, 1)$ *be a commutative* ω*-continuous semiring. To keep notation simple let* q *be a datalog query with one argument (it is easy to generalize to multiple arguments). For any* K*-relation* R *define*

$$q(R)(t) = \sum_{\tau \text{ yields } t} \Big(\prod_{t' \in leaves(\tau)} R(t') \Big)$$

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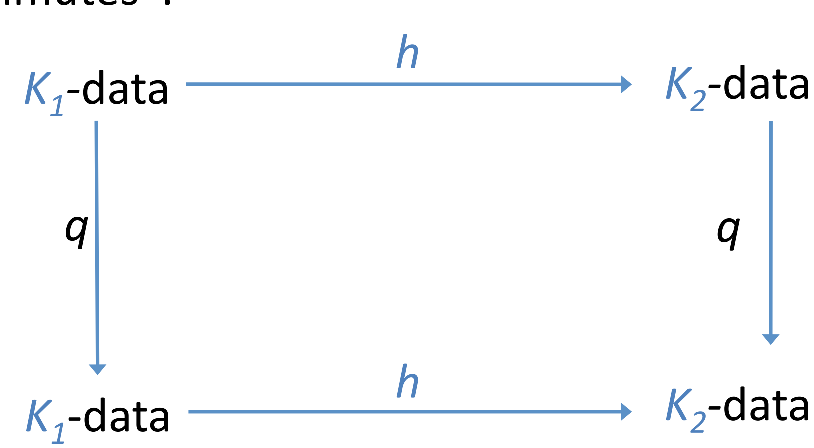

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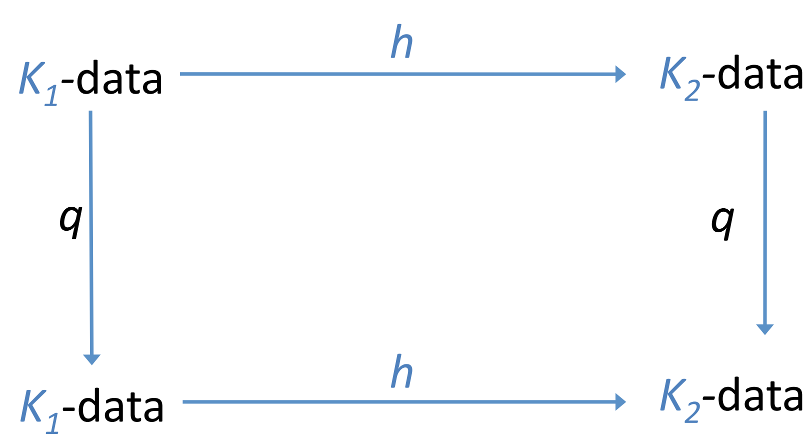

The Datalog provenance Semiring

Problem: there can be infinitely many derivation trees for one tuple

☞ infinite sums in annotations

In particular two kinds of infinite summations

- infinitely many copies of the same monomial → coefficients in $\mathbb{N}^{\infty} = \mathbb{N} \cup \{\infty\}$
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Formal power series:
basically polynomials with
infinite summation

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Definition 6.1. *Let X be the set of tuple ids of a database instance I. The* **datalog provenance semiring** *for I is the commutative ω-continuous semiring of formal power series $\mathbb{N}^\infty[[X]]$.*

Fixed Point Semantics

$Q(x,y)$:- $R(x,y)$

$Q(x,y)$:- $Q(x,z)$, $Q(z,y)$

(a)

R:

$a\ b$	2
$a\ c$	3
$c\ b$	2
$b\ d$	1
$d\ d$	1

(b)

Q(R):

$a\ b$	8
$a\ c$	3
$c\ b$	2
$b\ d$	∞
$d\ d$	∞
$a\ d$	∞

(c)

$\bar{R}$:

$a\ b$	m
$a\ c$	n
$c\ b$	p
$b\ d$	r
$d\ d$	s

(d)

Q($\bar{R}$):

$a\ b$	$\mathbf{x}$
$a\ c$	$\mathbf{y}$
$c\ b$	$\mathbf{z}$
$b\ d$	$\mathbf{u}$
$d\ d$	$\mathbf{v}$
$a\ d$	$\mathbf{w}$

(e)

$$\begin{aligned}
\mathbf{x} &= m + \mathbf{yz} \\
\mathbf{y} &= n \\
\mathbf{z} &= p \\
\mathbf{u} &= r + \mathbf{uv} \\
\mathbf{v} &= s + \mathbf{v}^2 \\
\mathbf{w} &= \mathbf{xu} + \mathbf{wv}
\end{aligned}$$

(f)

- Transform immediate consequence operator of Q into a union of conjunctive queries; here R ∪ (Q $\bowtie_{2=1}$ Q)
- Apply this RA query to $\bar{R}$ and $\bar{Q}$.
- Equate!

This leads to system of equations of polynomials in

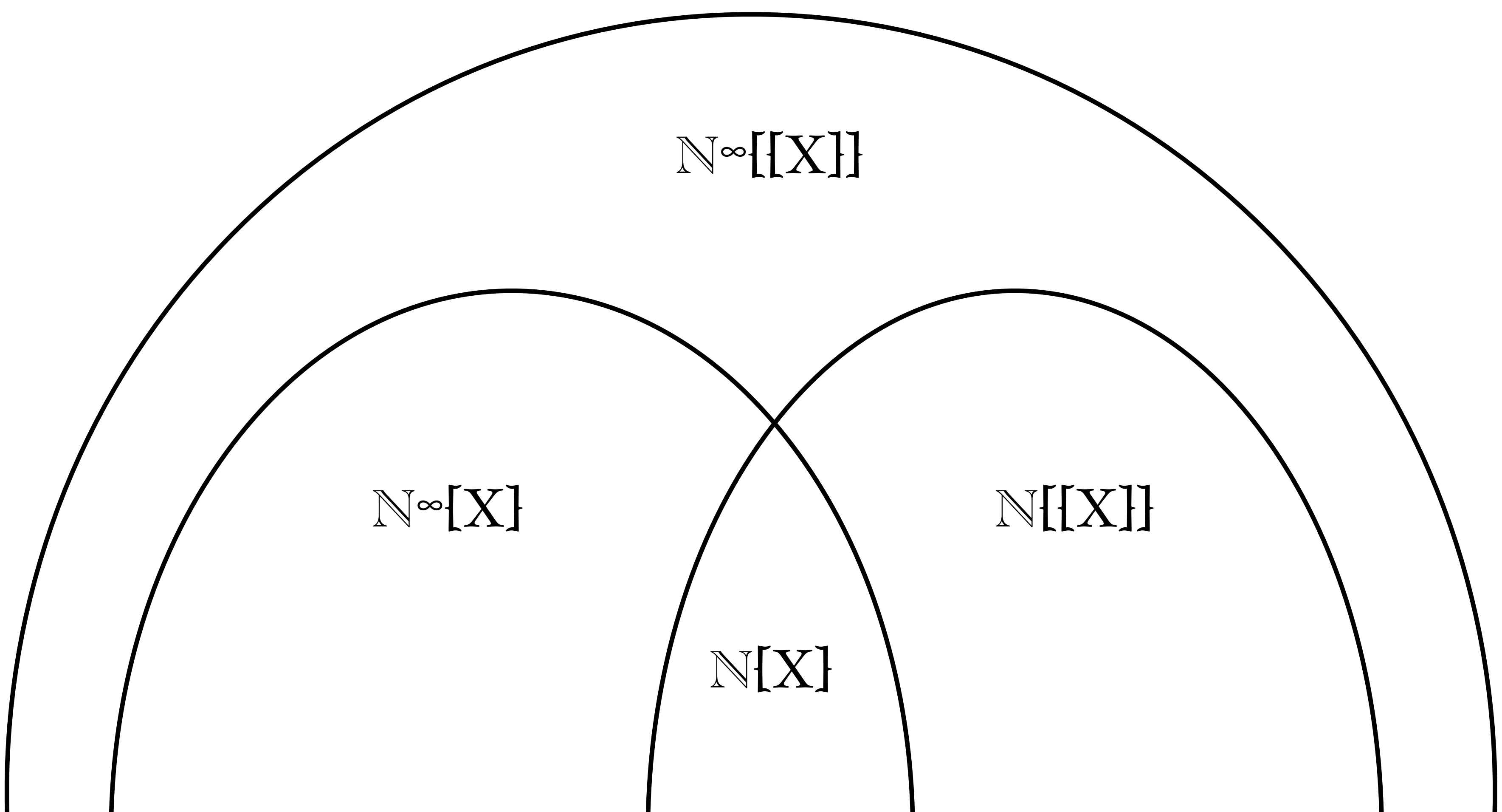
$$\mathbb{N}^\infty[[m,n,p,r,s]][\mathbf{x},\mathbf{y},\mathbf{z},\mathbf{u},\mathbf{v},\mathbf{w}]$$

As $\mathbb{N}^\infty[[m,n,p,r,s]]$ is omega continuos, these equations have least fixed points that can be computed.

Decidability

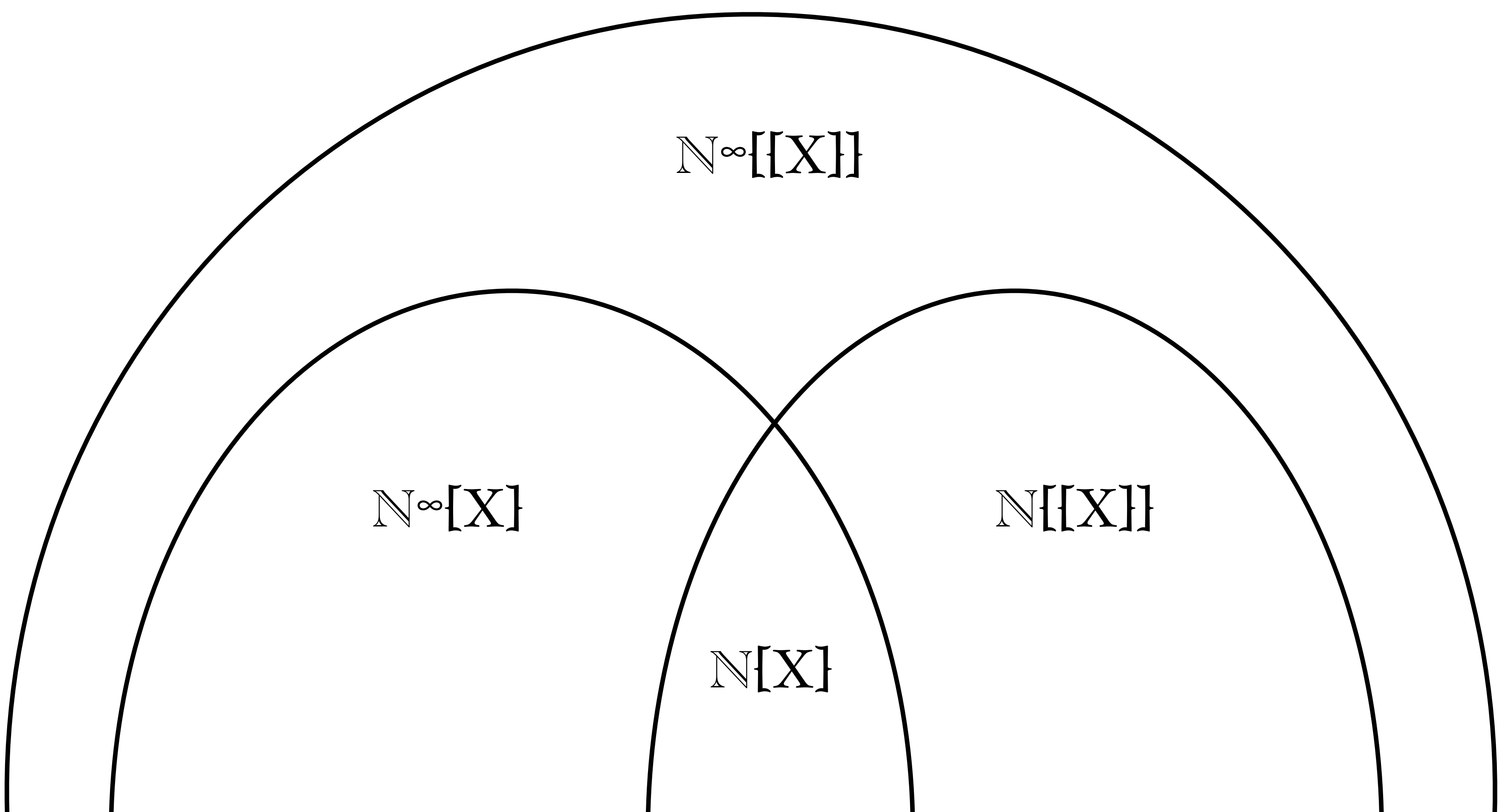
A tuple can have an annotations in any of the classes below.

It is decidable in which class the annotation of a tuple is.



Decidability: Case $\mathbb{N}[X]$

Claim: Let Q be a Datalog program, D a database, and R an relation in the intensional schema of P.
R(t) $\notin \mathbb{N}[X]$ iff t has a derivation tree T w.r.t. Q and D of height less than (# of atoms +2) that has a path with two occurrences of the same atom *a*.



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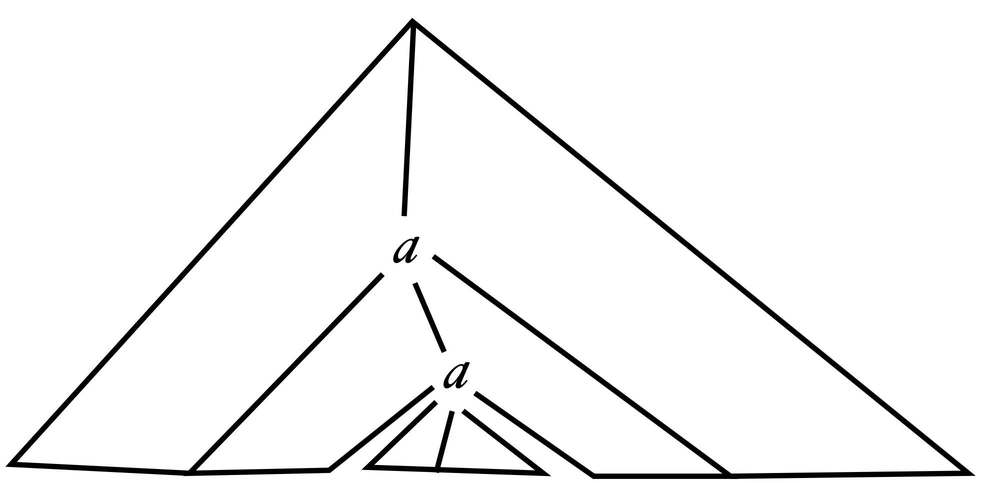
Proof: "⇐" Assume such a tree exists:



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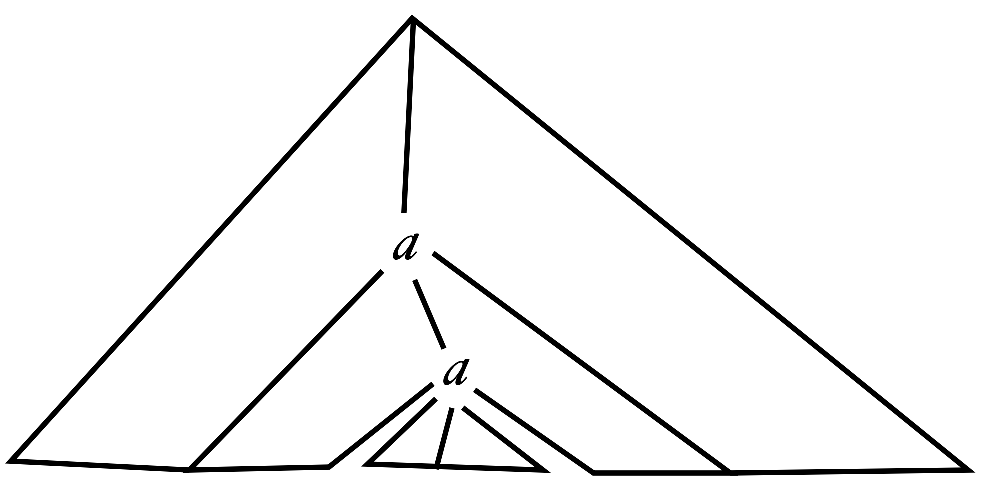
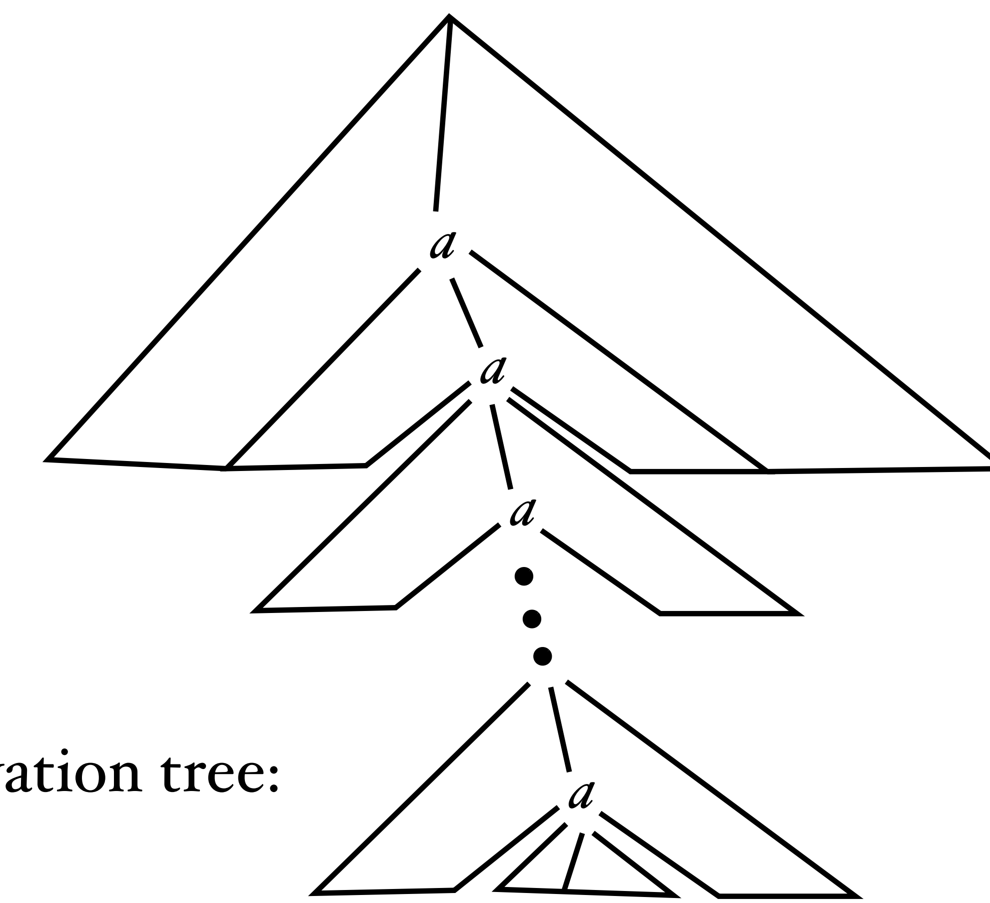


Then this is also a derivation tree:



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greater than
or equal

Thus there are only finitely many derivation trees.

Also decidable:

- given $t \in q(I)$, and a monomial μ, the coefficient of μ in the power series that is the provenance of t is computable (including when it is ∞).

- testing whether all coefficients are $\neq \infty$.

Not decidable:

- testing whether all coefficients are 1.

Conclusion

- A versatile framework for provenance computation.
- Specializes to many known systems for provenance.
- In a sense most general within frameworks that use Semirings.
- Provides semantics for positive datalog under rich semantics (e.g. bag semantics).

Thank You!