Now suppose $\mathbf{q}^*$ achieves capacity so that $C = I(\mathbf{q}^*)$. Consider

$$\sum_x q^*(x) \ln \frac{q^{(k+1)}(x)}{q^{(k)}(x)} = \sum_x q^*(x) \ln \frac{r^{(k+1)}(x)}{q^{(k)}(x)\left(\sum_{x'} r^{(k+1)}(x')\right)}$$

$$= -C(k+1) + \sum_x q^*(x) \ln \frac{1}{q^{(k)}(x)} + \sum_x q^*(x) \ln r^{(k+1)}(x)$$

$$= -C(k+1) + \sum_x \sum_y p(y|x)q^*(x) \ln \frac{1}{q^{(k)}(x)} + \sum_x \sum_y p(y|x)q^*(x) \ln Q^{(k)}(x|y)$$

$$= -C(k+1) + \sum_x \sum_y p(y|x)q^*(x) \ln \frac{Q^{(k)}(x|y)}{q^{(k)}(x)}$$

$$= -C(k+1) + \sum_x \sum_y p(y|x)q^*(x) \ln \frac{p(y|x)p^*(y)}{p^{(k)}(y)p^*(y)}$$

$$= -C(k+1) + C + \sum_y p^*(y) \ln \frac{p^*(y)}{p^{(k)}(y)} \tag{3C.27}$$

where

$$p^*(y) \equiv \sum_x p(y|x)q^*(x)$$

and

$$p^{(k)}(y) \equiv \sum_x p(y|x)q^{(k)}(x)$$

Again using inequality (1.1.8), we have

$$\sum_y p^*(y) \ln \frac{p^*(y)}{p^{(k)}(y)} \geq 0 \tag{3C.28}$$

and, from (3C.27)

$$C - C(k+1) \leq \sum_x q^*(x) \ln \frac{q^{(k+1)}(x)}{q^{(k)}(x)} \tag{3C.29}$$

Noting that $C \geq C(k+1)$ and summing (3C.29) over k from 0 to $N-1$, we have

$$\sum_{k=0}^{N-1} |C - C(k+1)| \leq \sum_x q^*(x) \ln \frac{q^{(N)}(x)}{q^{(0)}(x)} \tag{3C.30}$$

Again from inequality (1.1.8), we have

$$\sum_x q^*(x) \ln q^{(N)}(x) \leq \sum_x q^*(x) \ln q^*(x) \tag{3C.31}$$